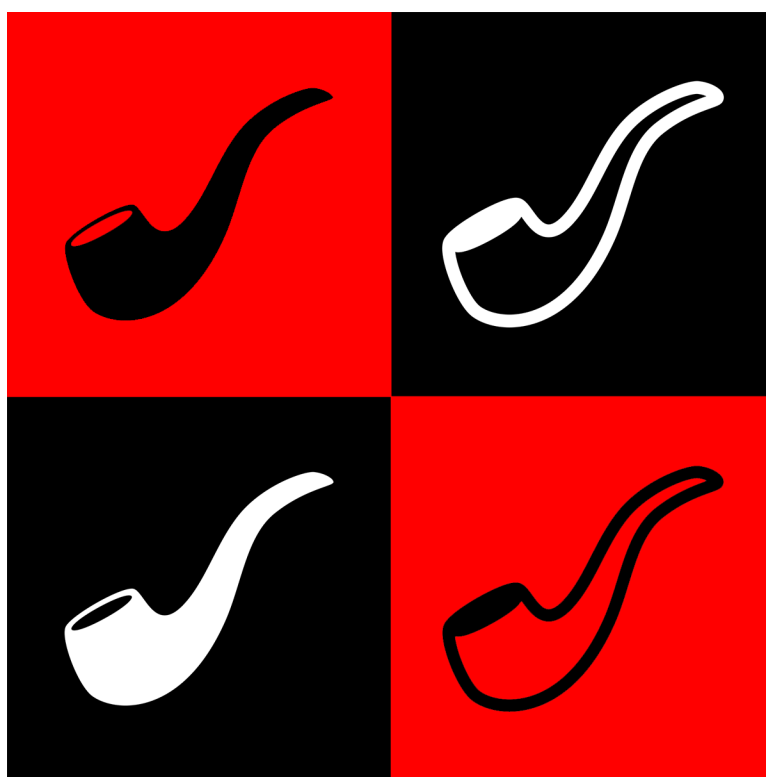# Sherlock Holmes Visits the Chameleon Chess Club

Further Adventures in Retrograde Chess Analysis
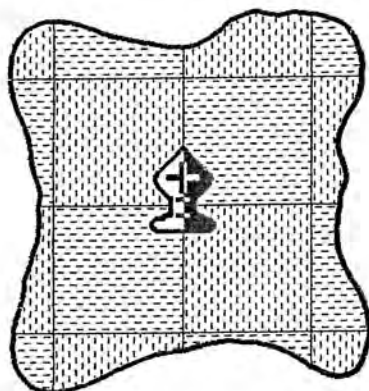
by

Bill Murden

with Forward by

Raymond Smullyan

Sherlock Holmes Visits the

Chameleon **Chess Club**

Further Adventures in
Retrograde Chess Analysis

By *Bill Murden*

Forward by Raymond Smullyan

To Fred Stahl, in appreciation of
his prodding me into computer literacy

Bill Murden

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FORWARD

This is a marvelous book of chess problems in what is known as retrograde analysis in which the solutions involve deducing what must have taken place in the past of the game. These intriguing problems have much the flavor of detective stories. This present volume is a perfect sequel to my Chess Mysteries of Sherlock Holmes and Chess Mysteries of the Arabian Knights; I cannot imagine a better one! Although the author calls it a "sequel," it can be read independently of my two volumes, since enough introductory material has been provided.

In this fascinating book, the author has definitely broken new ground. He has varied and advanced many of my themes of retrograde analysis to a remarkable degree. For example, as complementary to my monochromatic problems, in which no piece ever moves from a square of one color to a square of a different color, the author has introduced heterochromatic problems, in which it is given that on each move, the piece always goes from a square of one color to a square of a different color. This opens up an entirely new field of beautifully elaborate combinatorial situations. As another example, the author has cleverly combined some of his retrograde chess problems with my knight/knave (truth-teller, liar) logic puzzles.

The problems are really dazzling in their ingenuity, and I am delighted that retrograde analysis has taken such an unexpectedly imaginative and creative turn.

Raymond Smullyan
Elka Park, New York

AUTHOR'S PREFACE

A reader to whom "further" in the subtitle is meaningful would expect the author to be Raymond Smullyan. He is the logician who introduced puzzle afficionados to retrograde chess analysis. And he used Sherlock Holmes as the analyst par excellence, with Watson as his student and scribe. Smullyan's 1979 book, C*hess Mysteries of Sherlock Holmes*, justifiably ranks as a classic. The present book could look suspiciously like an unauthorized sequel. (Smullyan's *Chess Mysteries of the Arabian Knights* was a second book of retrograde analysis problems, but not truly a sequel to the first.) The present book even makes use of some of Smullyan's peripheral characters, specifically Lord and Lady Ashley.

I would not dream of presenting a book on retrograde chess analysis without acknowledging that it builds on Smullyan's pioneering publication. Much of the present content enlarges on or contrasts with Smullyan's foundations, and continuity between those foundations and the extensions is important. This is a separately authored sequel, but Raymond Smullyan has had an opportunity to preview it and, in the Forward, has expressed his acceptance of its appearance as sequel. Since this book stands on foundations fashioned by Smullyan, he is entitled to share in any kudos, while remaining free from blame for defects in any of its details.

That 1979 Smullyan work had a whole new world to explore. Smullyan ranged over the constraints, inherent in chess, that permitted some courses of play to be eliminated as logical impossibilities, with what was left necessarily true. He preempted the elegantly simple principles so completely that any follow-on volume is forced to emphasize more complex problems.

For the previously uninitiated reader, retrograde chess analysis attempts to answer questions about the history of a specific game. In the purest form, a diagram shows the present position, and a question is asked, such as:

What was the last move?

What piece was captured on square f5?

Was the missing Knight captured on the 18th or 19th move of the game?

Can Black castle? (Is it Black's move? Is he in check? Have his King and one
    Rook remained unmoved?)

Even Smullyan found it necessary to introduce additional constraints to arrive at fifty problems with fresh elements. He proposed the "monochromatic" game in which every move is between two squares of the same color. He set some problems on boards of ambiguous square colors, bringing move direction into the set of unknowns; one problem added chess pieces of ambiguous colors. A few of his players used what Smullyan described as ancient rules of chess, permitting a promoting Pawn to be exchanged for a piece of either color.

The physical awkwardness of his players gave Smullyan much ammunition. A frequent situation was a jostled board, with one or more pieces displaced from square centers and

ambiguously straddling lines between squares, or even corners that extended the uncertainty to four squares. Occasionally a piece would be knocked completely from the board and be replaced with a coin or other marker, leaving passersby to wonder at piece identity and/or color.

Finally, comments by players or observers may add to the information given by the chess position alone:

White gave odds of the Queen's Bishop.

Black castled four moves ago.

Pawns made all captures.

Such supplementary information adds further constraints and greatly extends the feasible range and richness of logical analysis.

To any reader not familiar with *The Chess Mysteries of Sherlock Holmes*, my advice is to find a copy, and work through it while deferring the present book. You will experience a sense of wonder, and the exhilaration of a new challenge, just as earlier readers did. And you will confront this fresh set of problems with an excellent understanding of what retrograde chess analysis is, and how to apply the basic constraints of chess to any problem in the genre.

* * * * * * * * * *

What will you find in the present book? It contains one major, conceptually new element -- the heterochromatic game, in which each move is between squares of opposite colors. The unique set of constraints in that game form allows a progression reminiscent of Smullyan's, with individual constraints allowing beautifully simple introductory problems. They also allow for the ready resolution of multiple ambiguities (five ambiguities of piece color and placement appear as early as the seventh problem in the book).

The book combines several of Smullyan's ambiguities, and mixes them with the new set. Conventional black-and-white boards signal use of conventional promotion rules. The board of ambiguous colors (chameleon board) signals player use of color-changing rules of promotion (chameleon promotion); avoiding the awkwardness of flagging the rules difference at each use. Not all such problems feature actual reverse promotions, the player agreement serving as an artifice for introducing uncertainty of move direction.

The heterochromatic game and the older monochromatic game (or the generally similar homochromatic form) take on new dimensions when set on the chameleon board, exercising constraints not important on the conventional board. The combinations permit a richer problem environment. Differences in meaning between monochromatic and homochromatic are flagged after the first problem herein, and reflect agreement between Smullyan and the present author.

Even on the conventional board, there are quantitative extensions from Smullyan's problems that almost amount to qualitative differences. A homochromatic game is shown

at an advanced stage, with no supplementary information; it is possible to identify White's third move uniquely! With supplements, that game form permits a detailed history of 10 captures: what captured each piece, where, and in what chronological order.

The en passant capture will rear its head occasionally, in contexts not explored by Smullyan.

Smullyan has written many popular books of recreational logic not involving the chessboard. A few problems. herein use "truth table" logic in preludes contributing to problem definition. One recognizes Smullyan's *What Is the Name of This Book?*, twisting it into *What Is the Game of This Rook?*.

On the average, the earlier problems are simpler than those later in the book. But the difficulty is deliberately varied throughout. A very small number of simple problems herein cover no ground left unexplored by Smullyan. They are included as review of elementary principles in preparation for extending those principles to new settings. Elementary problems in move parity of a pair of Knights, for example, pave the way for the broader applicability of move parity in problems involving multiple Knights or heterochromatic play.

For the benefit of readers who are new to retrograde chess analysis, and unable to locate the original Smullyan book(s), an Appendix treats briefly some fundamental principles, such as the counting of Pawn captures.

* * * * * * * * * *

In problem definitions, in the absence of cautions to the contrary, all comments by players or observers (and by Sherlock Holmes) are taken as absolute truth. It is sometimes necessary to filter out conversational fog to identify potentially crucial information. And truth must be interpreted literally; Watson often makes false assumptions about what comments imply, and the reader must guard against that tendency.

Problem solutions, also, are presented in conversational form. That makes them lengthier than they need be, but serves two important purposes. Watson often pursues alternative solutions that prove to be blind alleys or traps for the unwary. Some of those alternatives could appear attractive to some readers, who would remain unconvinced if the analysis ignored them.

More importantly, the conversational approach allows use of the Socratic method throughout. Holmes asks pointed questions for Watson to answer through analysis, each question leading closer to the solution. That device breaks a complex problem into a sequence of simpler subproblems for the less expert to pursue. A reader who doesn't know where to begin may get enough of a hint from Holmes's first or second question to carry the analysis to completion. Problems are removed from the "Pass/Fail" category, for greater reader satisfaction.

A final note concerns position identification. Chessboards are normally described by a coordinate system in which files are lettered a to h from left to right, and ranks

COORDINATE NOTATION
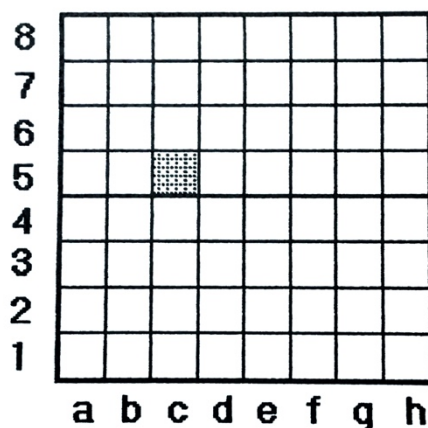
numbered 1 to 8 from the bottom up. Thus, square c5 is in the third file from the left and fifth rank from the bottom. But ranks and files may not be obvious on the chameleon board. For consistency herein, letters a to h identify columns from left to right, and numbers 1 to 8 identify rows from bottom to top of the diagram.

* * * * * * * * * *

The chess position graphics herein clearly are computer-generated. A graphics font of 42 array files represents only the six standard chess pieces plus the Unknown (question mark), in Black and White on squares of white, horizontally striped gray, and vertically striped gray. Unconventional effects (pieces of mixed color patterns and pieces displaced from square centers) are generated by program algorithms. Tailored array files for square borders supplement the centered-piece arrays and are drawn on automatically when pieces must appear to straddle those borders. Print resolution is 180 dots per inch in each direction, equality of vertical and horizontal resolution being essential to the chameleon board. In the middle 1980's, when the print program was developed, only Epson printers designated ESC/P or ESC/P2 offered that equality. The author's chessprint program was tailored to and is usable with only those selected Epson printers.

PAGE NUMBERS OF PROBLEMS AND SOLUTIONS

Example: the third problem, QUEENS GALORE, is on page 9. Its solution is on 146.

Use PDF Numbers to move quickly among problems and solutions. A PDF Number is a page number plus 10. Type a PDF Number in the white box in the marquee of Acrobat and hit Return.

A NEW CHESS CLUB

"Come, Watson!" cried Holmes when I answered my door. "There's a new chess club we must visit. Opportunities for retrograde analysis should abound there."

He refused to answer my queries until we had reached the club, where a subdued sign identified it as the "Chameleon Chess Club." Clearly alerted to my friend's planned visit, the club secretary, Goodwin, greeted us effusively upon our entrance.

Goodwin led us to an unoccupied table, where the position was as shown in the accompanying figure. Rather than the usual white and black squares, this board used two

1. CHAMELEON BOARD

2
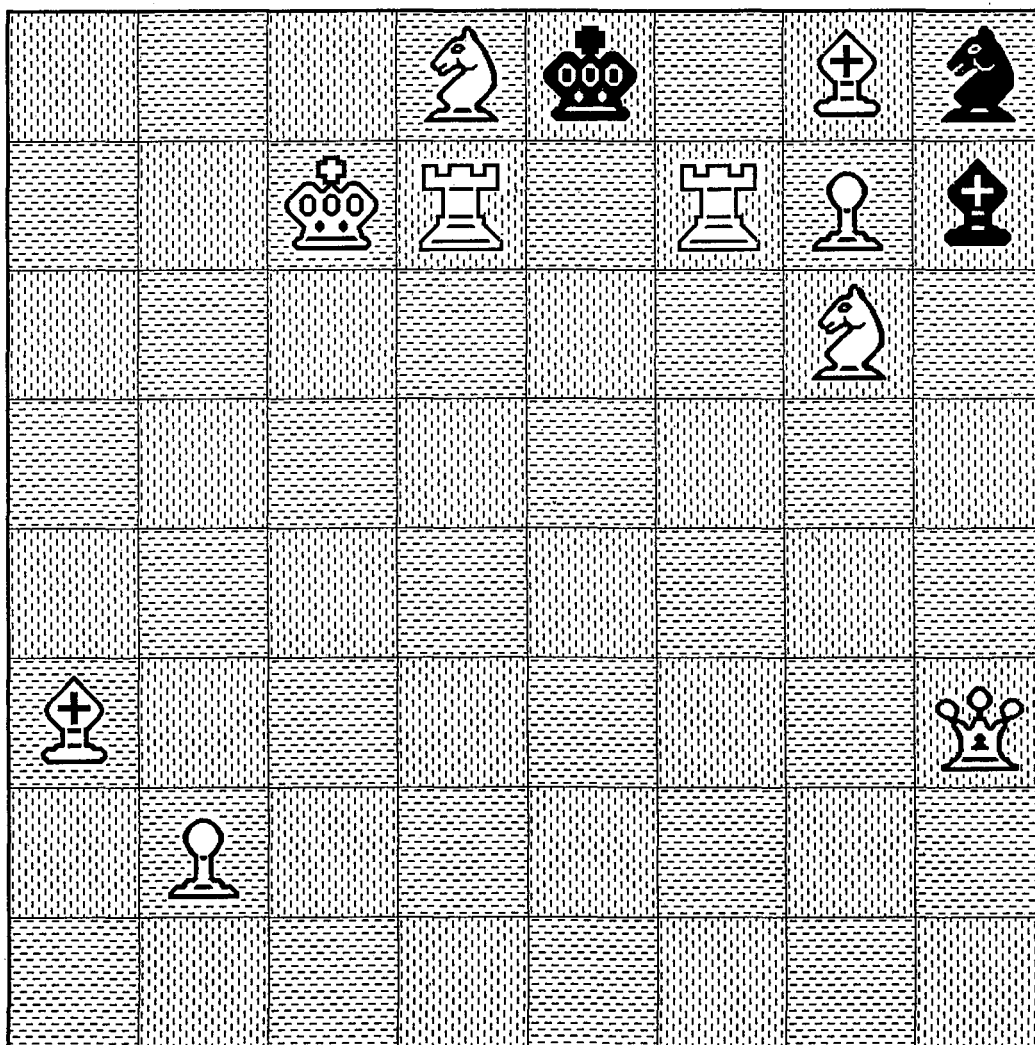
shades of grey, distinguished by vertical or horizontal striping.

"Most of our members like to play occasionally using the ancient rules of promotion, which allow a promoting Pawn to change color," said Goodwin. "Our club name derives from that color-changing capability. So that passersby can know which promotion rules are in effect, we use standard black-and-white boards for play with normal promotion rules, and chameleon boards, as we call these, with chameleon rules of promotion."

"Do your players always treat vertical greys as white squares?" asked Holmes.

"Good lord, Holmes!" I ejaculated. "You have already concluded that play is up and down rather than left and right. I didn't have the foggiest clue as to move direction."

"Nor do I, Watson," replied my friend. "But notice that if the board is rotated 90 degrees clockwise, what was a horizontal square at the lower left corner transforms to a vertical square at the upper left corner. From the players' viewpoint, the upper left corner of *this* board is a vertical grey no matter how the board is oriented. But, without knowing where the players sat, we don't know whether play was up-down or left-right. My question was whether all of the chameleon boards are of the same configuration. A board that always had a horizontal square at upper left would serve equally well."

Goodwin chimed in, "All of our chameleon boards are like this one, with the upper left square vertical. Players can always interpret vertical as white, horizontal as black."

I inquired, "Do your players depart from modern rules of chess in other respects?"

"No," Goodwin replied. "We welcome observers and wish to minimize any possibility of distressing them. If an observer understands the significance of the board choice, he can watch any complete game and never see what he could interpret as a rules violation.

"Some players choose to play under restrictive agreements. An example is homochromatic play, in which every move is between two squares of the same color. But it is clearly understood that the *legality* of moves between squares of opposite colors is not changed by the agreement to avoid such moves."

Holmes commented, "That distinction is particularly important in the definition of checking. For example, Knights do not move homochromatically; but a King is still prohibited from moving to a square where an opposing Knight would give check."

Just then Soames-Forsyte walked up and greeted us. "I can't imagine you find this obvious position very interesting. The game ended here. I announced mate in one move, and Galsworthy conceded."

"I note," said Holmes, "that there *was* a reverse promotion."

Without too much prompting, I was able to see the logic behind that conclusion.

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AUTHOR'S NOTE: [Solutions to this and subsequent problems appear at the back of the book, in the order of problem encounter, and identified by the same problem numbers and titles used for problem diagrams.]

THE AUTHOR PUTS HIS OAR IN: WHY HOMOCHROMATIC PLAY, NOT MONOCHROMATIC

As this book was being readied for publication, private communications between the author and Raymond Smullyan revealed a difference in our understandings of *monochromatic* games. Smullyan had invented and named the concept in *The Chess Mysteries of Sherlock Holmes.* There six problems dealt with games in which no move was between two squares of opposite colors; every piece remained on its original square color throughout the game. The word "monochromatic," meaning literally "one color," was

WHOSE RULES?
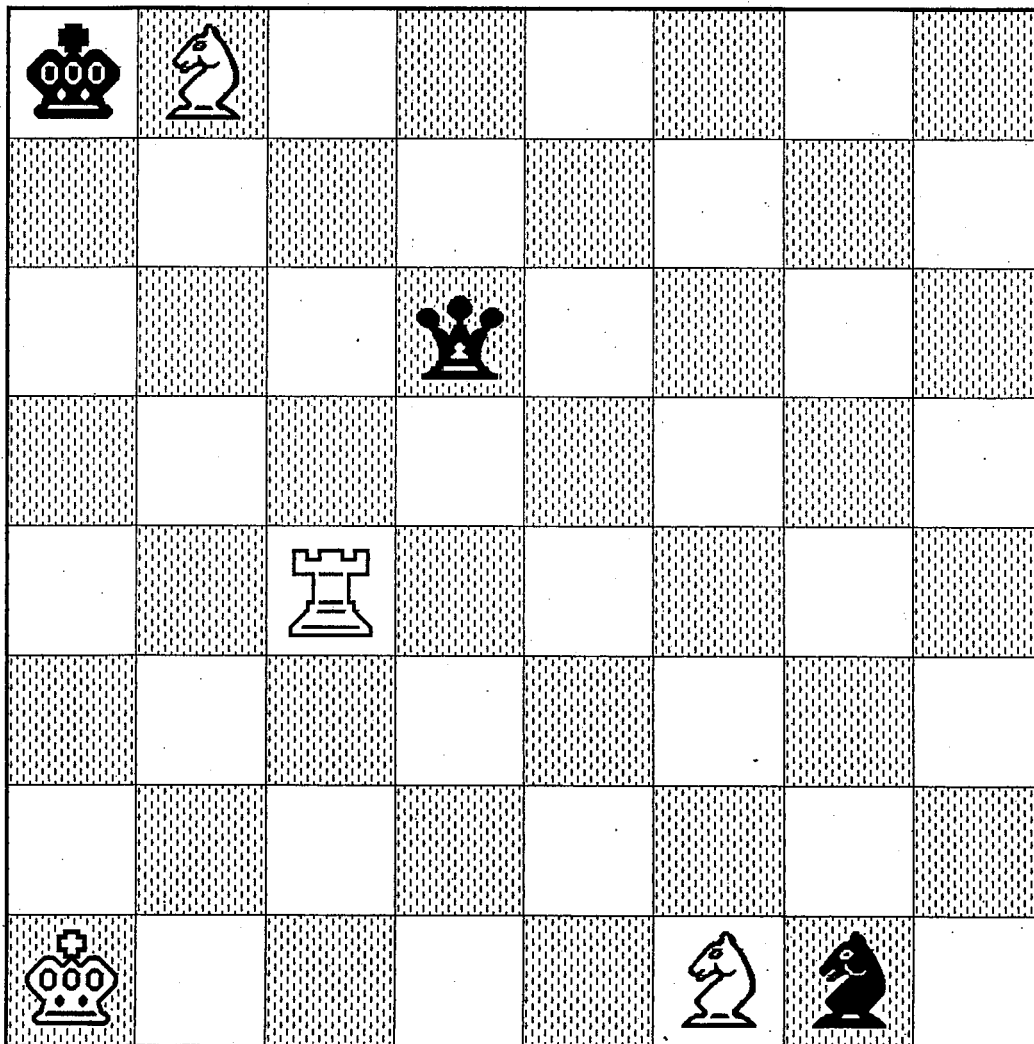
applied strictly to the individual move, and by extension to games made up entirely of such moves.

In some of the earliest Smullyan problems, monochromatic play was reported simply as possibly relevant historical fact, just as in another problem we might be told that neither King has moved. In later problems we meet people who *always* play monochromatic games, implying deliberation and agreement. If monochromatic play "just happened," we need not consider the possibility of rules changes. But, when players have agreed to play only homochromatically, the details of their agreement *may* need to be brought into the open.

The accompanying figure (WHOSE RULES?) can illustrate the dichotomy. Could the position have resulted from monochromatic play? If so, since Knights cannot move monochromatically, the White Knight on f1 has to be a promoted White Pawn, proving that White is moving down the board. The Black King originated on d1, and has traveled to a8 on white squares only. Its last move was from b7. It reached b7 from a6, or c6, or d7 through c8. The White Knight was on b8 when the Black King moved to a6 or c6 or d7. Wasn't that King move an illegal move into check by the Knight?

Normal chess rules would prohibit the indicated Black King move as a move into check. Those rules apply when monochromatic play "just happens." Do they apply when monochromatic play is by agreement? Raymond Smullyan says they do not, and it *is* his ball game. Smullyan reasons that the player agreement constitutes a single change in the rules of chess, accepting only monochromatic moves as legal. Check is automatically defined in terms of move legality. Kings are never captured, but the test of check is this: if the King were replaced by any other piece of its color, would the opposing piece be able to capture the substituted piece? If so, the opposing piece checks the King; if not, there is no check. In agreed monochromatic play with Smullyan's interpretation, the Black King would not have been in check on a6, c6, or d7, and could have reached its present location legally.

An alternative, favored by the present author, is to consider the player agreement as binding up to a point, but not as changing any rules of chess. The players have merely agreed that, *so long as it proves possible*, they will limit themselves to using a specific subset of the legal moves, the monochromatic moves. But the moves they avoid remain legal; the White Knight in the illustration could legally capture on a6, c6, or d7, thus would give check if the Black King were on one of those squares. This interpretation offers two advantages:

(1) One need not distinguish between games that just happened to be monochromatic and those that were limited by player agreement. The same rules apply in both cases.

(2) An observer who was unaware of the specific player agreement could, nevertheless, watch any complete game and never see what he would interpret as an illegal move or position (one of Goodwin's requirements in the Chameleon Chess Club).

Neither of the conflicting interpretations is illogical, and neither is inconsistent with Smullyan's previously published examples of monochromatic games. Both can make contributions to future retrograde chess analysis. But we must be able to know which

interpretation applies in any problem context. Smullyan has suggested a solution. We agree to reserve Smullyan's term *monochromatic* ("one color") for player agreements that are recognized as changing the rules of chess, and introduce the near-equivalent word *homochromatic* ("same color") for agreements that retain the conventional legality of all moves while additionally restricting play to the same-color subset of legal moves.

Because of the emphasis on "observer-friendly" play in the Chameleon Chess Club, *homochromatic* play, but not *monochromatic*, is found herein. Monochromatic play as now defined *can* produce positions far more disturbing than that above. Note the figure identified as GOODWIN'S NIGHTMARE, where the conventional viewpoint is that both Kings are in check, with White in quadruple check. Properly understood by

GOODWIN'S NIGHTMARE
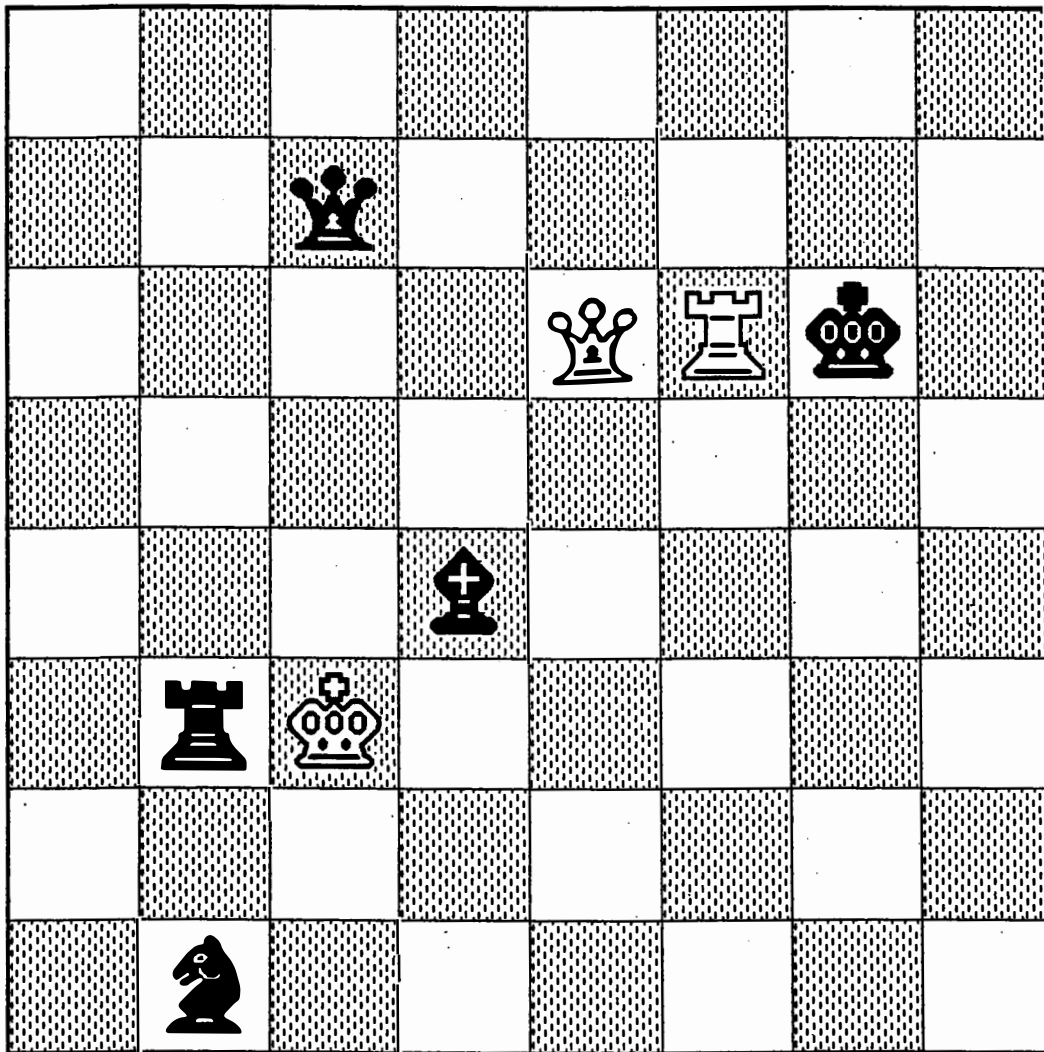
monochromatic rules, Black is not in check; White is in a legal double check by Bishop and Queen, the last move a Bishop move from c5 to d4.

Herein is also specific consideration of how players handle "as long as it proves possible," i.e., conflicts between fundamental chess rules and the commitment to same-color moves.

Now let us segue back to Watson's narration.

* * * * * * * * *

WATSON WONDERS

I spied another abandoned chameleon board on a nearby table, and spent a few minutes in cogitation before calling Holmes over.

"This position suggests the familiar question of whether Black can castle," I said. "I have worked through the logic for the standard board, but the possibility of a reverse promotion is causing me some difficulty."

Holmes appraised the situation, then said, "It might help you to talk through the same position on the standard board."

2. A CHAMELEON CASTLING CONUNDRUM
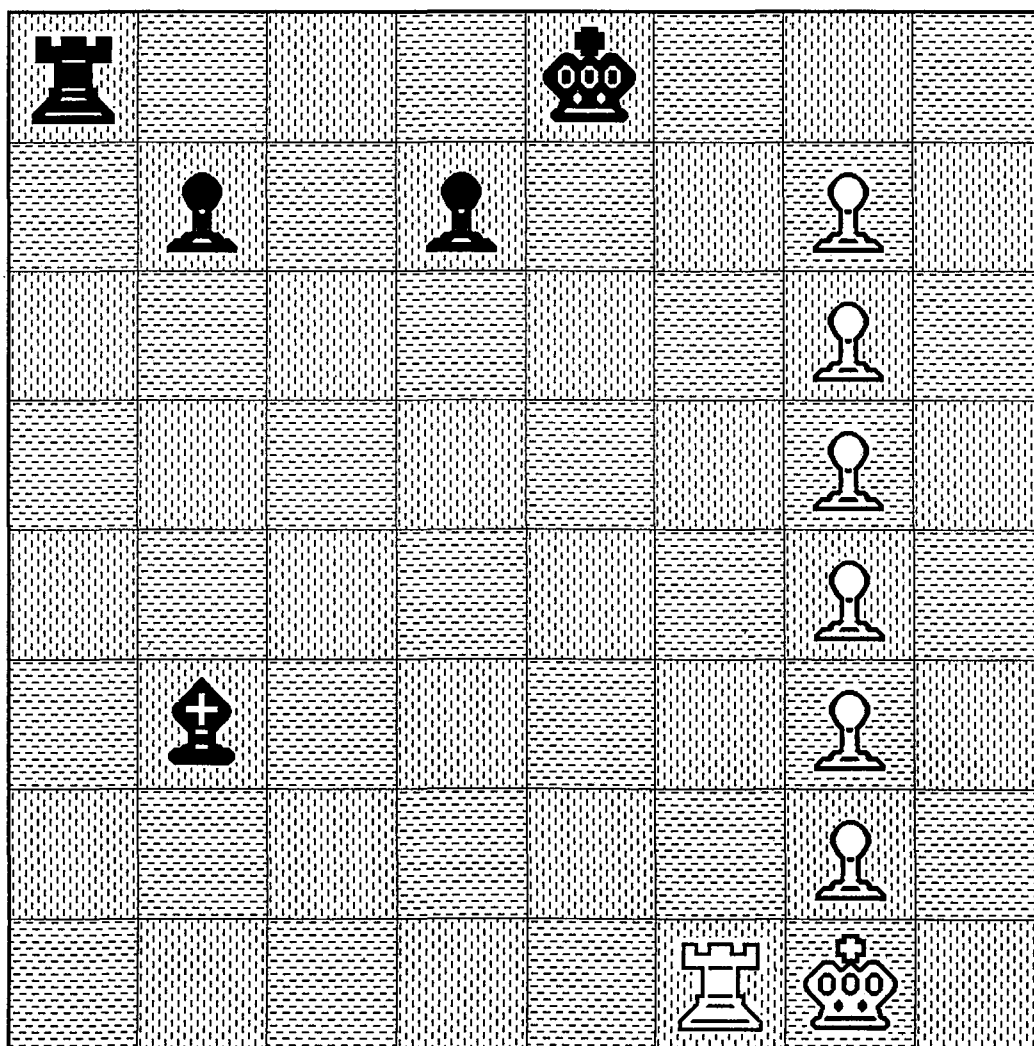
"Very well, Holmes," I responded. "On a standard board, if the upper left corner were black, we should know that motion was left-right. The Black King not being on its home square, there would be no possibility of castling. If the upper left corner were white, nothing about the position would rule out Black's moving *up* the board, in which case he could not castle."

Holmes concurred, adding, "That leaves the case of Black's moving down the board. What do you conclude for that case?"

I was rather proud of my analysis. "With Black moving down, the two Black Pawns have not moved, and the Bishop could not have left its home square at c8. It was captured there, and clearly not by a Pawn. The Black Bishop now on the board is a *promoted* Black Pawn. Only ten Black pieces remained available for capture by the White Pawns now on the g-file. And those Pawns had to make at least *eleven* captures: one each for Pawns from the h- and f-files, two by a Pawn from the e-file, three from the d-file, and four from the c-file. Black could not possibly be moving down on a *standard* board."

"Precisely so," agreed Holmes. "And what is different for the chameleon board?"

"One of the two missing White Pawns could have made a reverse promotion, creating a seventeenth Black piece, allowing the eleven captures required by the Pawn formation on the g-file. So I can't rule out Black castling on the chameleon board."

"Black Pawns from the a- and c-files would have to be captured by one or more of the White Pawns moving toward the g-file," said Holmes. "Does that fact give you pause?"

"No," I replied. "Those Black Pawns would have to promote before they could be positioned for required captures by White Pawns, but there are no obvious bars to such promotions."

"Correct!" said Holmes with some enthusiasm. "Just extend that line of thought, and you can answer the castling question for the chameleon board."

OVERPROMOTION?

As we were walking past a deserted table with a chameleon board, I was struck by an astonishing number of Queens. I pointed out the board to Holmes, and said jokingly, "I don't suppose there is any question of an underpromotion in this game."

We paused to count the Queens, finding three White and five Black.

"I can understand a rare reverse promotion," I said. "But with six Black Pawns still on the board, at least two of the Black Queens have to be promoted White Pawns."

3. QUEENS GALORE
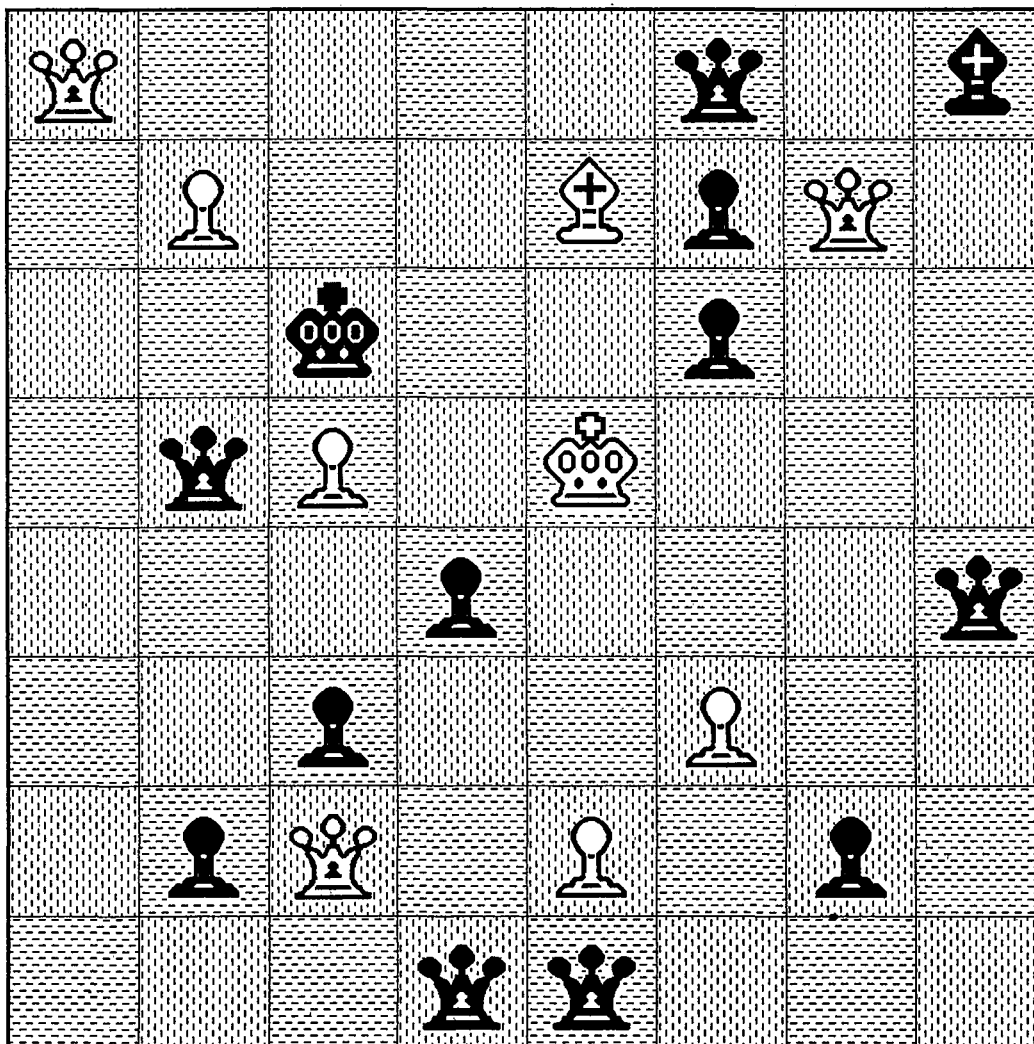
A voice to my rear said, "Actually, all six of the promotions to Queen were reverse promotions."

We turned and greeted Field, the speaker. "Marshal and I have been playing this game. Playing White, I had a very sound reason for the first reverse promotion. However, it seemed to fire Marshal up; before long he had promoted two Pawns, both to White Queens. I became caught up in the madness of the moment, and continued to make only reverse promotions."

I had been so engrossed by the plethora of Queens I had paid little attention to other aspects of the position. I now noted that White was in check, and commented on that fact to Field.

"Yes," he said,"I suppose we'll have to get serious through the rest of the game. Marshal has a commanding lead, and you could say that he has already settled down to serious battle with his last move."

In an aside to Holmes, I observed, "It's a little late to undo the damage of all of those Field promotions."

I then turned my attention to analyzing the check, but had to confess that I could not completely define the last move. I *could* identify the direction of movement and the piece that moved last, but needed help from Holmes in determining the square from which that piece moved.

What *were* those elements of the last move?

WATSON MISCALCULATES

A hail to Holmes stopped us at a table with this chameleon position. My companion introduced me to the players, Malory and Arthur. As they chatted concerning matters with which I was not familiar, I concentrated on the chess position. Malory, who had been playing Black, excused himself to replenish his cigars, and I took the opportunity to demonstrate what I had gleaned.

"Black can't castle," I explained. "One of the three White Knights has to be promoted. It was not a reverse promotion, with all eight Black Pawns still on the board.

4. HOW CAN THAT BE?
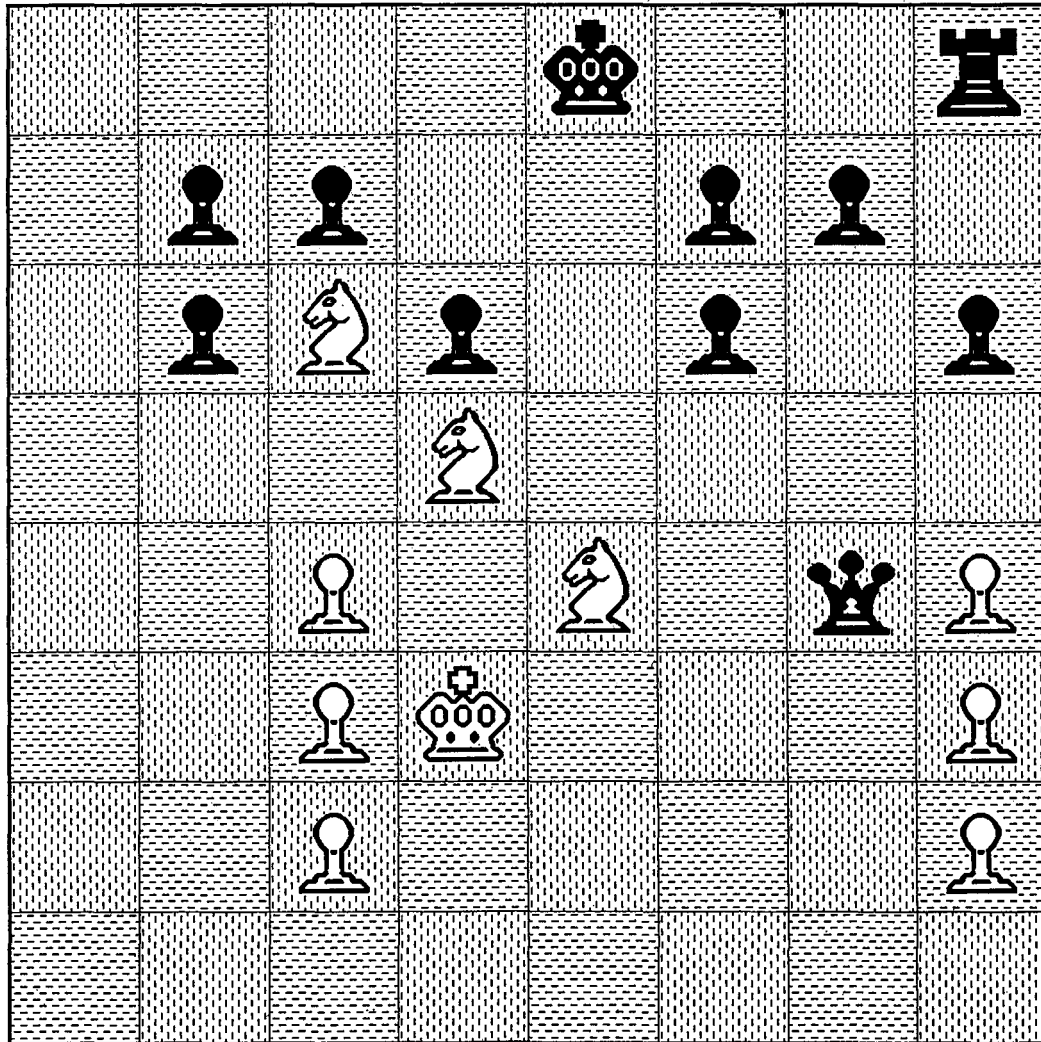
"One White Pawn on the c-file came from b2 with one capture; another came from d2 with one capture. One White Pawn on the h-file came from g1 with one capture, and another from f2 with two captures. The visible White Pawns, then, have captured all five missing Black pieces. A sixth capture was not available to a promoting White Pawn, so the promotion could only have been of the Pawn from a2 promoting at a8, or the Pawn from e2 promoting at e8.

"The promotion to White Knight did not occur at a8, or the promoted Knight could never have left that square; the Black Pawn at c7 has never moved, and the Pawn at b6 had to capture from a7 before a promoting Pawn could reach a8. Thus, the promotion to White Knight was at e8. The Black King had to vacate e8 to permit that promotion. The King having moved, Black cannot castle."

Arthur appeared about to respond, but subsided as Malory returned to the table. Arthur informed Malory, "Watson has been drawing some intriguing conclusions from my Knights."

"Oh," said Malory to me, "Arthur does have a reputation for being masterly with his Knights. But, as you will have observed, he can't manage to hold onto his Queen."

Malory thereupon resumed his seat as Black, and immediately castled!

"That move *was* legal, Watson," said Arthur, majestically.

Where did I miscalculate?

AN ALL-WHITE CHAMELEON

It was a delightful surprise to spy Lord and Lady Ashley, whom we had met on a cruise, seated at a chameleon board. We walked over to greet the couple.

"How nice to see you, Mr. Holmes, Dr. Watson," Lady Ashley said vivaciously, "Are you still pursuing your interest in retrograde chess analysis?"

"Decidedly so," Holmes assured her. "The Chameleon Chess Club promises to expand our horizons."

5. WHITE'S LAST MOVE
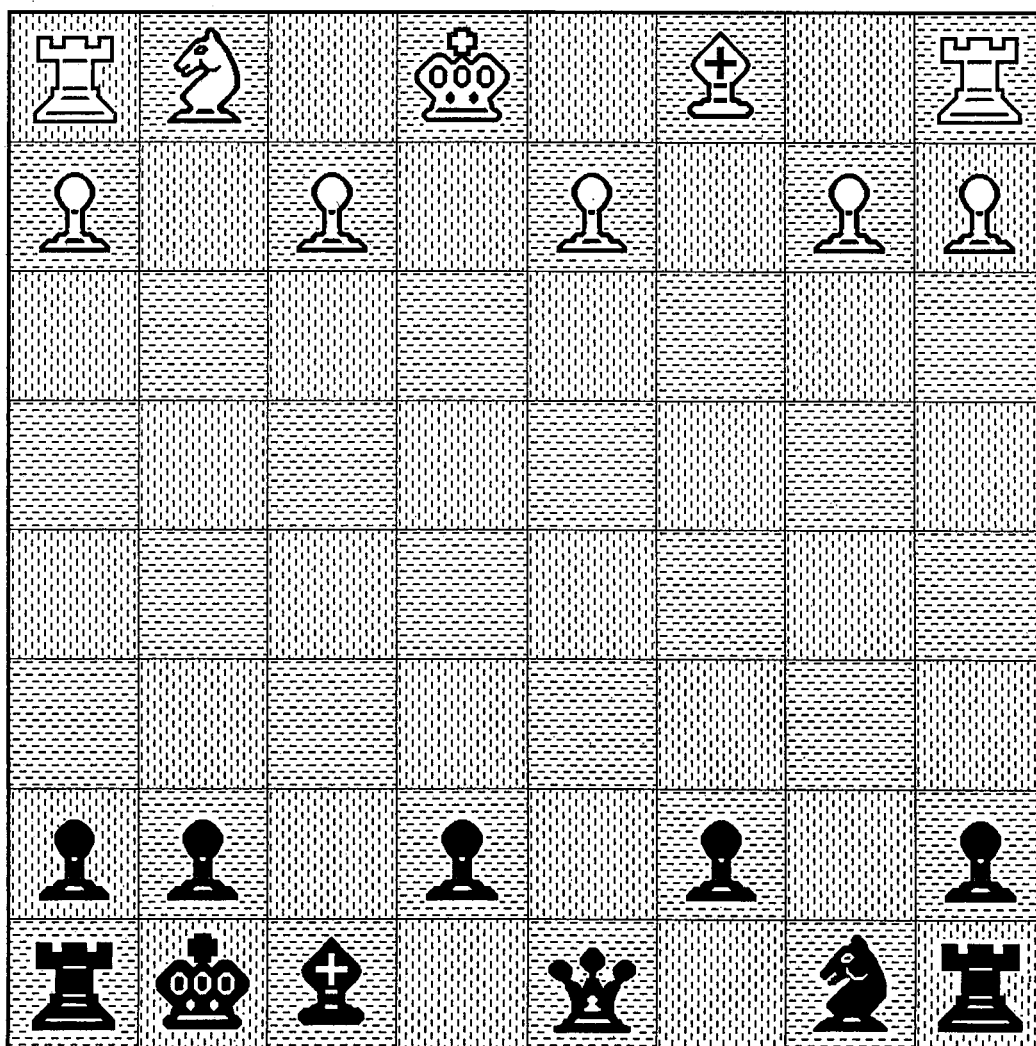
"But it may narrow yours," I interjected. "I remember the two of you as playing only monochromatic games. And it would not be possible to play monochromatic games, as that term is now understood, within Chameleon Club etiquette."

"No," she agreed, "But we could probably have continued playing without significantly changing our old understanding, while calling our play homochromatic rather than monochromatic. However, the Chaneleon Club has brought us out of our old shell, almost forcing us to accept greater variety of play."

Lord Ashley spoke for the first time, "Still, the present game has been one that could be categorized as either homochromatic or monochromatic, since no play has forced us to come to grips with differences in checking between the two forms. I prefer to think of this as the ultimate monochromatic game; that term means literally 'one color,' and all moves of this game have been from and to 'white' squares, as represented by vertical grey."

"Now that *is* unusual," Holmes acknowledged. "You know, the word *monochromatic* never prompted me to consider an opposite, but the more recent usage of *homochromatic* does cry out for an opposite and complementary *heterochromatic* game, with all moves between squares of opposite colors."

"Right!" exclaimed Lady Ashley. "And I believe we can show you examples of heterochromatic play here in the Chameleon Chess Club. But, first, what do you make of this interrupted game? For example, can you tell whether the last move was a capture?"

Holmes pondered for several minutes, to my great exasperation, since I had quickly spotted the only possible last move. Finally, he shook his head, "There is one firm conclusion I can draw, but I can't determine the last move and don't know if it was a capture."

"For Heaven's sake, Holmes," I blurted. "The answer is staring you in the face. Except for the unmoved Pawn on h7, the Black King is the only piece still on the board that *can* move on white. It obviously moved from c2 to b1, capturing the last White piece that moved on white."

"And what White piece was that?" Holmes asked quietly.

"Why, it was... Oh, no! No White piece moving homochromatically could have reached b1 while the Black King was on c2. There is no possible last White move!"

"Watson," Holmes chided, "you tend to think in terms of black and white, while this problem involves shades of grey."

When I realized the significance of that comment, I moved nearer to a solution. But Holmes had to guide me through the last couple of steps.

AMBIGUOUS PIECE COLORS

Holmes suggested that I chat with Goodwin, while Holmes accompanied Lady Ashley in searching out heterochromatic games with potential interest for retrograde analysis.

Goodwin told me, "All of our chessboards are two-sided, with the normal board on one side, the chameleon board on the other."

I reminisced, "Holmes once ran into a problem in which the chess set, itself, was of two ambiguous colors. Has your enthusiasm for chameleon play extended that far?"

6. WHOSE HUE?
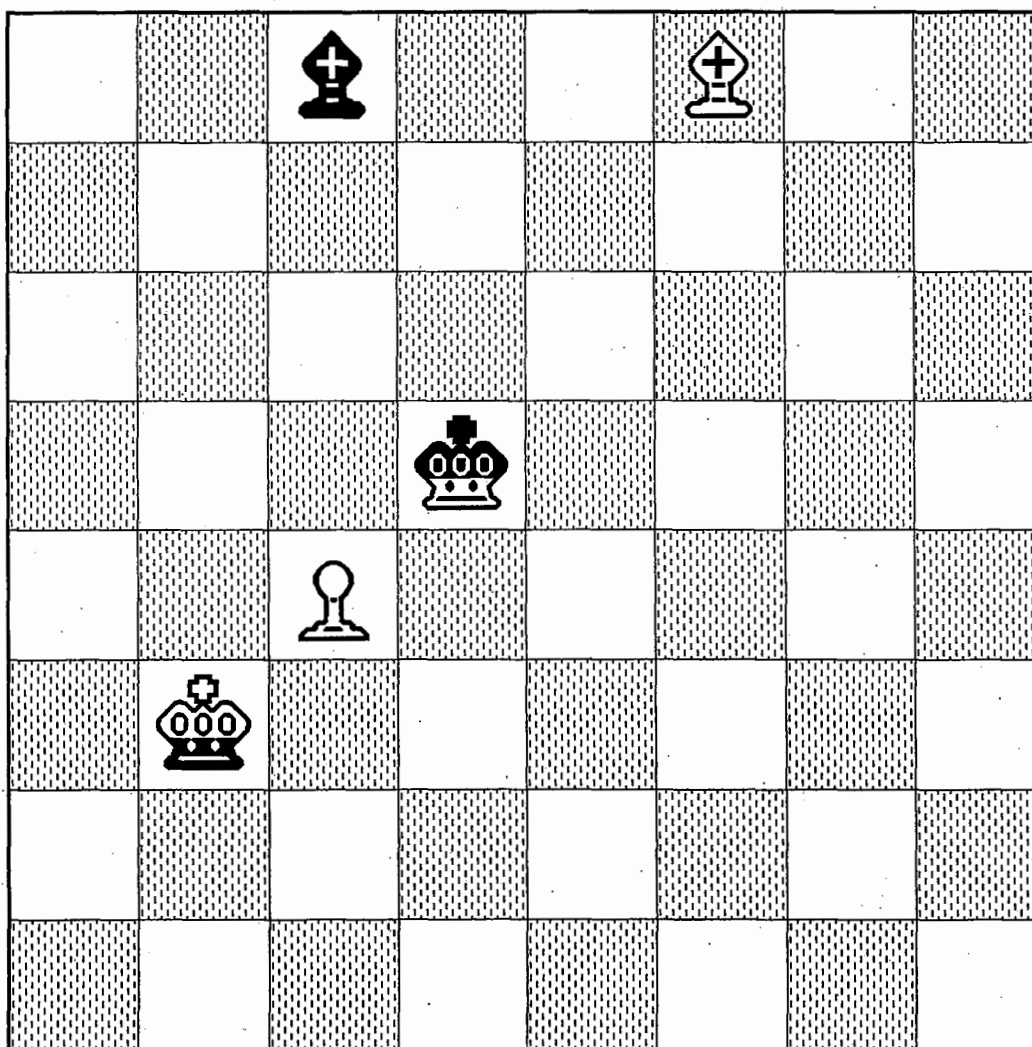
"A few members insisted on having such chess sets available," said Goodwin, "but even fewer have persisted in using them. I should say that they are now used primarily as temporary replacements for fallen pieces."

He showed me one set of ambiguous colors. One pattern combined a White upper half and Black lower half, and one the reverse combination. "When both are used," said Goodwin, "players refer to the colors as White-Up and White-Down."

A second set combined left and right halves of opposite colors, described by Goodwin as "White-Left and White-Right." I idly rotated a piece in my hand, and was mildly astonished that the appearance did not change with the viewing angle; White-Left did *not* transform to White-Right with rotation. Goodwin shrugged off any conceptual difficulty, "Once you have seen paintings with eyes that follow you about a room, you simply accept and admire the artist's work."

In a third set, consecutive quarters around the clock were of alternating colors. "White Up-Left and White Up-Right," said Goodwin. He noted my furtive rotation of a Bishop, and commented succinctly, "Same artist."

When Holmes and Lady Ashley returned, Goodwin excused himself to attend to other duties. I was led to a table that contained the abandoned game diagrammed above, Holmes solemnly assuring me that play had been heterochromatic. Curiously enough, that first exposure to heterochromatic play involved Kings of the ambiguous White-Up and White-Down patterns.

Lady Ashley said, "I am informed that the last move was a check, clearly by the White Pawn. But, not knowing the move direction, it isn't clear which is the Black King, in check."

"I haven't given much thought to heterochromatic play yet," I began. "But it is perfectly obvious that Bishops can't move heterochromatically. One of the two Bishops is on its home square, and the other is necessarily promoted. If we knew which is the promoted Bishop, the move direction would be established, and we would know which King the Pawn is checking."

"That is the essence of the problem," Holmes assented. "And which Bishop do you conclude is promoted?"

"With so few pieces on the board," I replied, "I can't believe it is possible to prove which is the promoted Bishop."

Holmes, who presumably was likewise encountering his first heterochromatic problem, pointed out the compelling logic, which involved the piece that is giving check.

CHECK WITH UNMOVING BISHOPS

Holmes steered our small group toward another table, where play was in progress. As we approached the table, we heard an exuberant cry of "Check!" from one player, but were too late to see what piece he had moved.

Lady Ashley introduced the players, Drumm and Phyfe. Drumm, who had boomed out the check, confirmed that theirs had been a heterochromatic game. He was seated at the less cluttered side of the board pictured below, a lone White Bishop the only piece on his first two ranks.

7. FIVE TO RESOLVE
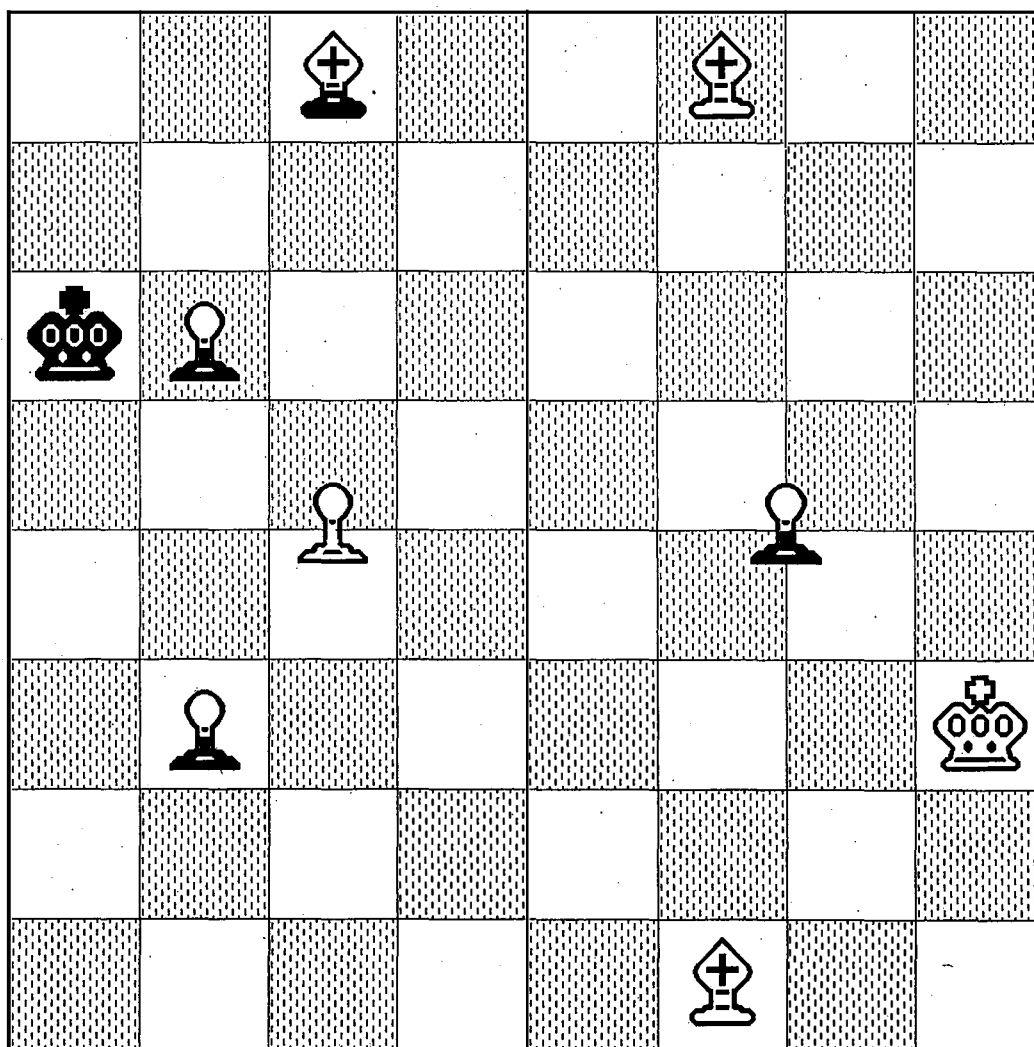
Phyfe's side of the board featured two Bishops, one White and one of ambiguous color. Of the four Pawns on the board, three were of ambiguous colors, and two were displaced, ambiguously straddling lines or corners.

Phyfe complained shrilly, "Drumm startled me, and I jostled the board."

"More than once, it would appear," said Holmes, "from the number of replacement pieces you have had occasion to use."

I observed, "Since Bishops cannot move heterochromatically, one of the White Bishops is a promoted White Pawn."

Holmes added, "I note that the same ambiguous pattern is used for pieces of both colors. So, resolving the color for one piece does not resolve it for all pieces."

I asked, "How can you tell with certainty that both piece colors are represented?"

Holmes reminded me, "A Pawn cannot capture, so cannot leave its file except by being captured. Two Pawns on the same file, as on the b-file here, are necessarily of different colors."

He continued, "And those principles of heterochromatic play are all you should need in order to resolve all of the ambiguities and identify the last move."

WHICH PAWN?

Sutter and Mills were standing by this table as we arrived, providing no clue to move direction in the chameleon game. With relatively few pieces remaining on the board, I felt that I should be able to uncover some nugget of information without too much digging.

As Holmes chatted with the players, I perused the board. It was obvious that White had just moved, the White Rook on e1 giving check. The Rook could not have made the last move, which could only have been from another square on the e-file, where it would have given imaginary check. The check, then, was discovered; of the Pawns still on the

8. EN PASSANT
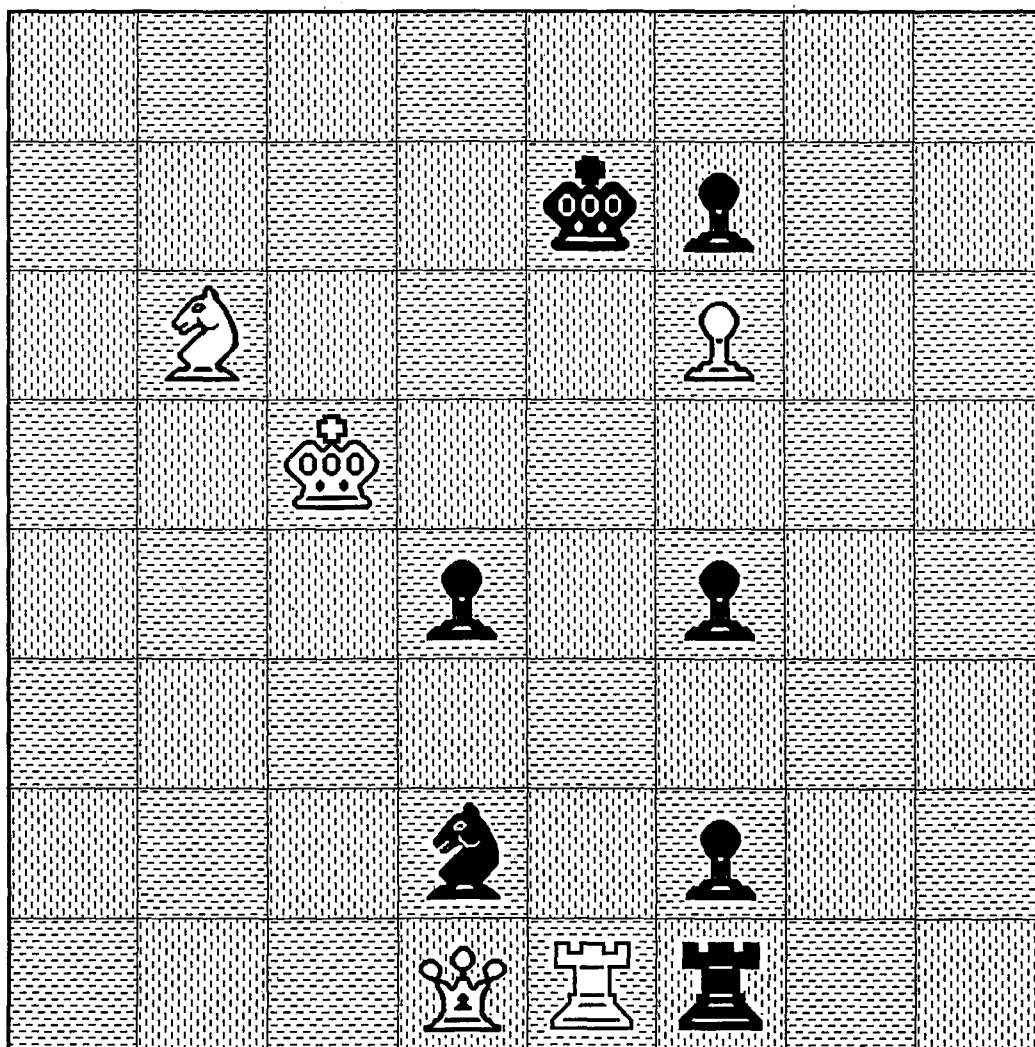
board, only the White Pawn on f6 could have moved to discover the Rook check. That Pawn could have moved from e5 with a capture, or from e6 without capturing.

I was about to announce these findings when Sutter spoke, "I don't suppose it is obvious, Watson, that one of the Pawns on the board has made an en passant capture."

That comment promised to eliminate the noncapturing last move. "No," I admitted, "it was not obvious that an en passant capture had occurred. But, knowledge that there *was* an en passant capture uniquely identifies the last move."

Holmes, for the first time, examined the game position. Then he laughed heartily. "Oh, Watson!" he chided. "You should be able to identify the last move without being told that an en passant capture has been made. Now you can add another simple problem element; identify the Pawn that made the en passant capture."

"Elementary, my dear Holmes!" (The phrase had a pleasant taste in the mouth.) "An en passant capture puts the capturing Pawn on the opponent's third rank. If White is moving up the board, the White Pawn on f6 is on the opponent's third rank; but it did not capture en passant, since the Black Pawn on the f-file is still present on f7.

"If White is moving left, two Black Pawns, those on f2 and f4, are on the opponent's third rank. But neither captured en passant on the last move, since that would have been a Black move into check. The only other Pawn that could possibly have captured en passant is the White Pawn on f6. White is moving to the right, and the Pawn captured en passant from e5 to f6. Before that capture, both the White Pawn on e5 and the Black Pawn on e6 blocked check by the White Rook. So the check was a doubly discovered check, removing two interposed pieces with one move. *That* is a nice touch."

"It would have been a nice touch," Holmes agreed, "if it had happened. But it didn't.

"What phrase is used to summarize the American organization of government, Watson? The answer should put you on the right track."

READING DIRECTIONS

As Holmes and I approached this table, the Ashleys were making their adieus to Cavendish and Priestley. The Ashleys then moved away from us, the other two toward us. As they passed, I heard Cavendish say to Priestley, "We should get together for a drink. Water, of course."

Priestley smiled broadly, and I laughed aloud, compounding Holmes's obvious perplexity, revealed by his vaporous stare. I reacted to his unspoken question with a casual comment about the chemistry between old acquaintances.

9. HOMOCHROMATIC?
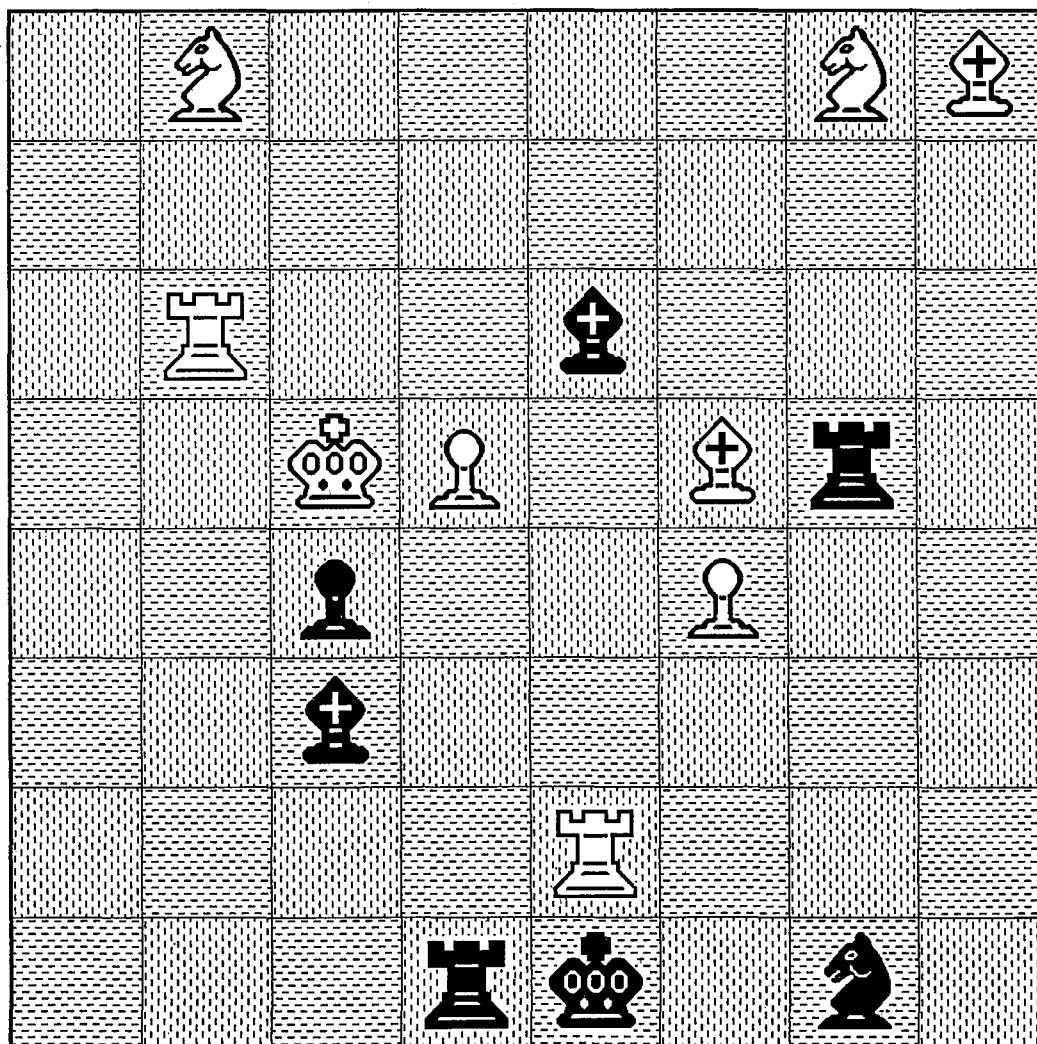
Looking over the now-abandoned position, I recognized familiar elements and said "Well, Holmes, the direction of movement is pretty clear, isn't it?"

Holmes's befuddlement gave way to amusement as he suggested, "Why don't you run through the logic that makes it so clear."

Ignoring his changed expression, I explained, "Though no longer a certainty, it is a reasonable assumption that the Ashleys have played a homochromatic game. The Knight positions are impossible with left/right movement. The three Knights and four Rooks are on squares that could be reached with original pieces only if White is moving down. If White were moving up, all seven pieces would have to be promoted, requiring at least 28 captures by Pawns, while only 16 pieces are missing."

"Frankly, Watson," said Holmes, "I had deduced nothing except that White had made the last move. Oh, yes! And that the game had *not* been homochromatic. That fact does not rule out play by the Ashleys, but I imagine that Cavendish and Priestley were the players."

"Holmes," I exploded. "I suppose there's no absolute proof that play *was* homochromatic. But how can you possibly prove that it wasn't?"

SERRIED RANKS

One of the heterochromatic games to which Lady Ashley led us was this game, on the chameleon board. We recognized the players, Wilde and Shaw, and greeted them. They responded earnestly and flamboyantly, respectively.

Black, with two Queens, appeared to have a distinct advantage. Still, depending on the move direction, which I had not yet determined, White could have one or more Pawns only a move or two away from promotion.

"The board was jostled," the habitually restrained Wilde apologized. "One Pawn of

10. AMBIGUOUS PAWNS
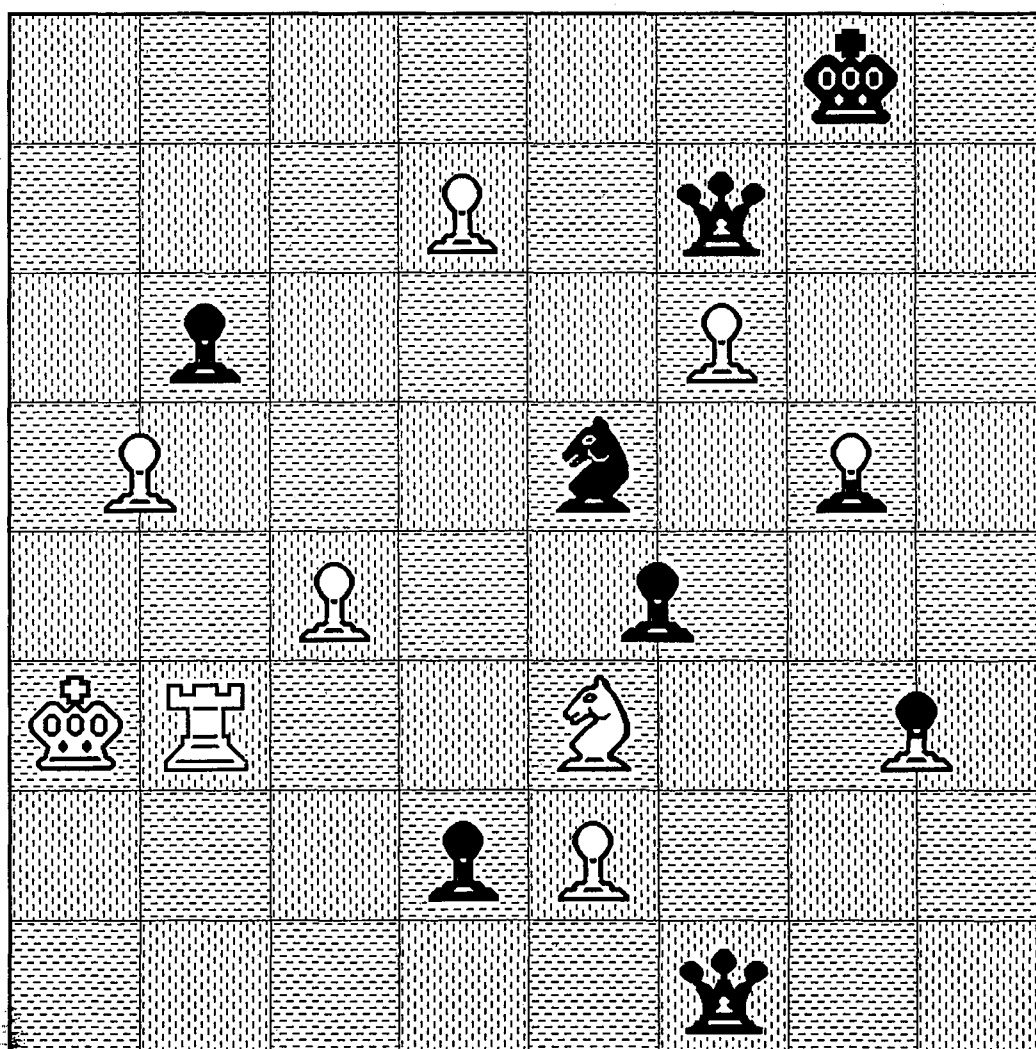
each color is displaced somewhat from the center of its square. Two other Pawns fell from the board and have been replaced with Pawns from spare sets. I see that one of those also needs adjustment. If you will give us a moment, we will set the stage so you can have an unblemished picture of the verities."

Holmes, who had examined the position closely, held up a peremptory hand, "Actually, the board as is may serve better to illustrate some aspects of heterochromatic play. If you will bear with us, Watson and I will attempt to resolve the multiple ambiguities, using retrograde chess analysis. I *will* ask you to confirm that each of the displaced Pawns is partially in the correct square."

Shaw chimed in with an affirmative response, but acknowledged the reasonableness of the question, "You never can tell, when the apple cart has been upset."

Holmes nodded appreciatively, "In that case, resolving the ambiguities is simply a matter of minding the P's and Q's."

I hoped he was right. A White Pawn needed to be placed on either a5 or b5, and a Black Pawn on e4 or f4. A Pawn on g5 could be of either color. Another Pawn of ambiguous color could be on either g3 or h3. And the direction of movement was up in the air. "Holmes," I pleaded, "give me a clue as to where to start."

"A reasonable starting point," he said, "is the restriction of Pawn movements in heterochromatic play."

A SLANTED SOLUTION

Darwin and Wallace were seated at this table, Wallace in the White position. A White Knight on d6 was checking the Black King. Wallace stretched out his hand toward the Pawn placed ambiguously between g3 and g4, obviously to adjust its position, since it was not White's move.

"Please," Holmes requested. "Hold off on adjusting the Pawn position until we have had a moment to absorb the situation."

"At least," I remarked, "there is no question of whether Black can castle."

11. RESTING PIECES
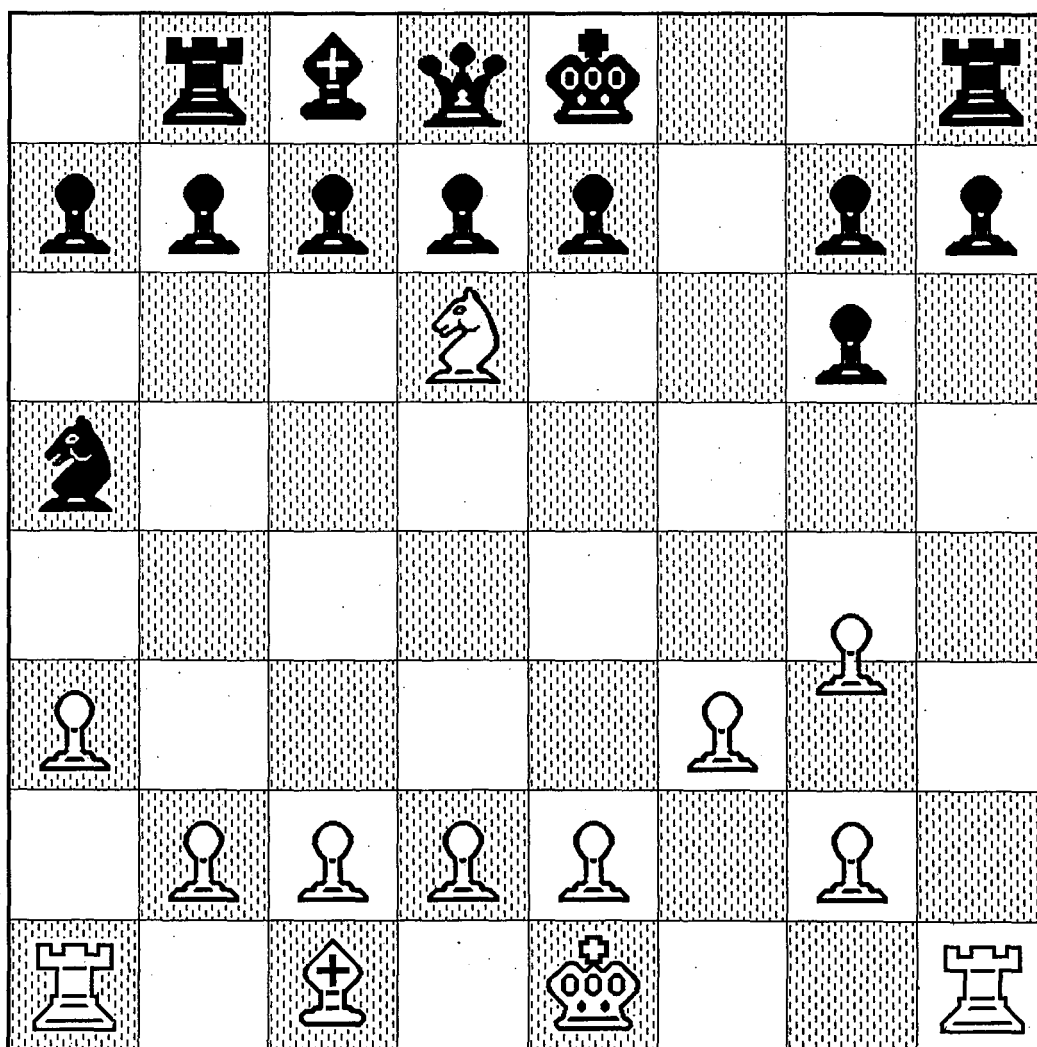
"Not now," Darwin agreed, but went on to observe. "Each of us had the option to castle on the last move, but neither of us considered that a natural selection."

"Even with the castling situation known," Holmes said, "a tidy little problem remains. Considering that we have some missing pieces and one ambiguously located piece, I might phrase the problem as identifying all final resting places."

"That's a relief," said Darwin. "For a moment I thought you might propose to establish the origins of these pieces."

"Possibly in a different observational environment," Holmes commented. Turning back to me, he cautioned, "Although most of the elements are straightforward and familiar, this problem does have a different slant."

LEAPS OF FAITH

From around a corner, a voice I recognized as that of Collins intoned, "The Lord is my shepherd; I shall not want." A ringing "Check!" followed almost immediately.

As we drew closer, Alexander said glumly, "I should have trusted my judgment. I could have castled on my last move, and am sorry I didn't."

I asked Collins, "What is the occasion for the quotation?"

That worthy replied, "In my earlier years I was forced to memorize biblical passages.

12. COUNTLESS KNIGHTS
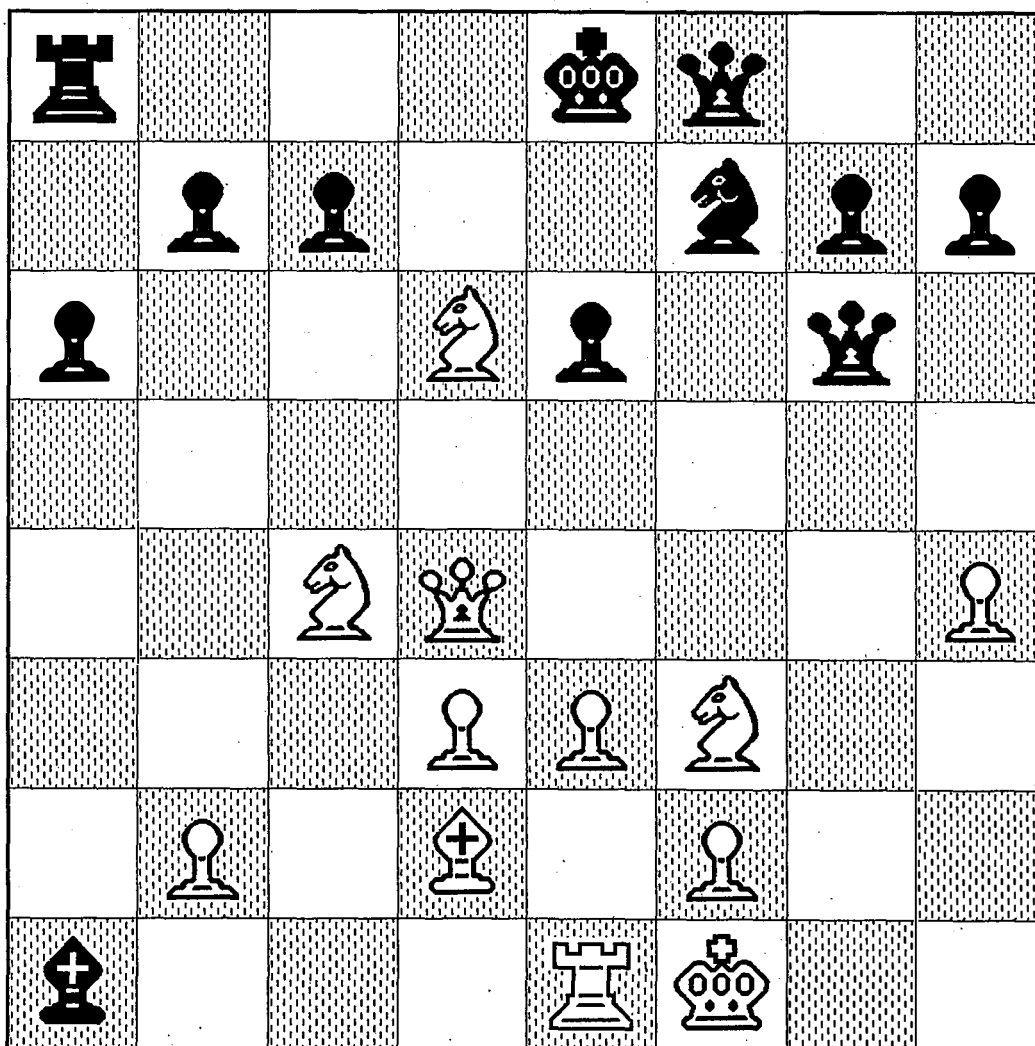
I have been been keeping track of my Knight moves in this game by running through the Psalms in order. I am confident you recognized the 23rd Psalm.''

"Of course," I said. "But by giving that count, you may have deprived us of a retrograde analysis problem. Sometimes it can be quite challenging to determine whether Knights have made an odd or even number of moves; move parity, we call that.''

"Watson," said Holmes. "Collins quoted that Psalm before calling the check, possibly even before making the last Knight move that created the check. Can you be confident he was counting the *imminent* Knight move rather than the *last* he had completed?''

"You mean," I managed, "that he may now have made either 23 or 24 Knight moves?''

"Precisely," Holmes nodded. "And if you will examine the position closely, you may agree that resolving that question requires more than usual care.''

I soon agreed that the problem might be beyond me. "Holmes," I complained, "I learned to count Knight move parity by noting whether the two Knights were on the same or different square colors. How do I apply that criterion of 'same or different' to more than two Knights of one color?''

"You look for an alternative phrasing of the criterion - one more amenable to generalization. Try redefining move parity in terms of a single square color and variable numbers of Knights on that color," he hinted. ''

"It appears," I suggested. "that any of the three missing White Pawns could have promoted, and perhaps more than one of them. I am not comfortable with a third White Knight. The possibility that a second White promotion created a *fourth* Knight, now missing, can only make the problem even more complex.''

"Your implied last question is an excellent place to begin the analysis," Holmes suggested. "How many White Pawns have promoted?''

A CONTINUING PROBLEM

Lady Ashley was leading us in a search for further heterochromatic games. At table after table she peered at the position and shook her head. "Their game wasn't heterochromatic this time."

We followed her as she wound among tables, paused to ask a question, and finally said, "Here. We have a heterochromatic game to show you, once more on the chameleon board."

The players, Brown and Foreman, stood to greet us. "We were debating what to do

13. FALLEN QUEEN
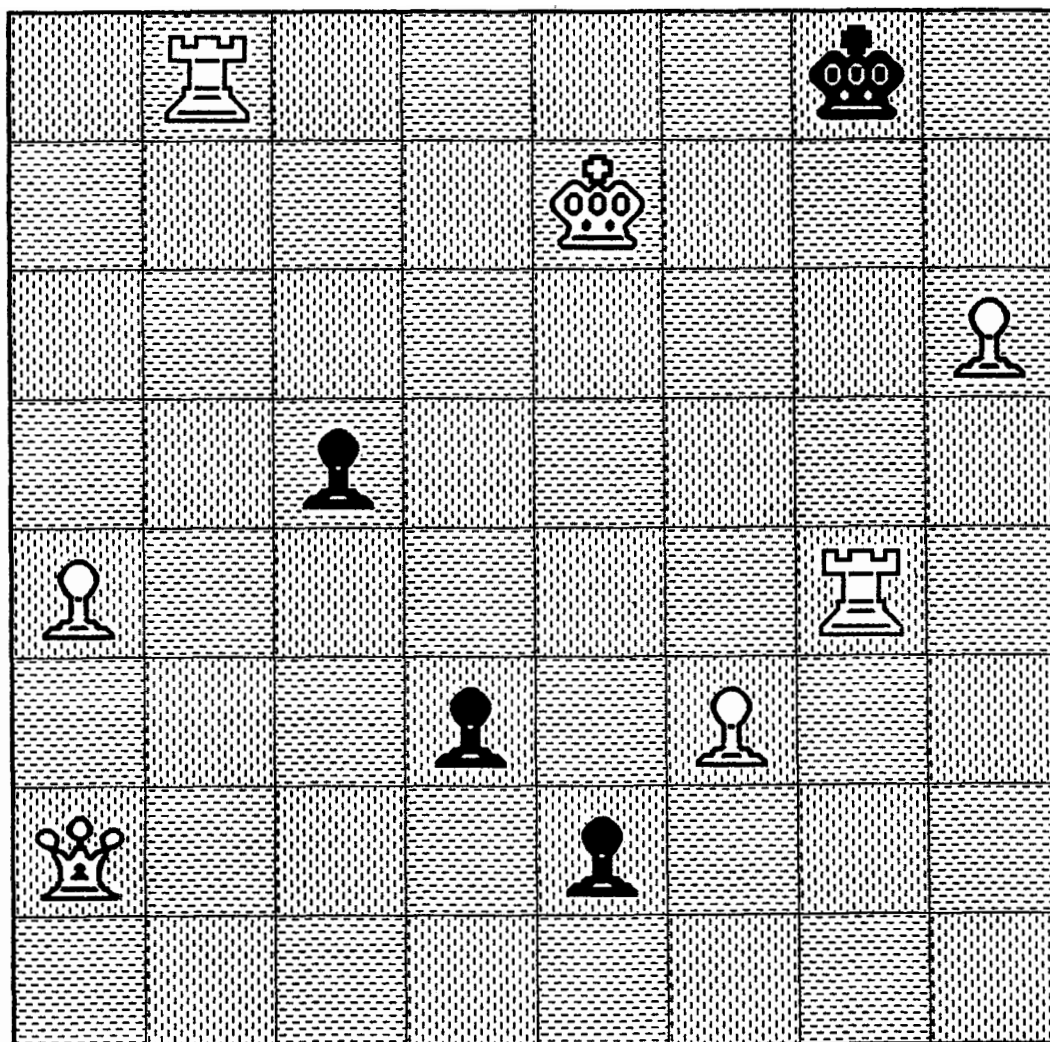
about this game," said Foreman. "As Lady Ashley informed you, we have been playing under an agreement to limit ourselves to heterochromatic moves, each move being between two squares of opposite colors."

"Yes," Brown chimed in. "And one piece has just fallen from the board - the Black Queen. We were having a spirited debate over whether to barrel along or discontinue the game."

"Surely," I said, "You can keep track mentally of the one fallen piece."

"That isn't the total problem," said Foreman, and would have continued but for an interruption by my companion.

Holmes, who had been drinking in the position during this exchange, said "The direction of movement contributes to the players' concern. However, placing the fallen Queen properly exercises an aspect of heterochromatic play, and that problem can be solved without pinning down the direction of movement. Would you like to have a go at the problem, Watson?"

I had distilled the essence of the situation, and the solution came to me rather quickly. Of course, it *was* a simple problem.

Still, Holmes had to explain why placement of the fallen Queen was not the total problem.

IS CASTLING EVER ALLOWED?

Early in the evening, Holmes and I stopped to chat with Stanley and Livingstone, who had just arisen from a game with this position.

"We are pleased that you have discovered the Chameleon Chess Club, gentlemen," said Stanley. "The variety of play will offer fresh problems to maintain your zest for retrograde analysis."

"It already has," Holmes acknowledged. "In addition to the chameleon board itself, we have been introduced to heterochromatic play. I am confident that we have not yet

14. PLACES FOR ALL
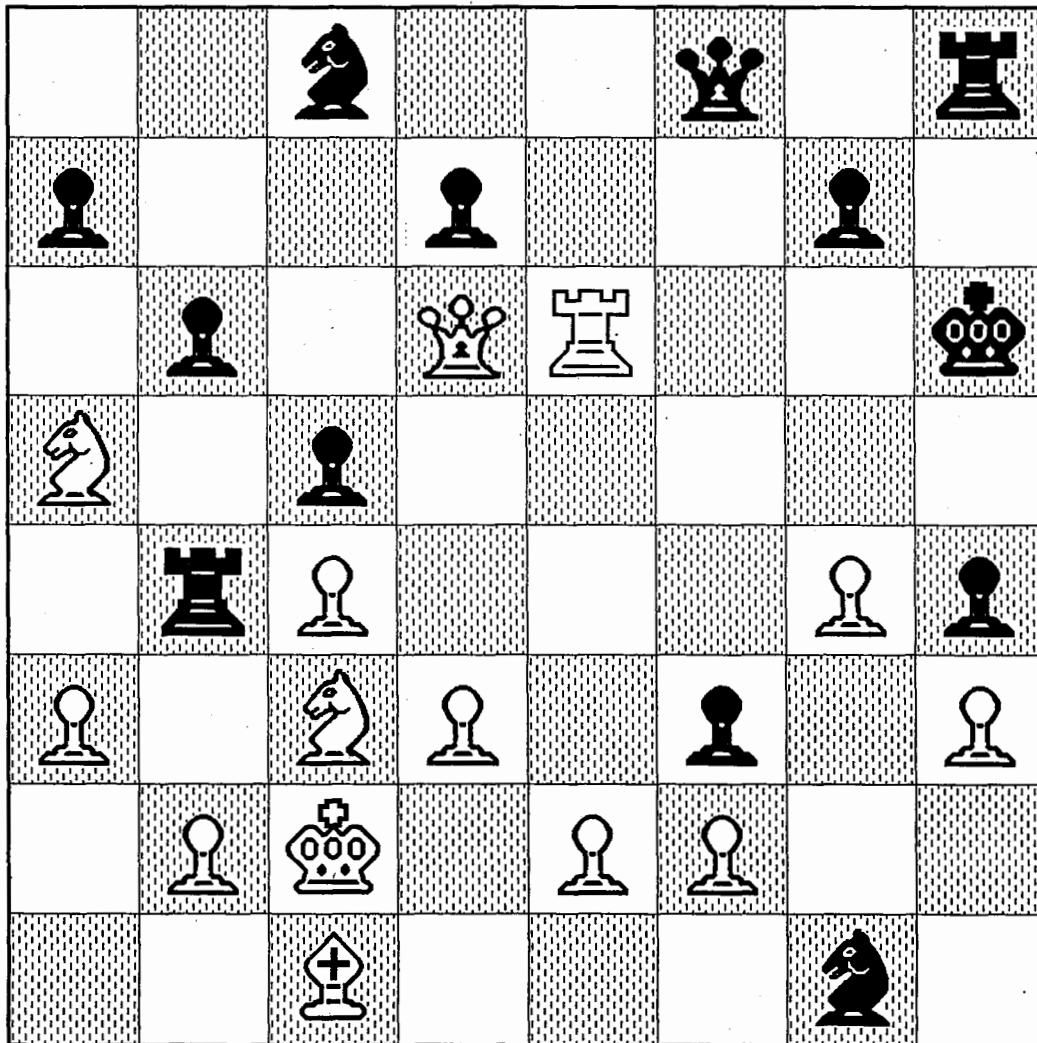
explored the full scope of problems permitted by that game form."

"*Our* game was heterochromatic," said Livingstone. "If that form of play were newer to us, this game would have raised a question: Can castling occur in heterochromatic play?"

"Livingstone, I presume you have discovered an answer," Holmes mused. "King-side castling is homochromatic. But Queen-side castling is ambiguous, the King moving homochromatically and the Rook heterochromatically. What did you conclude?"

"With other enthusiasts, we reexamined the definition of a heterochromatic move as one between squares of opposite colors. Logically, "between" applies to two alternatives or two limits. 'Between you and me and the gatepost' is colorful but illogical. Castling involves four squares, and definition in terms of 'between' seems inadequate."

"Were you able to agree on an expanded definition that does address castling?" I asked.

"We recognized two basically different views consistent with the understanding of heterochromatic movement of a single piece. We could require *each* moving piece to change square color, barring castling in heterochromatic play. Or we could call a move heterochromatic when *one* piece changes its square color, and admit Queen-side castling.

"It may surprise you, Holmes, to learn that *you* influenced the final decision. You had mentioned to Lady Ashley that there should be a heterochromatic game as a complement to the homochromatic form. A philosophy don stated that, to be logically complementary to the homochromatic game, the heterochromatic game must allow every legal chess move that the homochromatic form excludes, and vice versa. We found that argument compelling, and agreed that Queen-side castling must be considered an allowable heterochromatic move."

"I am thankful that you did," said Holmes. "Although I don't see exactly how it would arise, retrograde analysis would be poorer if any form of chess forbade castling."

"I apologize, Holmes," said Stanley. "I was about to tell you whether castling actually occurred in this game, and might have spoiled your pleasure in analyzing the play."

"Please don't apologize," Holmes urged. "The allowability of castling is knowledge we could need at some point. I *do* want to know whether any odds were given in this game."

The players assured us that no odds had been given, made parting comments and went for refreshments. Holmes and I proceeded to explore the position they had left.

"Can you tell whether either player castled?" I asked.

"White clearly did not, since the original Bishop on c1 has always been there, preventing Queen-side castling. It is impossible to determine whether Black castled. Nevertheless, the discussion does point the way to a more satisfying basis for defining heterochromatic play, which places Queen-side castling on a stronger foundation.

"And that foundation is pertinent to the solution of a different problem. Where was each missing piece captured?"

BISHOP'S INFORMATION

As Holmes and I approached a table with a chameleon board, Shannon moved his White Rook from b4 to g4. Then he and Weaver paused to greet us.

"Our game has been heterochromatic," Shannon communicated.

"Then the Bishop at h1 is promoted," I observed.

"Yes," Weaver gave the completely predictable answer. "Each of us has promoted at least one Pawn."

15. EACH HAS PROMOTED
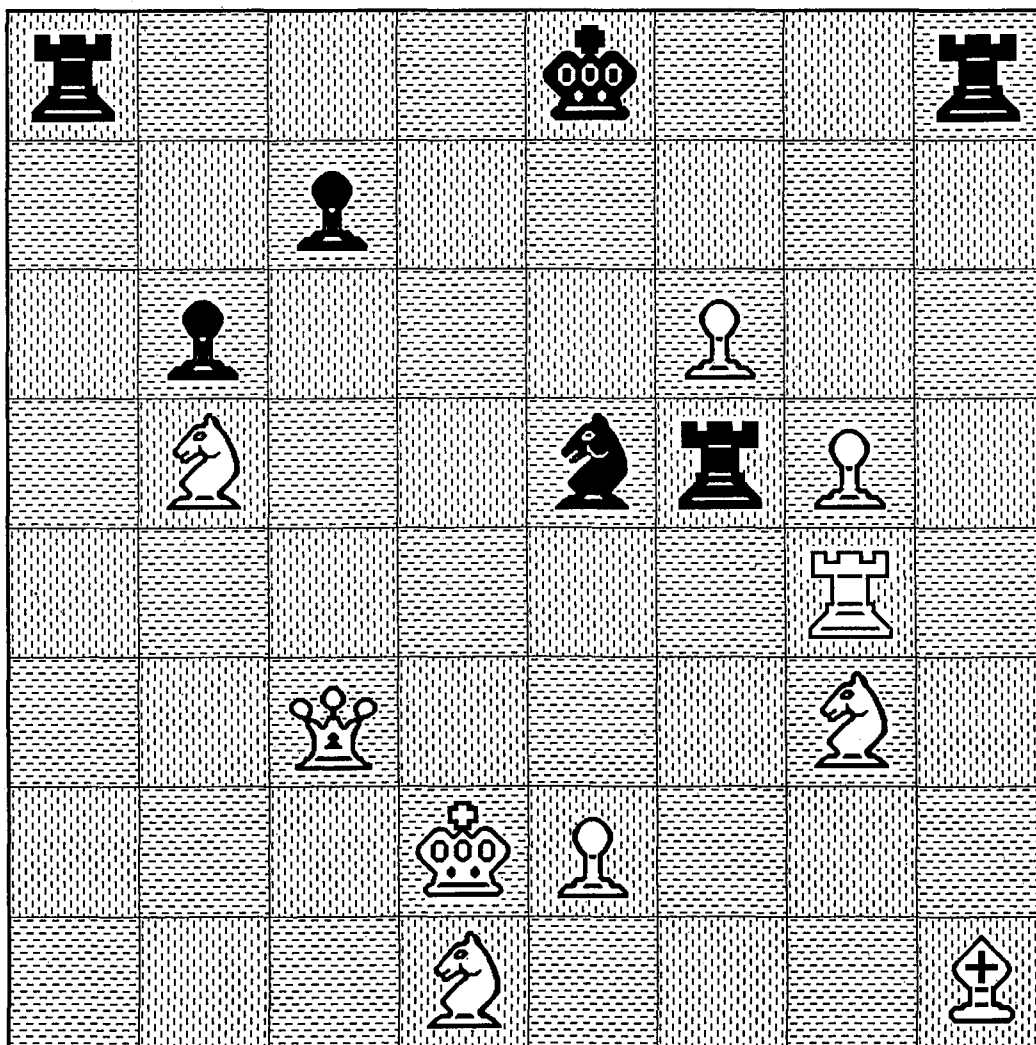
Holmes mentioned with approval a learned article on which the two players had collaborated. Shannon, in turn, complimented Holmes on helping to put heterochromatic castling into perspective.

"The present position," I interjected, "with Black having the move, raises the interesting question of whether Black can castle. Of course," I backpedaled, "it is not absolutely certain that Black is moving down the board."

Weaver would have responded to the implied question, but Holmes held up an admonitory hand. "Please, not another single bit of information. Let us allow Watson to examine the position more thoroughly. I am confident he can answer his own question, through retrograde analysis."

That was my cue to delay no longer.

ODD COORDINATES

Lady Ashley insisted on leading us to a corner table. "There is a heterochromatic game that I know will interest you."

Arriving at her destination, she scanned the board, on which a Black Knight was perched ambiguously at the corner of four squares, and said, "No move was made while I was away, so the game remains heterochromatic."

It was unclear which player had the move, though proper placement of the Knight well might answer that question. One of the possible locations would place White in check by

16. KNIGHT SITE
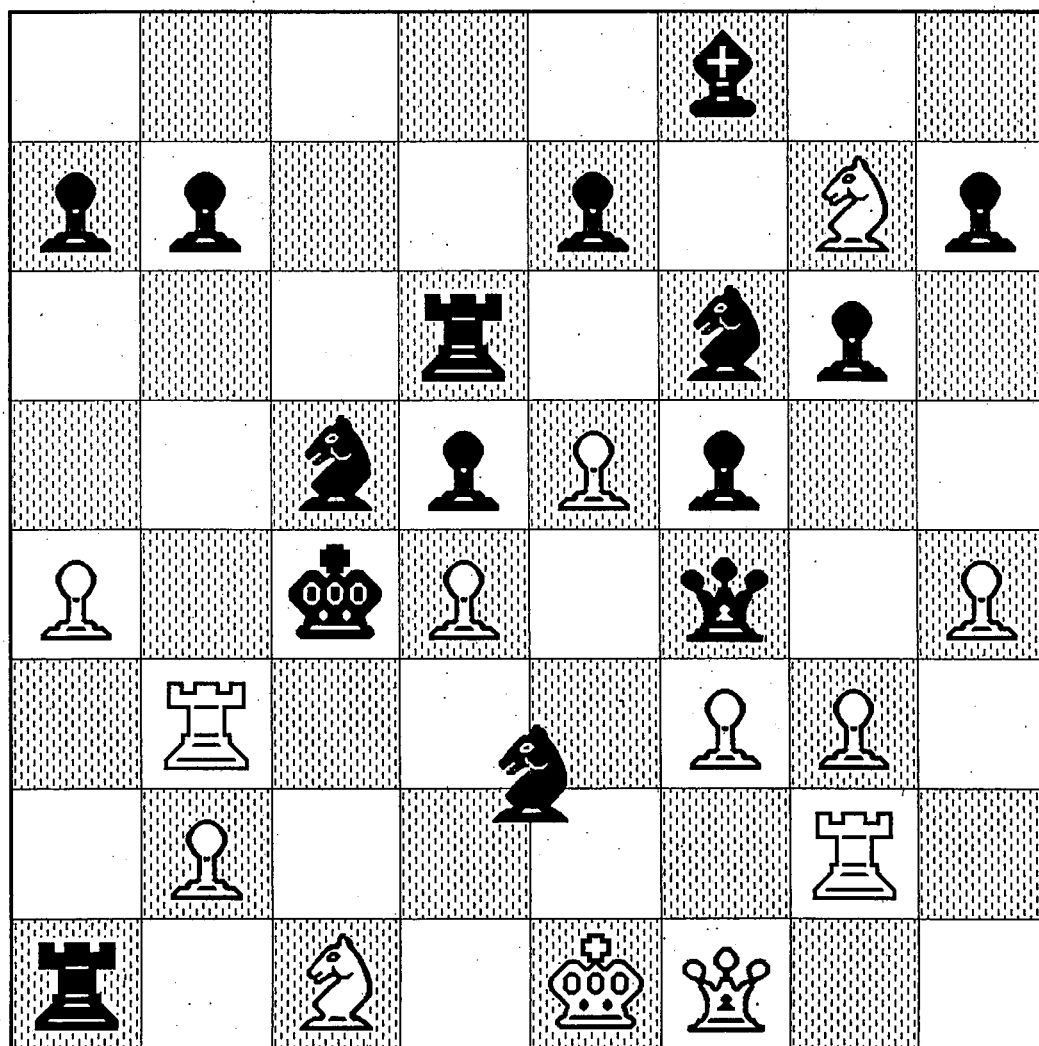
the Knight, and two would leave Black checked by the White Queen on f1.

Rand, one of the players, mapped out the situation, "White gave the Queen's Bishop's Pawn as odds."

His opponent, McNally, added, "We were about to adjust the displaced Knight. Perhaps you would like to do so, Holmes?"

"Thank you for giving me that latitude." Holmes looked as if he were reading a book, his eyes scanning the board row-by-row. His survey completed, he nodded in satisfaction and nudged the Knight to the center of one of the four squares.

"I did not expect that location," I said. "It seems odd."

My friend's reply gave me the key to duplicating the balance of his analysis: "It would be odder if it were even."

BLACK CAN CASTLE

Eliot and Sand were pondering this position on the chameleon board. A marker on f7 provided no clue to the color or identity of the piece it represented.

I commented to Holmes, speaking softly to avoid disturbing the players, "Assuming the direction of motion is the obvious direction, the Black King occupies its home square, and Black Rooks are at the two home Rook positions."

Holmes replied, also sotto voce, "With Pawns on both bounding columns, play cannot be left/right. And the number of available captures is insufficient to allow White to be

17. WHAT IS THE QUESTION?
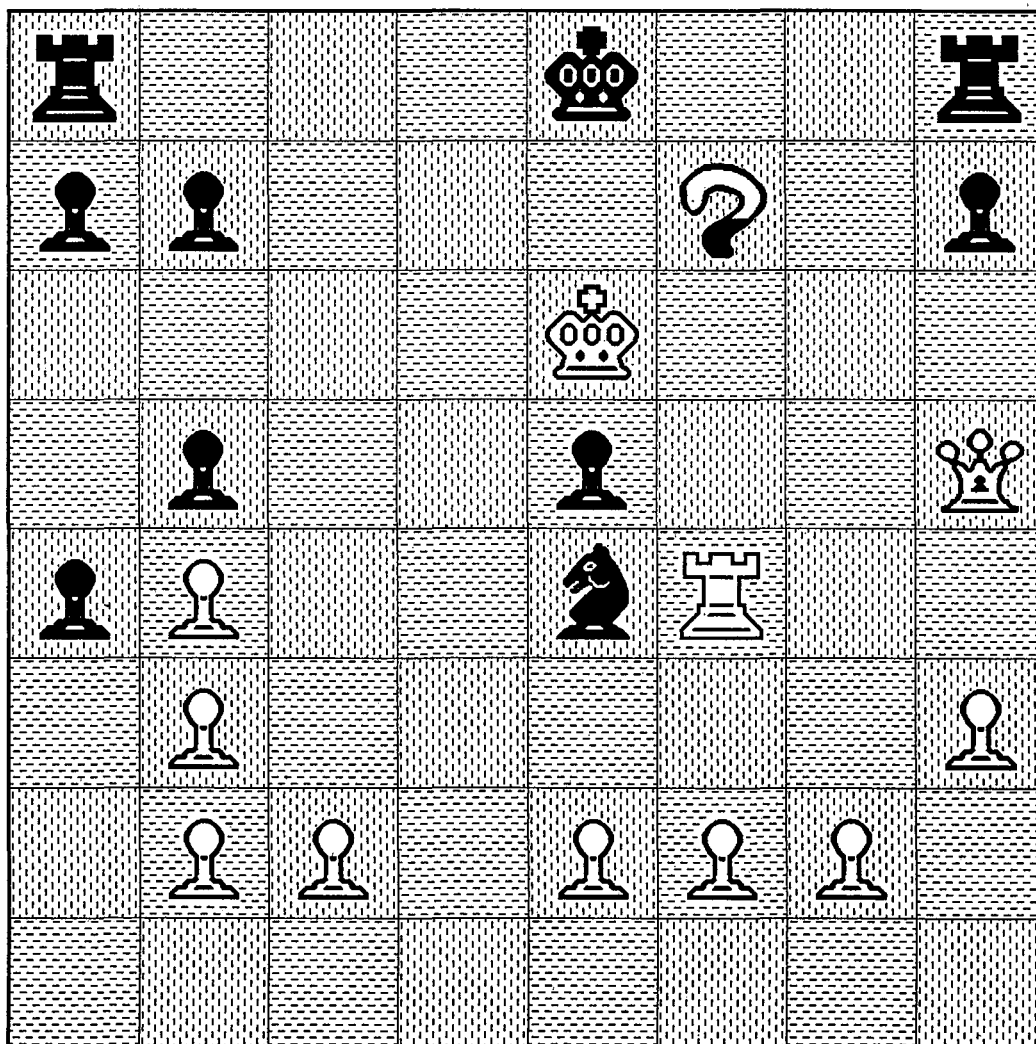
moving down the board, no matter how the bypassing captures are divided between the two sides."

"Then I wonder if Black can castle?" I whispered to Holmes.

He responded, "That could depend on what piece the marker represents. And, remember, a Black Pawn may have promoted."

We must have spoken more loudly than I had supposed. Eliot, playing Black, looked up and remarked, problematically, "As a matter of fact, gentlemen, I *have* promoted a Pawn. Moreover, I can castle on either side."

Holmes commented dryly, "Well, by George, that clears up all of the questions, including implicitly what piece the marker represents. But you might want to identify the piece explicitly, Watson."

NAME, RANK AND FILE

As we approached this table, Tennyson grabbed wildly at something near the floor. Noticing us, he said, "The Rooks are blown about the sky ... Or cast as rubbish to the void."

"You can't take Tennyson literally," said Locksley. "In the course of castling, he knocked a piece from the board; it wasn't necessarily a Rook, and it's on the floor rather than blowing around the sky."

Holmes examined the board while I tried to help Tennyson and Locksley find what

18. WHAT FELL FROM WHERE?
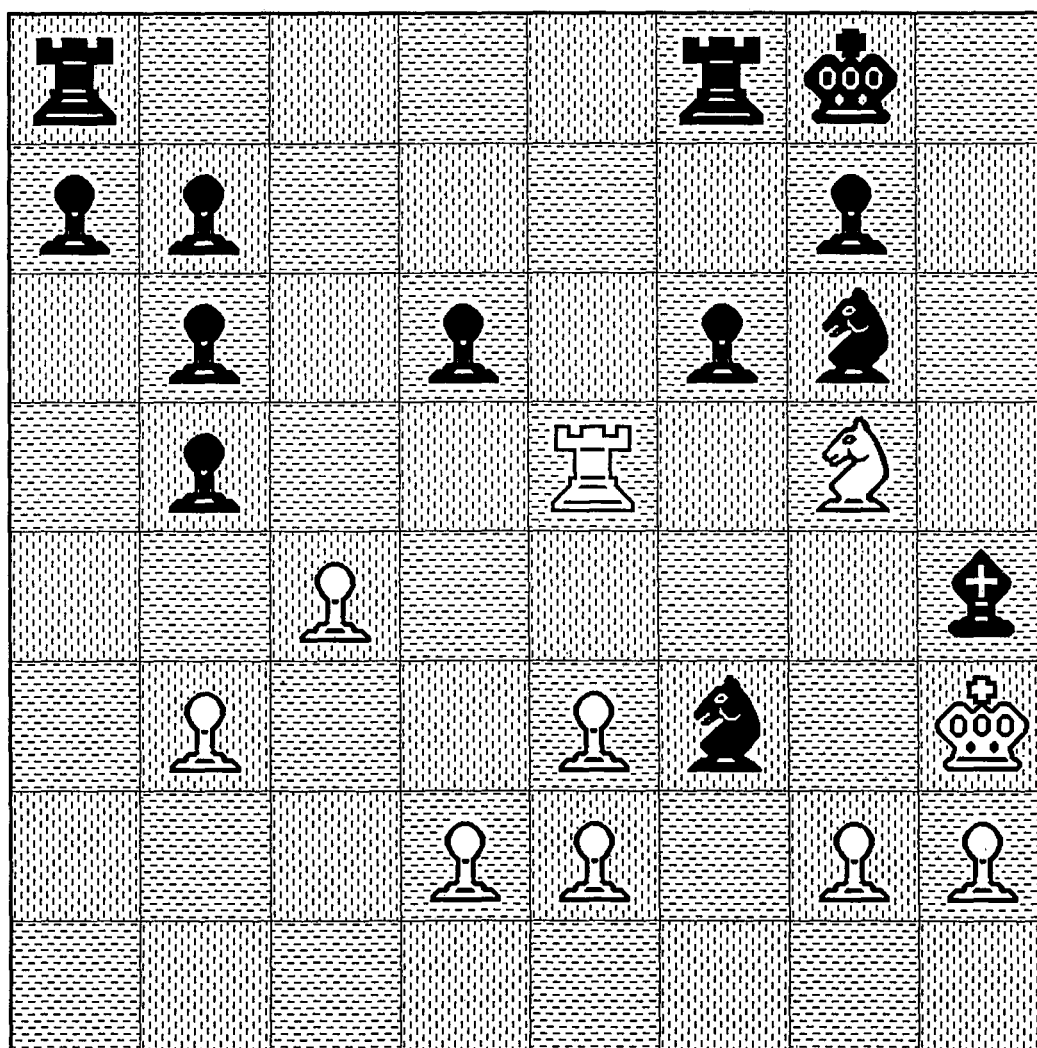
had fallen. Tennyson said, "I stretch lame hands of faith, and grope, and gather dust and chaff." Locksley and I were less colorful, but equally unsuccessful.

"We had best use a coin to track the piece for the rest of the game," said Locksley.

"I will provide the coin," said Holmes. "And, if you can suspend play for a few minutes, I should like to see if Watson can place the coin on the proper square, and identify the piece it represents."

"There methinks would be enjoyment," said Locksley. "Watson, read the wide world's annals, you, and take their wisdom for your friend."

Holmes, eyebrows raised, silently handed me a coin, and I studied the problem of its placement and identity.

DIRECTION POINTERS

Lord and Lady Ashley were standing by the chameleon board, conversing with Poundstone. They interrupted their discussion to greet Holmes and me.

"I assume this has been another homochromatic game, Lady Ashley?" I inquired.

"I really don't know, Dr. Watson," she replied. "This is a game Mr. Poundstone was playing."

"With Rutherford," said Poundstone. "But, in truth our play *was* homochromatic.

19. PROMOTED AND DEPARTED
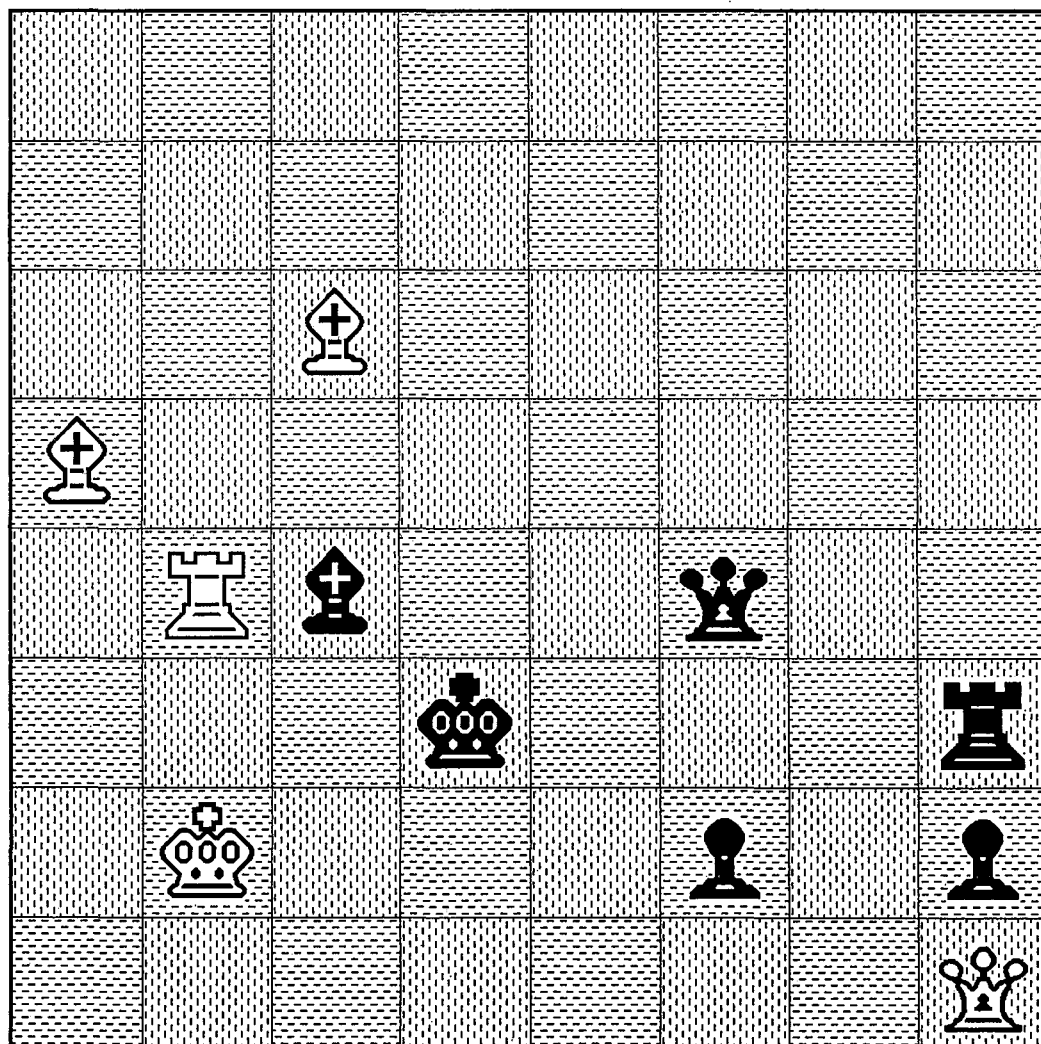
Still, I'm afraid I was merely going through the motions, and remember no specific details that might interest you.''

"Not even the promotion of a Pawn, and subsequent capture of the promoted piece?'' Holmes prompted.

"Have you been talking with Rutherford?'' asked the astonished Poundstone. "Yes, those events happened. But it is inconceivable that you could know so without being told.''

"I am astonished also, Holmes," I said, "though not at your ability to deduce a promotion, of course. But, with so many pieces on the board now, it does seem farfetched that you can be certain a capture was specifically of a promoted piece, rather than an original piece that was subsequently replaced through a promotion."

Holmes remained unperturbed. "Watson," he said, "all should be apparent once you have determined the direction of movement."

"At least, the Pawn on h2 rules out left-right movement," I said.

"Not necessarily," Holmes insisted. "If Black otherwise could be moving right, the position could reflect an incomplete last move. The last move could have been a capture from g1 or g3 by that Pawn, the move not yet completed by exchanging the Pawn for a piece. But there is a more reliable basis for limiting the possible move directions."

CAN BLACK COPY?

Wells, as White, castled as we approached the table where he was playing against Ellison. "I wonder if Ellison can castle," I whispered to Holmes.

I found it gratifying that Holmes required more than a quick visual scan to satisfy himself with the answer to that question. "Considering his last move," said Holmes, "is the key to what he can do on his next move."

"The Knight on a1 did not make the last Black move," I stated. "It could have moved only from b3 or c2. White Pawns occupy both squares, and did so before White's last

20. BOTH CASTLE?
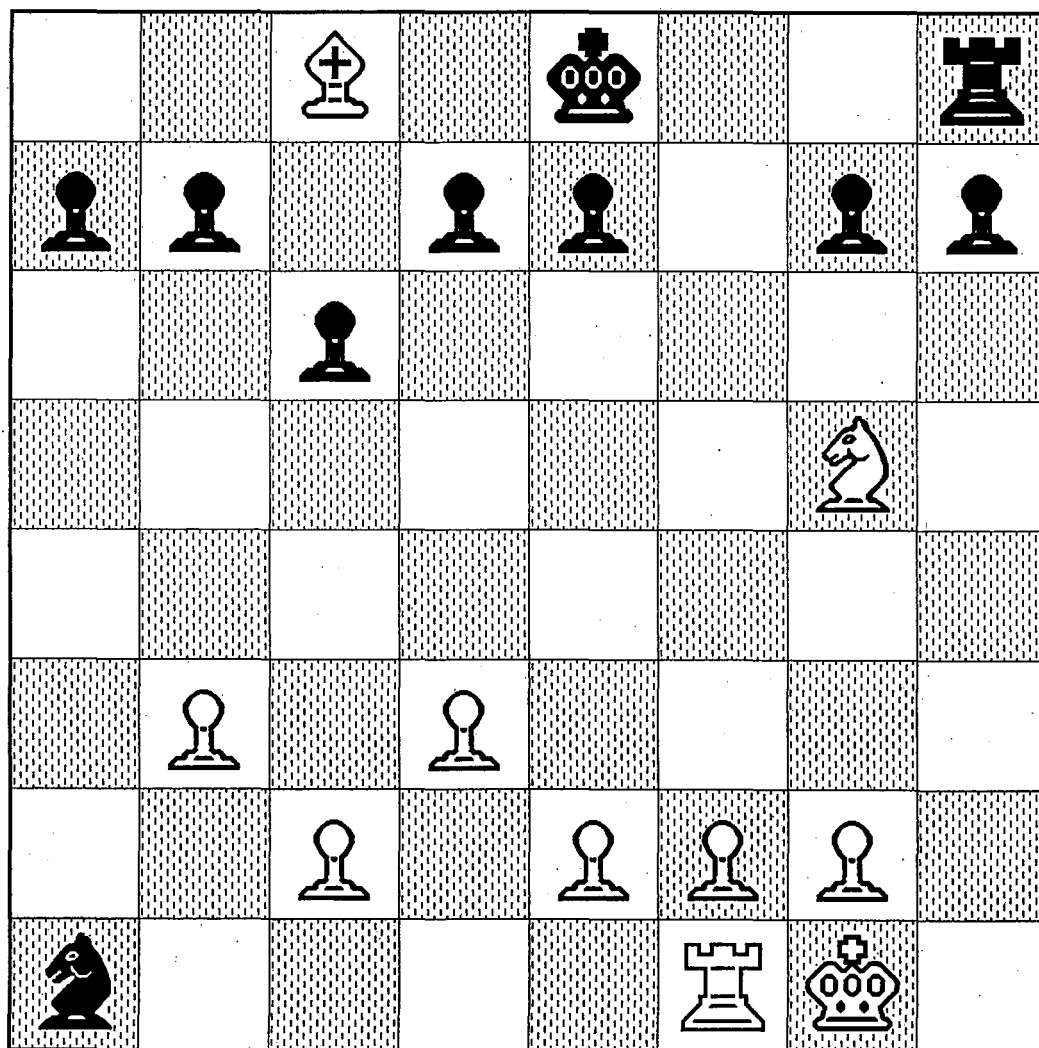
move, which was castling.

"The Black Pawn on c6 did not make the last Black move. The White Bishop on c8 obviously is a promoted White Pawn. An original Bishop could not reach that square; moreover, the original White Bishop on white squares was never able to leave f1, but was captured there. The Black Pawn from c7 had to move before the promoting White Pawn could reach that square en route to promotion.

"That leaves only the King or Rook to have made the last Black move, and castling is ruled out."

"Not so fast, Watson," said Holmes. "I am not 100% certain that the last Black move was by the King or Rook. As the poet says, 'Send your soul through the invisible.'"

I looked suspiciously at Wells and asked, "Has any piece fallen from the board without being replaced?"

"Why, no," he replied. "What a strange suggestion."

I had to look elsewhere for the invisible.

LAST MOVE SEEN

Lady Ashley had informed us that Cholmondely and Taliaferro frequently played heterochromatic games. I spotted them at a distant corner table just as Taliaferro was completing a Queen move, and persuaded Holmes that we should explore their game.

As we drew nearer, it became obvious that the White Queen move had been to b4. After we exchanged greetings, I commented, "I saw the end of that Queen move; am I correct in believing it was a capture?"

Cholmondely glumly (I apologize for that rhyme) confirmed that his strength had just

21. CAPTURE AND CASTLE?
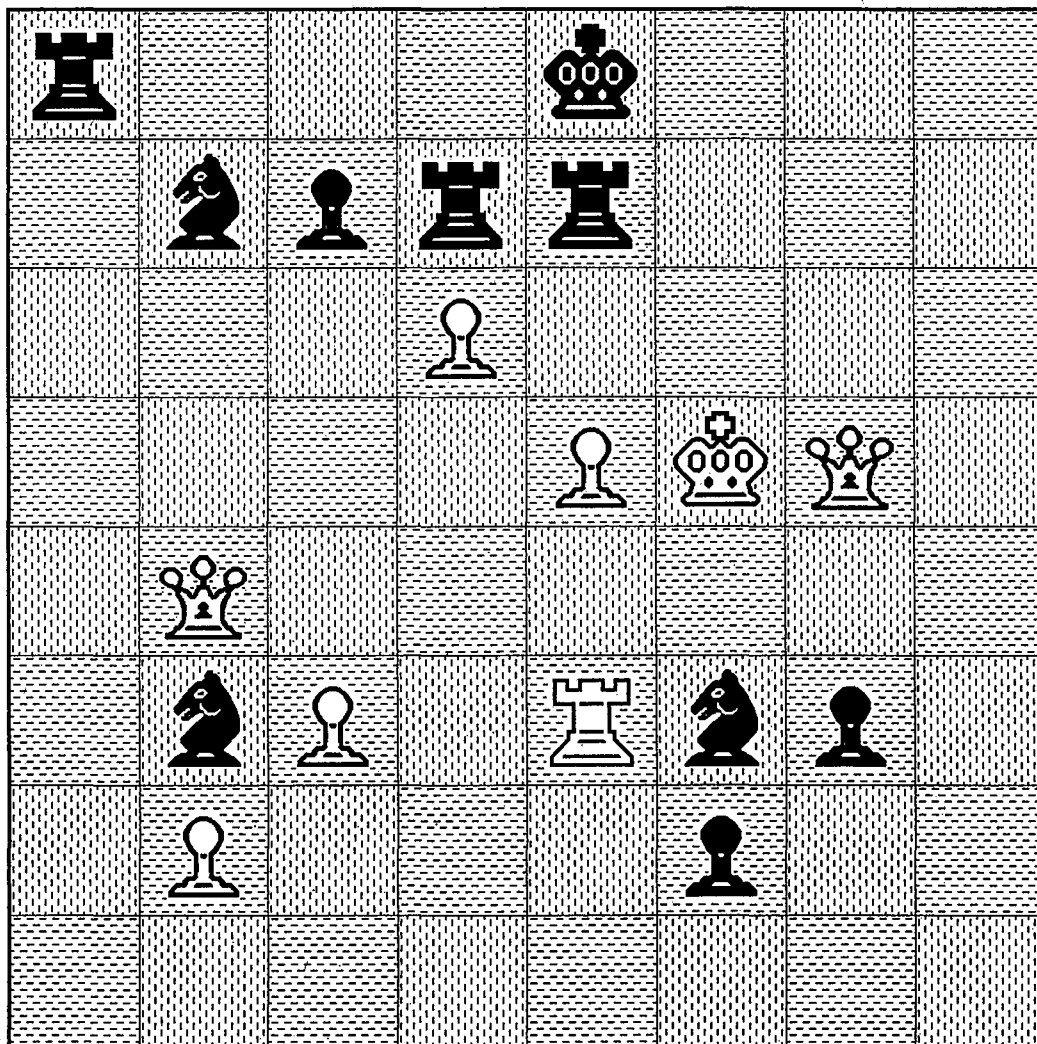
been depleted, and was stopped by Holmes before giving more information.

"Was your play heterochromatic, gentlemen?" Holmes inquired.

"Yes," said Taliaferro, "with no odds given."

I noticed that the Black King and its Queen's Rook were on their home squares, and said, "If castling is to take place in this game, it clearly will have to be by Black. Of course, with the game so advanced, castling may no longer be legal, to say nothing of its advisability at this time."

"Actually," Taliaferro pronounced, "I did castle much earlier in the game."

Holmes forestalled any comment from Cholmondely by stating, "Watson, the question of legality of Black castling is answerable by retrograde analysis. And a byproduct of that analysis will be the identity of the Black piece captured on the last move."

A SUDDEN DEPARTURE

Eastman stood by a table, giving Land a pat on the shoulder as the latter departed. "Called away suddenly," Eastman snapped; "I asked him to let me know what develops the instant he has the full picture."

"I hope it's nothing too negative," I murmured. Holmes was focusing on the chess position on the chameleon board. Eastman chose not to give the subject further exposure, but addressed the game in progress.

"I see that Land jostled the board in his haste," Eastman said. Enlarging on that

22. POSITIONING MOVE
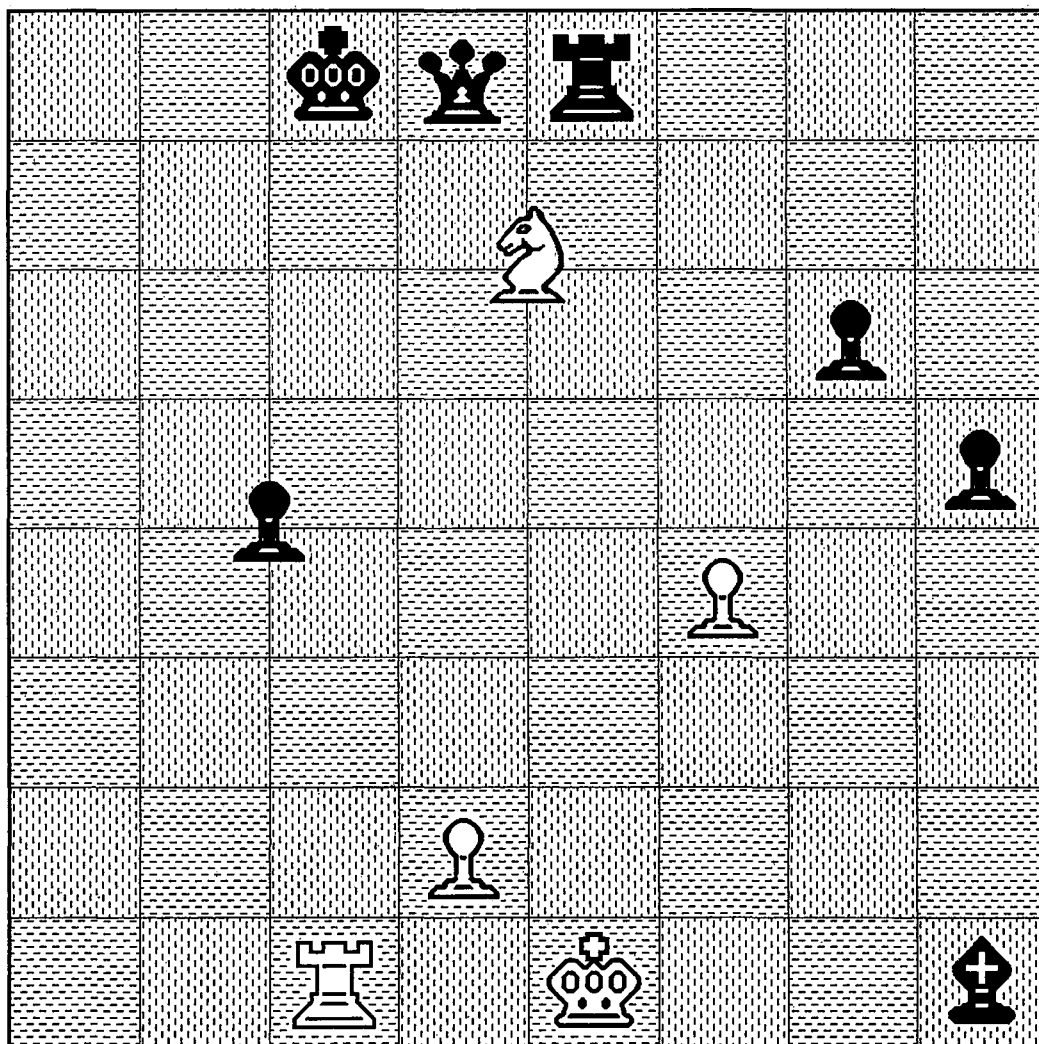
comment, he pointed out, "A Black Pawn and White Knight are displaced, though each is partially in its proper square. Perhaps you can project where they belong."

I shifted my field of view to the board. Depending on the location of the Black Pawn, the Black King might or might not be in check by the White Rook on c1. And the White Knight might or might not block a check of the White King by a Black Rook on e8. "One of the two displaced pieces," I noted, "must block a Rook check. Resolving their placements could well define who has the move."

Holmes fidgeted, frowned slightly, and asked, "Was the game played with restrictive conditions?"

Eastman shone light on that subject, "Yes. Play has been heterochromatic."

"Then, Watson," said Holmes roguishly, "you have the cart before the horse. Determining who has the move can lead to resolution of the placement ambiguities."

A QUEEN HAS MOVED

Kinsey, playing White against Pomeroy, castled as we stopped beside a conventional board. I observed to Pomeroy, "Clearly, you also have castled."

"Yes," Pomeroy agreed. "And not long ago. But I have made four moves since castling, even moving my Queen."

Said Kinsey, widely known to be Pomeroy's superior, "In the event that you are trying to analyze the game, you should be informed that I gave odds of the Queen's Knight and Queen's Rook."

23. WHEN DID KNIGHT FALL?
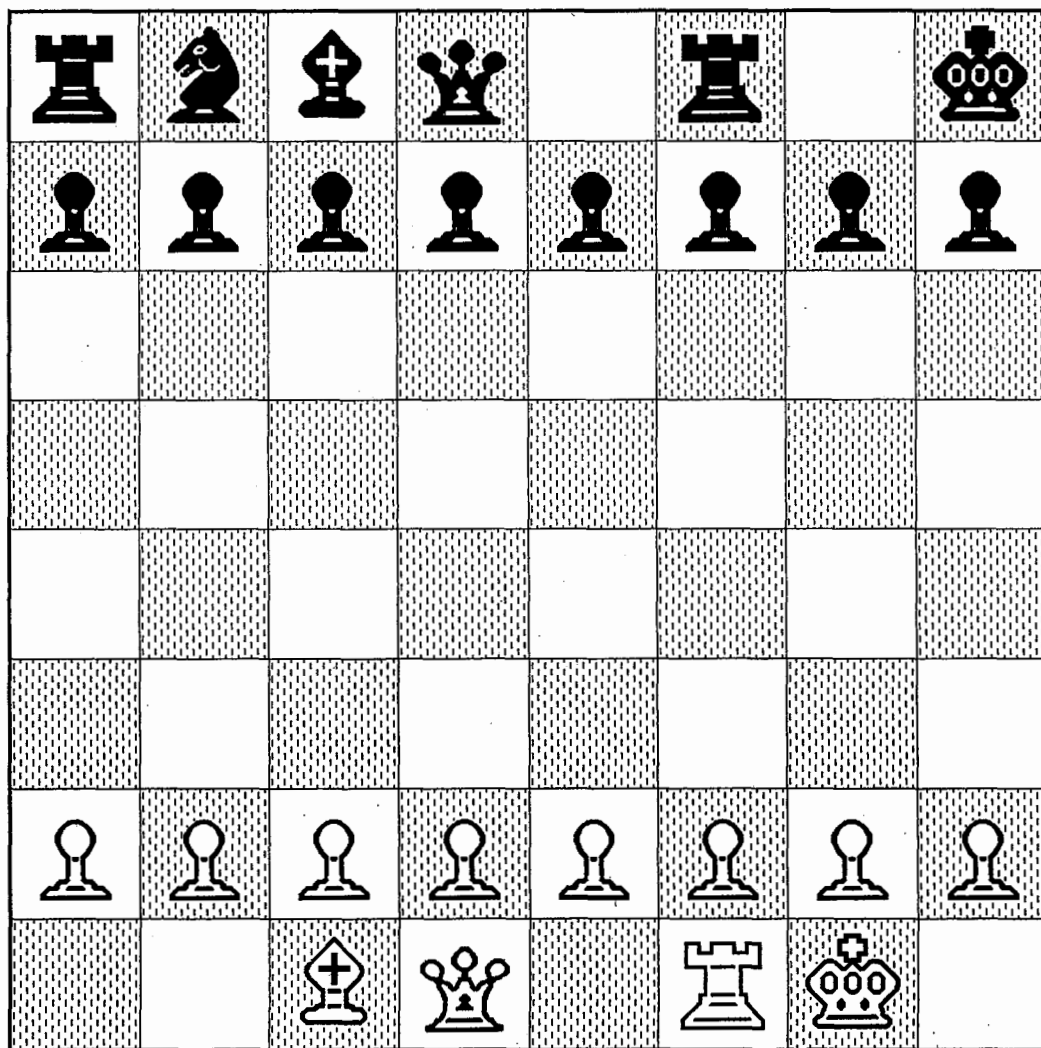
Holmes surveyed the position and shook his head, "There doesn't seem to be much basis for retrograde analysis here."

"That's too bad," said Kinsey, rather waspishly. "There is one aspect of play on which our memories are a little uncertain. We know that the first capture of a Knight occurred on either the 17th or 18th move of the game, and hoped that you might be able to tell us which."

Holmes perked up like a Dalmatian hearing a fire klaxon. I muttered, "You might say the game is ahorse."

His piercing eyes darted over the board, appearing to linger over the pieces on Black's home rank. Before long, he nodded in satisfaction and said, "You could have asked many questions about past events that would have embarrassed me. But I am able and willing to satisfy your curiosity over one Knight.

"Although he will encounter an intriguing element of novelty here, Watson has had much pertinent experience. Let us see if *he* can rise to the occasion, before I address the challenge."

PAWN OFF

As we neared a table with a chameleon board, Landor was peering under the table and inquiring, "Where art thou gone, light-ankled Youth?"

His opponent, Hood, explained condescendingly, "Landor is incapable of simple prose. A Pawn has fallen, some would say thrown from its eminence."

We volunteered to assist in the search, but Hood shrugged fatalistically, and said, "Make no deep scrutiny. A coin can replace the piece." Perhaps lacking a coin, he made no effort to suit action to word. Or perhaps Holmes had urged delay, with a silent signal.

24. WHAT COLOR?
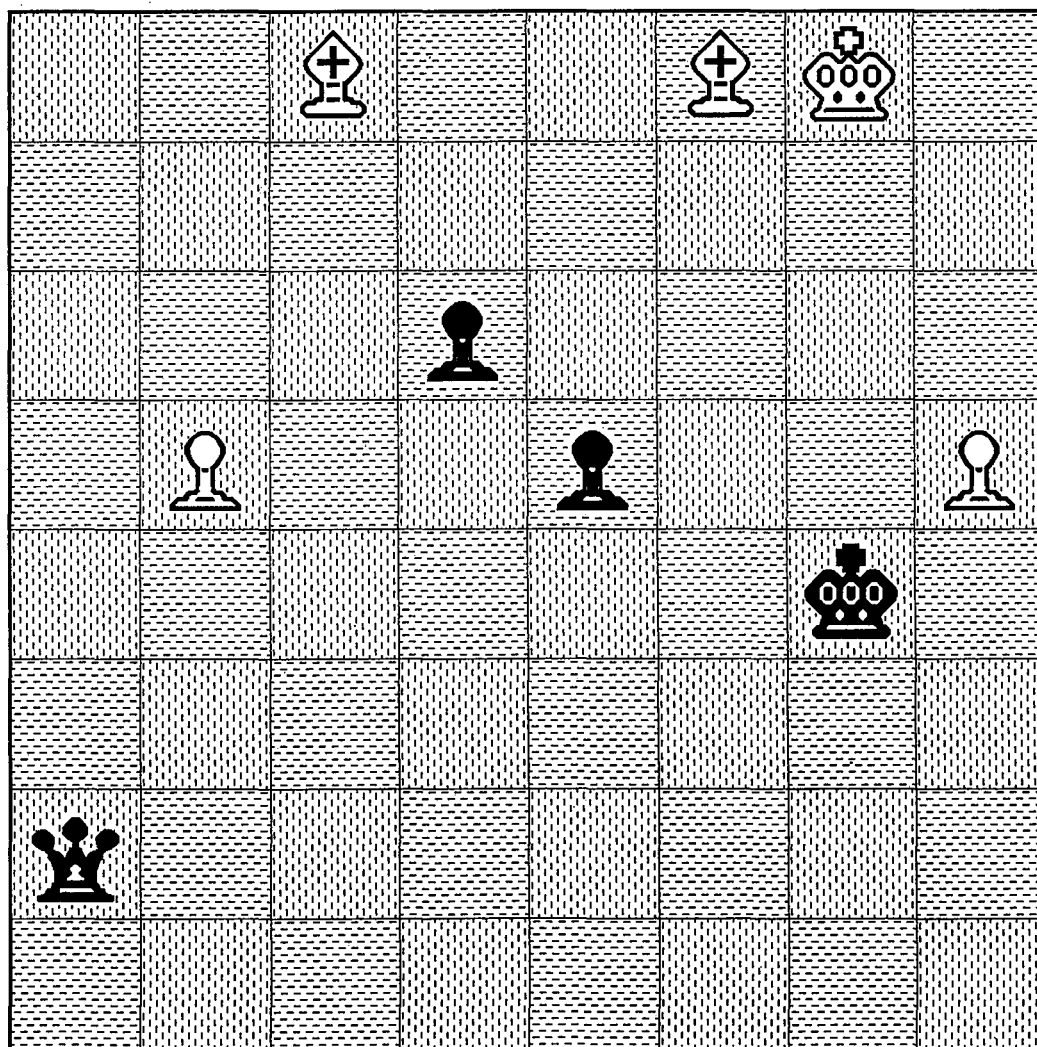
Landor, abandoning his search, informed us, "The game has been heterochromatic, and it is my move that has been postponed."

"A respite however brief," said Hood.

With so few pieces remaining on the board, it was not at all certain who was playing White or what the direction of movement was. Perhaps those matters would become clear once a marker for the fallen Pawn was in place.

I was about to suggest that the players use a captured Pawn to replace the one that had fallen. Before I could get the words out, Holmes said, "Watson, you should be able to determine what color the fallen Pawn is, and who is playing White."

"I am not even certain about the direction of movement," I protested indignantly.

Holmes roared with laughter, "I didn't ask you about that."

A MOVING ARGUMENT

Lady Ashley, who had left in search of other promising games, rejoineded our group, but waited patiently until Holmes had completed the description of his analysis and accepted the compliments of the players. Then she said, "Mr. Bayes and Mr. Savage are playing a heterochromatic game that might interest you."

She led us, winding between tables, until we spied Bayes and Savage. As we neared the table, Savage played a Black Knight to square e1, capturing a White Rook.

After Lady Ashley ascertained that play had remained heterochromatic, Bayes greeted

25. GET THE NUMBER
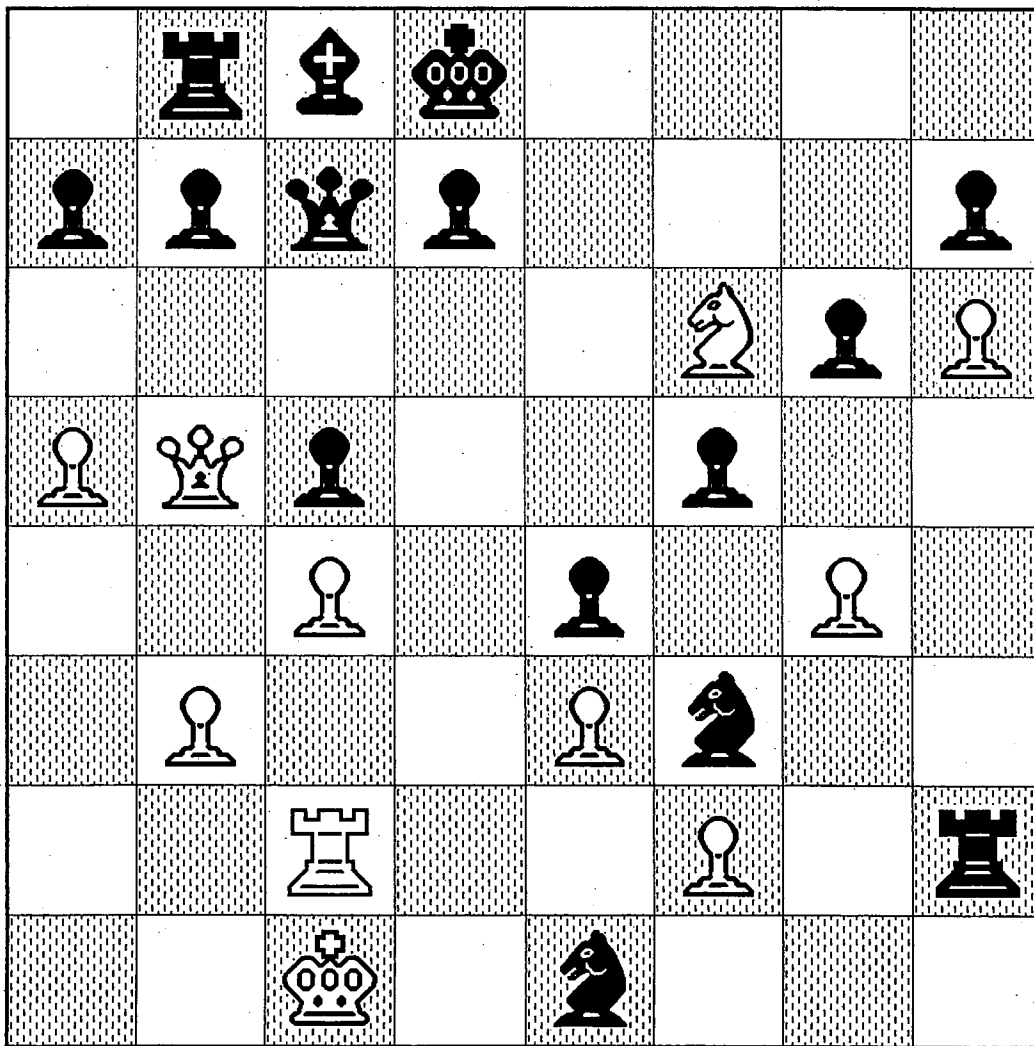
us with a wry smile, "That 33rd move greatly decreased the probability that I shall win." He *was* down by a Rook, a Knight, and a Pawn, not counting the immobile Bishops.

Savage retorted, "Your assessment of the probability is more accurate than your count of the moves. I make that the 35th move of the game."

They argued for a few minutes, each yielding some ground. At last they agreed that the number of moves had certainly been more than 32, and fewer than 36. Turning to Holmes, Bayes asked, "Can retrograde analysis resolve the uncertainty in the number of moves, or at least increase the likelihood of either hypothesis?"

Holmes had surveyed the position during the argument, and was quick to reply, "In this case, it can eliminate all doubt. However, I can conceive of an almost identical position that would make the resolution impossible."

I looked in vain for any special characteristics of the position. In desperation I asked Holmes, "How small a difference would make the problem insoluble?"

He replied, "If the Black Pawn that has advanced to g6 were still on g7, I should be unable to specify the precise number of moves."

I was completely unable to comprehend how the movement of that one Pawn made the problem soluble. Holmes had to explain.

DID THE COLORS CHANGE?

Lord and Lady Ashley stood by this table, conversing animatedly with Hunter and Gaither. I gleaned from their conversation that Hunter's interests were wide-ranging, while Gaither's centered around home and fields.

Aside from using the chameleon board, the most obvious feature of the position was that a Black Queen on h8 checked the White King on d4.

I spoke to Lady Ashley, "In view of your great interest, I suspect this game may have been either homochromatic or heterochromatic."

26. NUMBER UPPED?
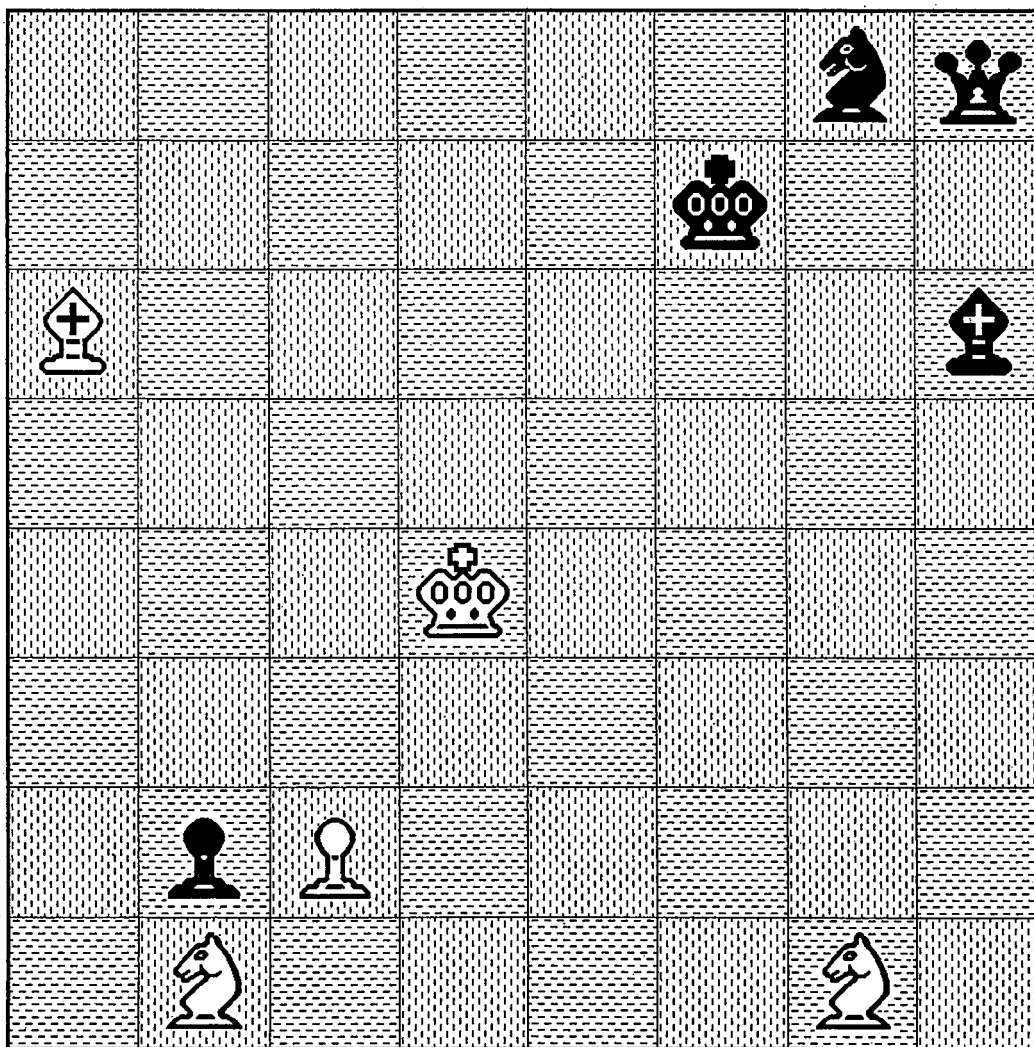
She smiled, "You are absolutely right, Doctor Watson. And, by now, I should expect you to be able to tell which."

I soon nodded in satisfaction, "Yes, one of those possibilities can be eliminated rather quickly."

Holmes interrupted before I could run through the simple analysis, "A slightly more difficult problem, Watson, is showing how many Pawns have promoted and whether any promotions were of the color-changing variety."

IMPOSSIBLE MUTUAL CHECK

Longstreet was standing alone beside this chameleon board, as if over a map of some hard-fought battle. Something about it caught my eye, and I took hold of Holmes's sleeve to stop him from striding past.

"Is there something of interest, Watson?"

"There is something that is completely *impossible*, Holmes. Look! Regardless of the move direction, White is checked by one of the two Black Pawns, on c3 and e5. At the same time, Black is checked by the Rook on a6."

27. THERE IS A SOLUTION!
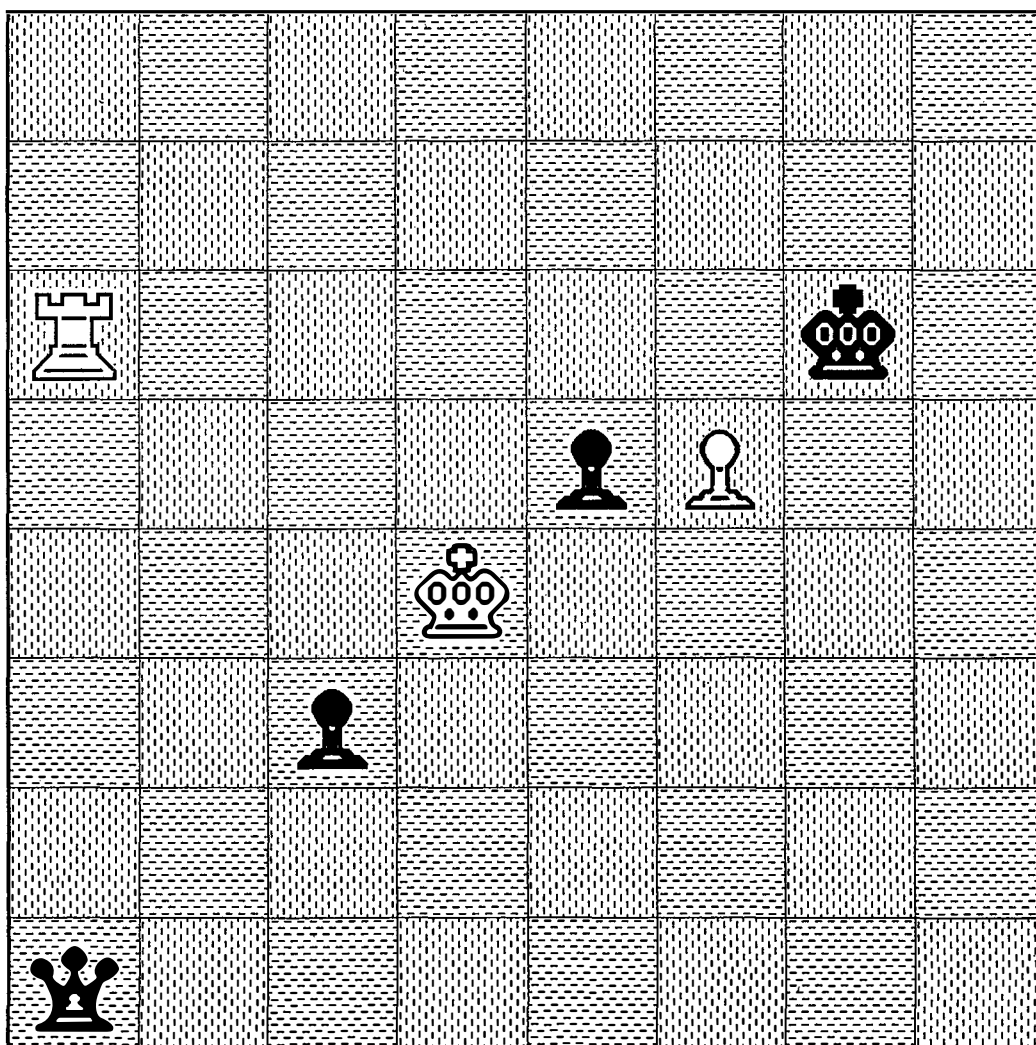
"Curious, indeed," Holmes agreed. "But there may be a reasonable explanation. Did you contest this set-to, Longstreet?"

"Yes, against Meade."

"Was he, by any chance, called away suddenly?" asked Holmes.

Longstreet said, "As a matter of fact, Meade *did* receive a dispatch of extreme urgency, and left forthwith. The position being what it was, I called after him, 'I concede your victory.'"

"Thank you," said Holmes. "You see, Watson, there *was* an explanation that can keep the play legal."

The situation would have seemed fairly straightforward if some piece blocked the apparent Rook check, leaving only White in check, by one of the two Black Pawns.

"I take it, Longstreet," I said, "that in his haste, Meade took away a piece that belongs on the board."

Holmes's face showed some consternation, which turned to relief when Longstreet responded, "No, quite the contrary. No other piece belongs on the board."

"And with that, Watson," said Holmes, "the move direction and the last move by each player can be uniquely determined."

MORE NEWLY ORDAINED

Lady Ashley introduced me and Holmes to two wiry gentlemen with magnetic personalities, Faraday and Olmstead.

When asked, Faraday assured us, "Our game has, indeed, been heterochromatic, as all of our current games are. We find the rhythmic alternation generates a pleasing flow, attracting us to a higher frequency of play. Our resistance is low, and it is only with the greatest reluctance that we leave the club, later and later. But perhaps that's just a phase."

28. CHECKING ACCOUNT
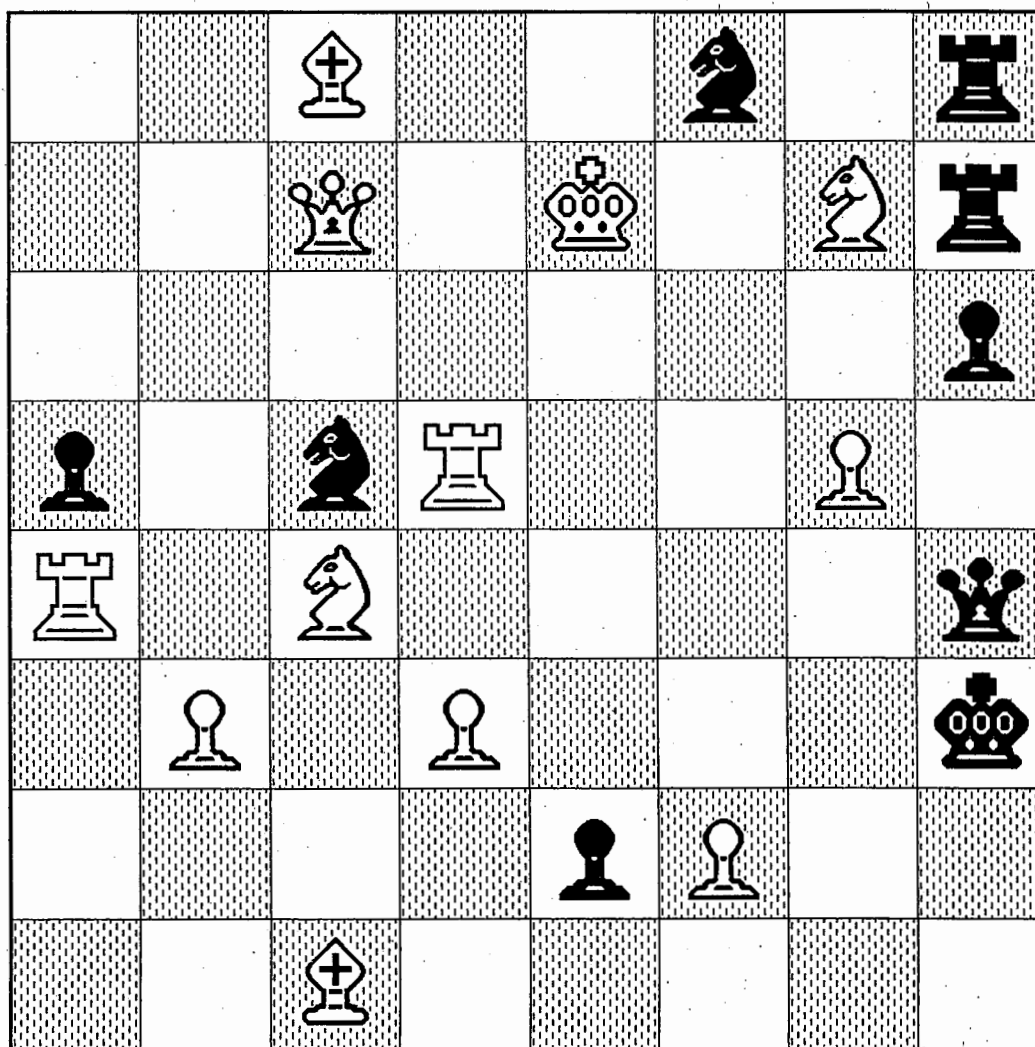
"Your concern may be well-grounded," said Holmes. "However, this position seems to have some potential for retrograde analysis. Watson, what strikes your eye about it?"

"The position is one of check, of course. I noticed White Bishops on c1 and c8," I said, "and realized that one Bishop had to be promoted. But no file contains Pawns of both colors, and I see no other obvious clues to the direction of motion. It appears impossible to determine which Bishop is promoted."

"Watson," Holmes smiled, "It *is* possible to identify the promoted Bishop. You mentioned the key to the solution before you described the problem."

CAPTURE AND CHECK

Cross and Bowman were pondering this chameleon board, and paid minimum attention to our arrival. I noticed a Black Pawn carelessly placed between b5 and c5, and a White Pawn between f4 and g4. A Bishop on e4 was of ambiguous color, and clearly checked one of the two Kings, White on b1 or Black on g6.

"The situation could be double check," I pointed out to Holmes. "The Black Rook on b8 may or may not check the White King, depending on the placement of the ambiguous Black Pawn. The White Queen may check the Black King, depending on placement of the

29. FROM WHERE?
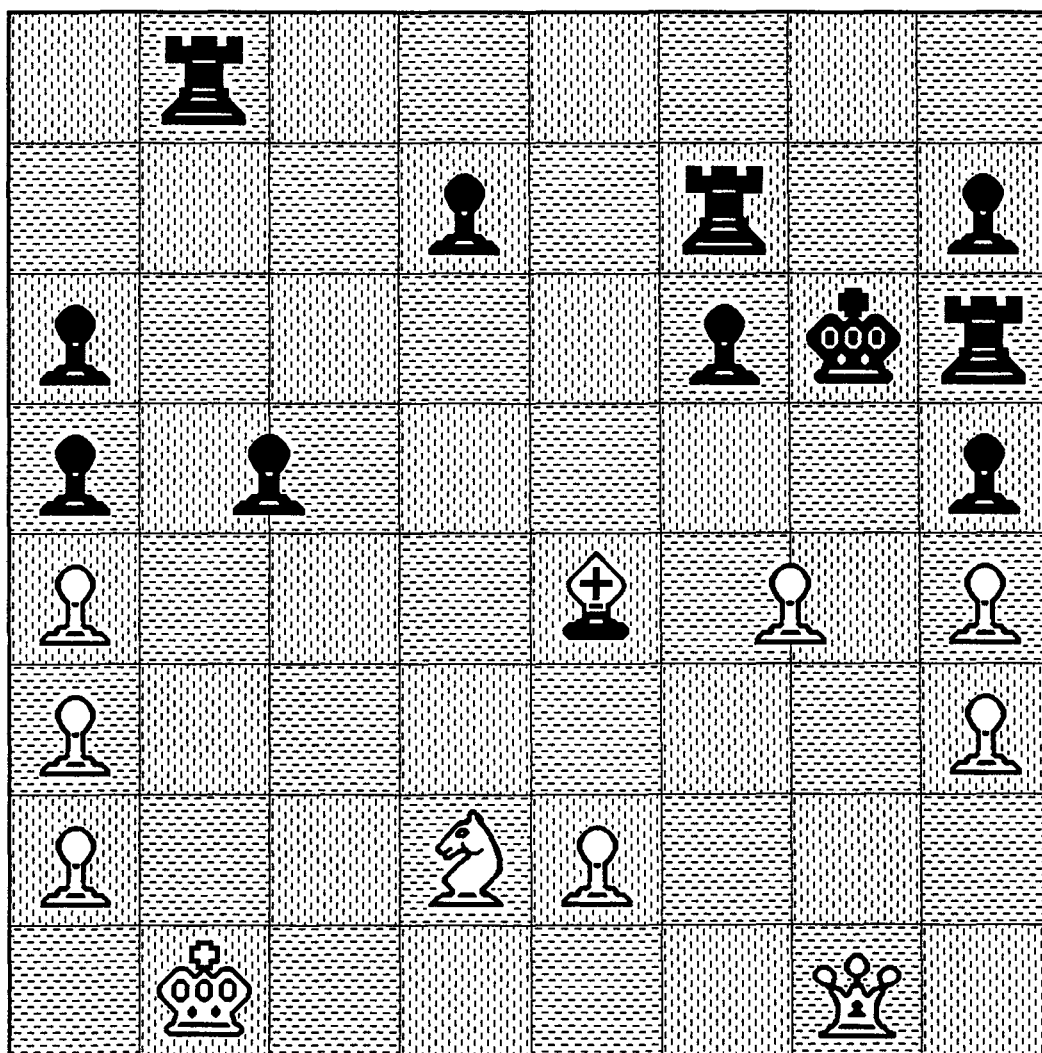
ambiguous White Pawn."

He nodded, "And I assume you have noticed the three Black Rooks."

"Each side is missing one Pawn," I noted. "The promoting Pawn could have been of either color, promoting with one capture."

Cross and Bowman conversed, their voices lowered, before Cross addressed us, "We were wondering whether straight-shooting required us to say something about the last move, or whether you could deduce the facts on your own."

I did not anticipate the words with which Holmes responded, "I hope the information in question is that the last move was a capture."

"Why, yes," said Bowman with a distinct twang, eyebrows arched. "Then you *could* tell!"

"Not with certainty," said Holmes. "But the problem is so much tidier with a last-move capture. We can now resolve the color of the ambiguous Bishop, and the locations of the ambiguous Pawns. For the piece that moved last, we can also identify the square from which it moved."

Cross drew back and tensed, as if to dispute that claim. But he said only, "We have no quarrel. Give it your best shot."

JOSTLED AGAIN

Lady Ashley directed us to another table where, she assured us, we would find a heterochromatic game. The players, Butler and Dorman, were scrambling servilely about the floor and peering under the table as if for dust.

"Has the board been jostled?" I asked.

Dorman admitted, "Yes, gentlemen. Butler did it; one piece fell from the board."

Butler said, sounding simultaneously aloof and deferential, "I would not have felt right

30. THE ROAD TAKEN
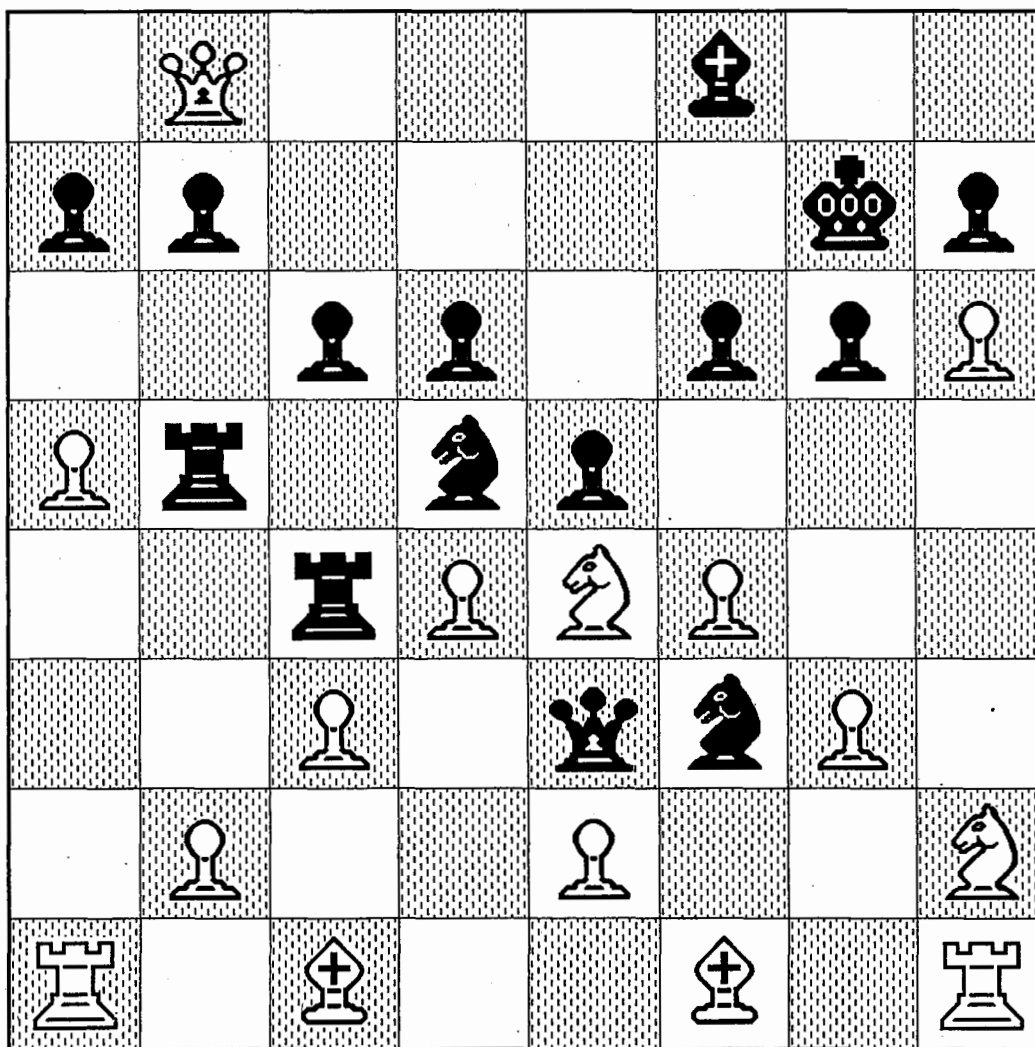
about continuing with the table even slightly awry."

"Can you confirm our understanding that your play has observed heterochromatic discipline?" Holmes inquired.

"Scrupulously so," Butler replied.

With most pieces still on the board, I thought I should be able to identify the fallen piece. 15 Black pieces were showing. There was no Bishop on c8, and the Bishop could not move heterochromatically. I was about to name the Bishop as the fallen piece, and c8 as its proper location.

Holmes saved me from that embarrassment by asking, "Watson, can you tell where the fallen White King belongs?"

ILLEGIBLE DIRECTIONS

Although no players were at the table, I could not think of this chameleon game as abandoned. Rather, it had been concluded by the obvious checkmate. A Black Bishop on g8 was pinned by a White Rook on f8, and could not be interposed to negate the check delivered by the Rook on h6.

I told Holmes, "Despite the checkmate, I am having a great deal of difficulty in determining the direction of movement, and I am not certain of White's last move."

"Don't feel bad, Watson," he consoled me. "It is impossible to determine either. But,

31. DUAL PROMOTIONS
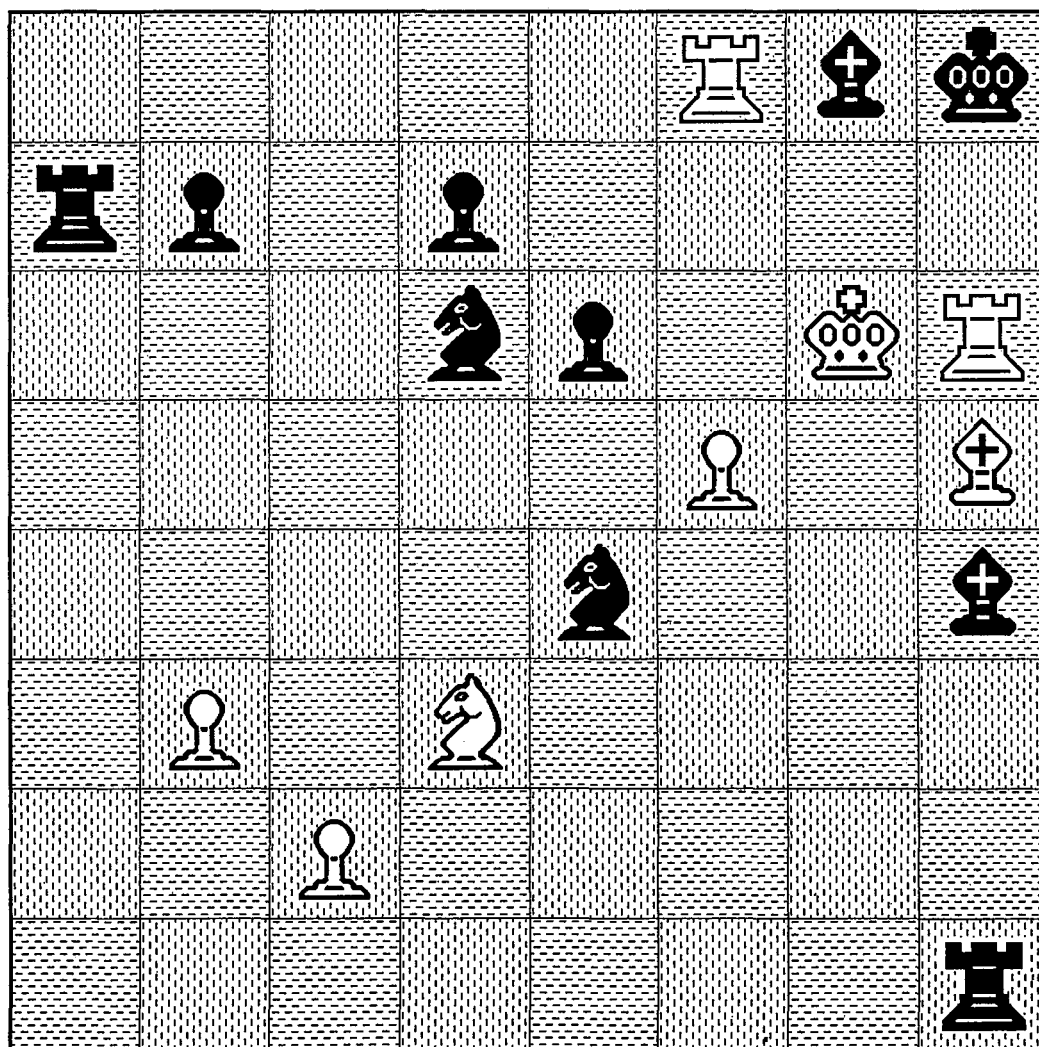
assuming that no piece has fallen from the board (so the apparent checkmate is real), I *can* conclude that there have been at least two promotions, and that at least one promoted piece is still on the board."

"Which piece is *that*?" I asked curiously.

"I really don't know," Holmes answered.

"Do you know its color?" I asked.

The reply was a terse, "No."

Clearly, there had been underpromotion, since no Queen was now present. I wondered whether either promotion had been a reverse, color-changing promotion.

But I was reluctant to pose further questions that Holmes might be equally unable to answer. I shrugged philosophically and said only, "I will see if I can prove two promotions."

MITER SQUARE?

Barrow and Fairbanks were moving about the floor, albeit with glacier-like slowness, obviously searching for a fallen piece. Upon noticing me and Holmes, the walrus-mustached Fairbanks volunteered that they were searching for a fallen Bishop. They soon abandoned the search and resumed their seats, Barrow on the Black King's side.

"Aside from the fallen Bishop," Holmes asked, "has there been anything unusual about your game?"

"Not at all," Fairbanks responded coolly. "It was a very conventional game, each of

32. UNMASK THE BISHOP
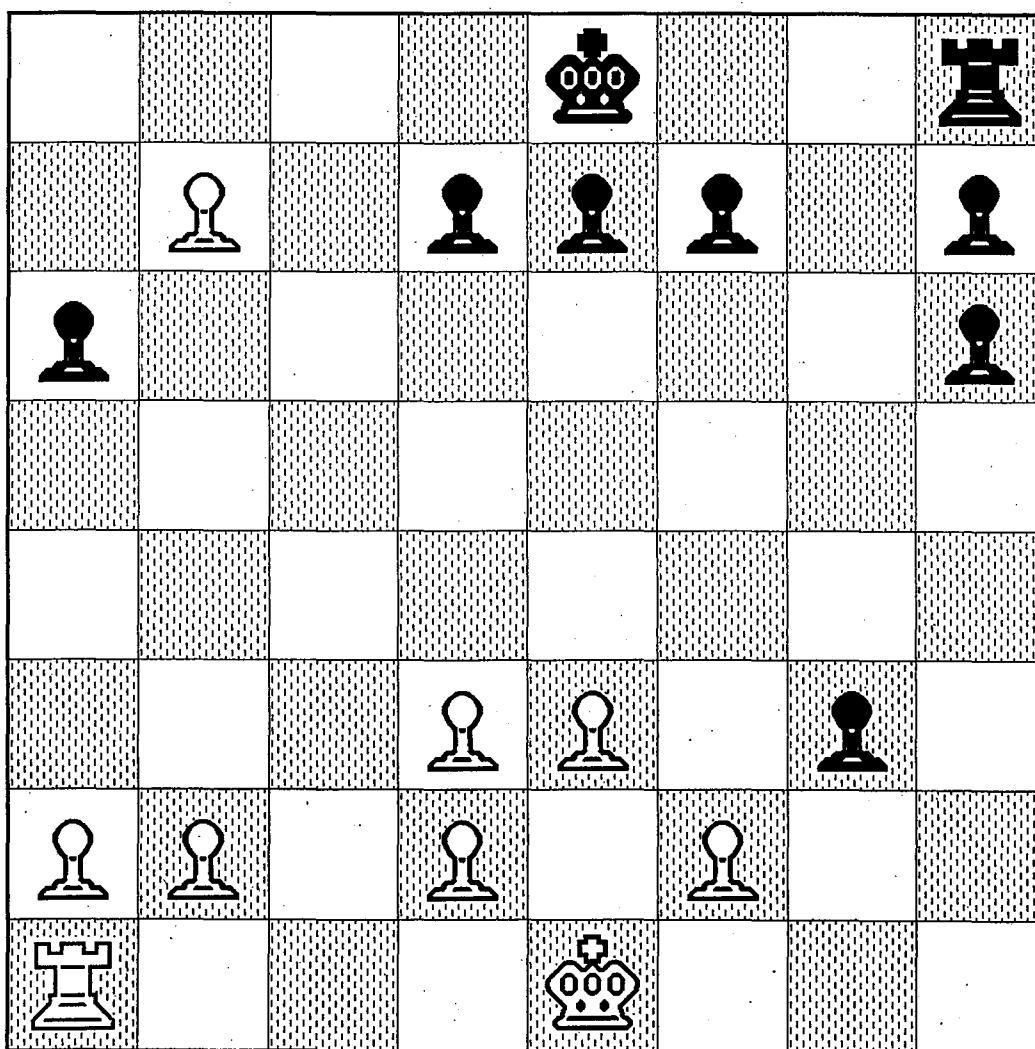
us opening with a two-square Pawn advance."

"Even the final move," Barrow observed distantly, "was not earthshaking, but only the commonplace capture of a Pawn."

Holmes, whose eagle-eyes missed little, commented, "Even so, we might find a little sport in trying to determine the color of the fallen Bishop, and the square on which it belongs."

PROVE PROMOTION

I counted the pieces on this abandoned board and remarked to Holmes, "There has been a lot of movement in this game, with very little material lost. Only one White piece and three Black pieces have been captured."

Holmes stared at the board briefly, and said, "Even fewer, Watson. Clearly, at least one piece has fallen from the board. And, with no more than three pieces captured, there may have been a promotion."

"It isn't like you, Holmes, to be so tentative," I said. "With one White Pawn missing,

33. ROOK DOWN
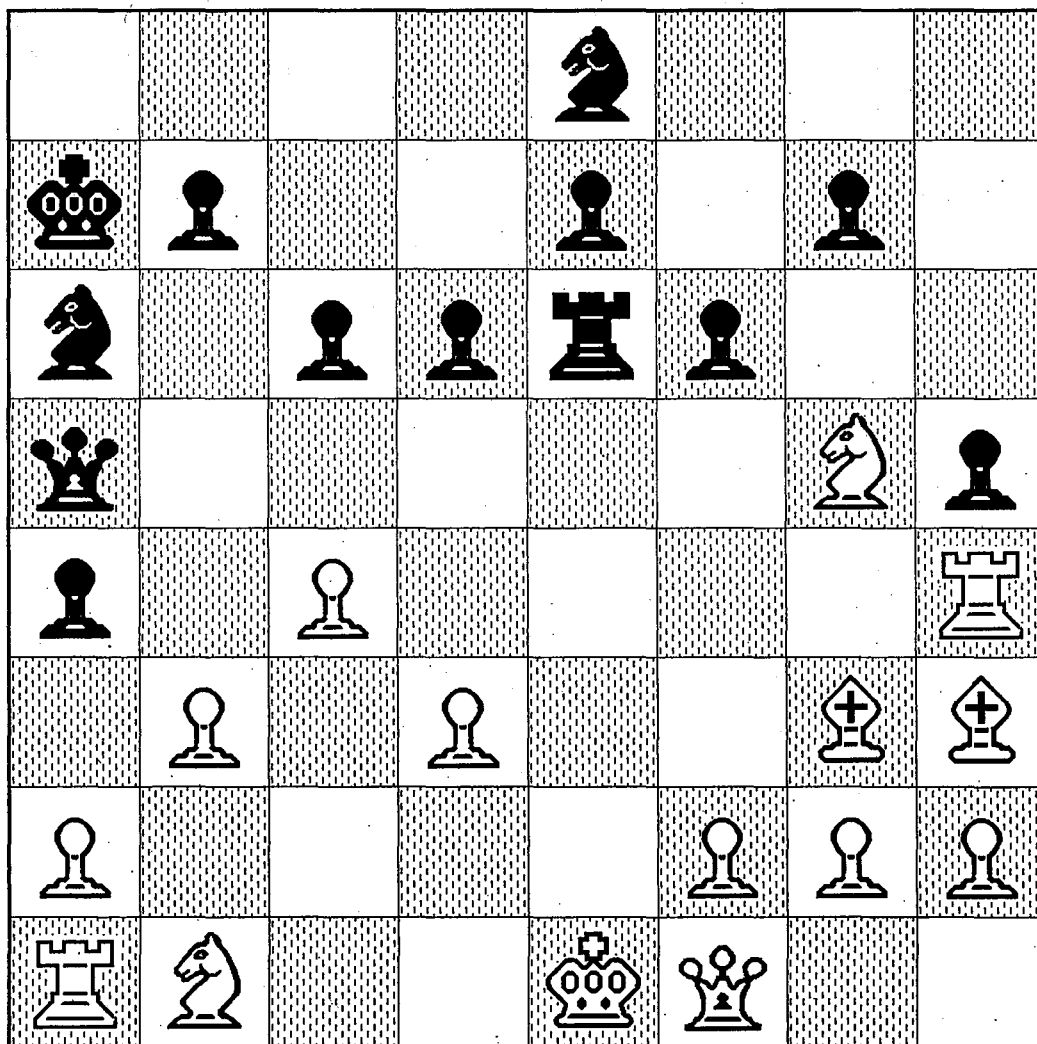
it isn't remarkable that there *may* have been a promotion." I intended to press him for proof of a fallen piece, but was sidetracked by the return of the players, Gildersleeve and Gatsby.

Gatsby greeted us and said, "We interrupted this game when a Rook fell from the board. Have you, by any chance, found it?"

"Sorry, no," said Holmes. "But I *have* concluded that there has unquestionably been a promotion." I glowered at his sudden shift from the *possibility* to the *certainty* of a promotion, but said nothing. After all, Gatsby *had* confirmed the conclusion that a piece had fallen.

"Great!" Gildersleeve nodded and smiled affably, "I suppose the giveaway was the impossible double check?"

"The impossible double check together with the Bishop locations," Holmes answered.

I concentrated on the Bishop positions; if a Rook blocked the apparent check from the Black Queen, one Bishop could capture the Black Rook to negate its check. Before I could announce that initial finding, Holmes unloaded the whole problem on me.

"Watson," he cajoled, "why don't you try proving that a promotion occurred?"

PARTLY CLOUDY

We stoppd to speak with Rider and Haggard, who were standing by a table holding this chameleon board. Widely traveled, both had heard of Holmes's analytical strength, and wondered what insight he could unearth from the position.

"Our game has been heterochromatic," Rider informed us.

After an unusually lengthy perusal, Holmes said, "Without being told that play was heterochromatic, I could not even have deduced that a promotion has occurred."

34. PROMOTED TO TRAVEL
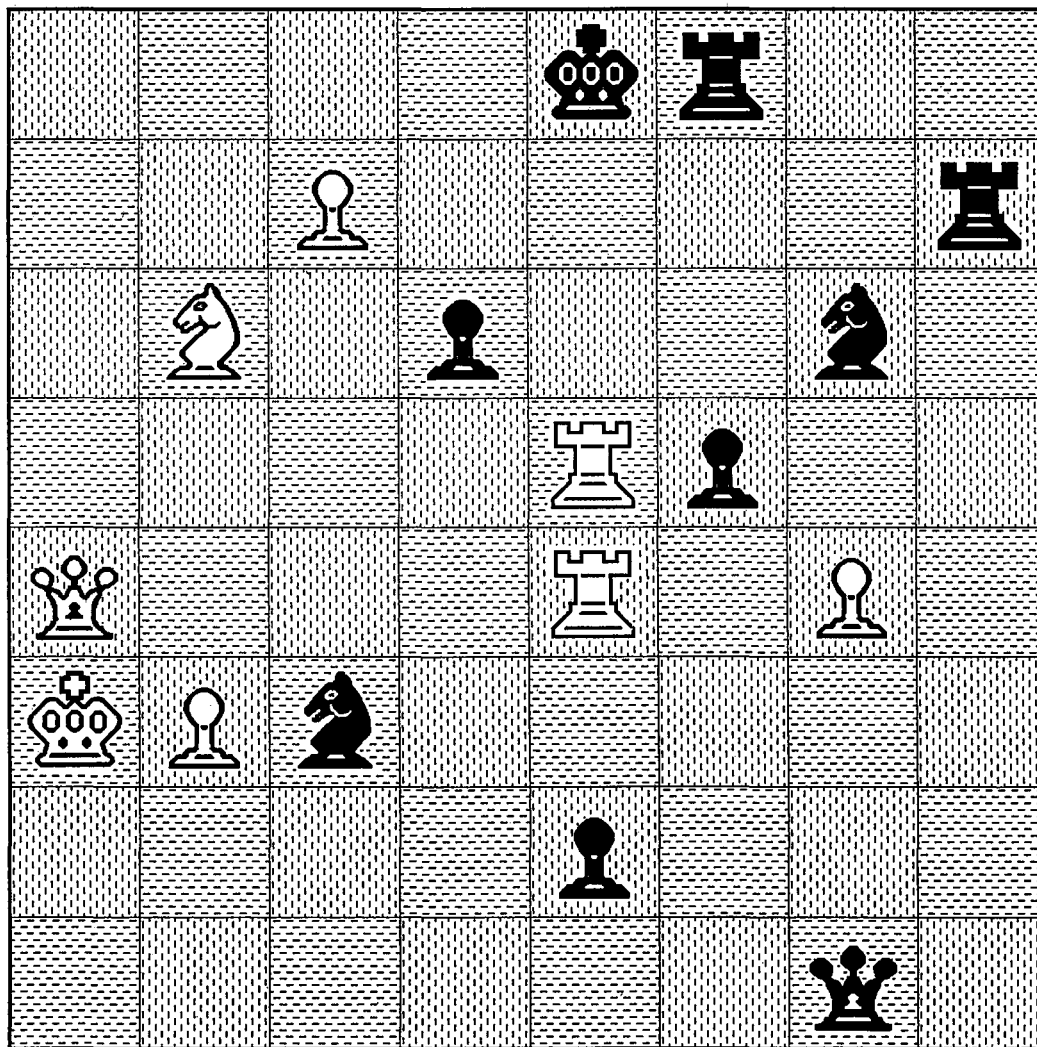
"That seems to be a promising start," said Haggard. "Is the promoted piece still on the board?"

"I don't know," Holmes admitted reluctantly. "But the promoted piece *did* move after promotion."

"Was the promotion a reverse promotion?" I asked. Holmes paused even longer before repeating, "I don't know."

"Perhaps you can tell us if it was an underpromotion," Rider suggested. Once more, Holmes answered, "I don't know."

"Surely you can determine the move direction," I insisted.

"All four move directions are possible," Holmes answered.

"With so many unknown elements," said Haggard, "It would take King Solomon's mind to establish the facts. I am going to have to challenge you to prove that a promotion did occur."

"And beyond that, I added, "I must challenge you to prove that the promoted piece subsequently moved."

FLAG THE CAPTURE

Lady Ashley approached a table, spoke briefly with the players, then motioned for me and Holmes to join her.

"This game has been heterochromatic," she said. "Messieurs Coleridge and Southey were about to search for a piece that has fallen from the board."

"Gentlemen," insisted Coleridge, "please don't offer to help with the search. It will go fast enough with two of us. In fact, intervention by a person named Sherlock might make me forget what I'm doing."

35. PAWN OR BISHOP?
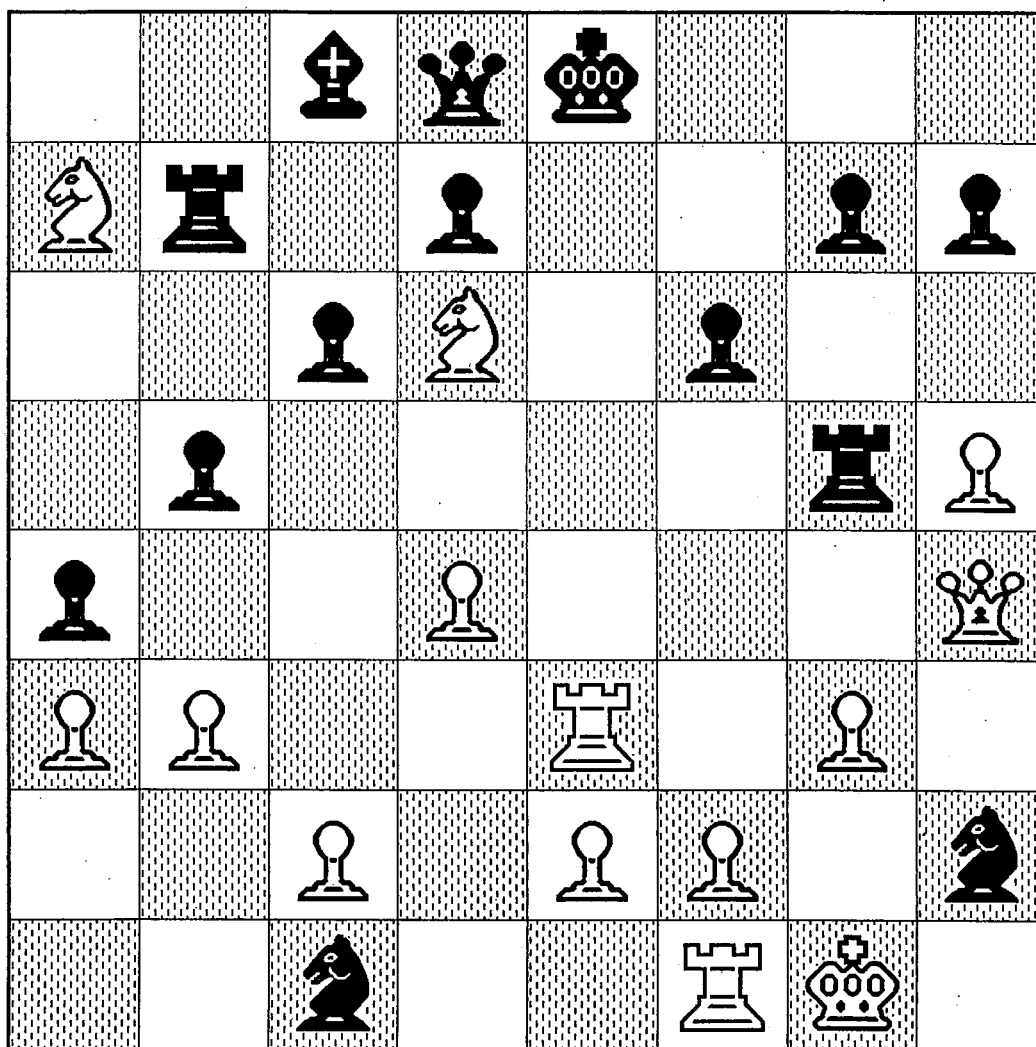
Southey added, "We are familiar with your enthusiasm for retrograde analysis. Perhaps you can identify the fallen piece, and the square where it belongs, before we retrieve it. By the way, no odds were given."

Holmes nodded cordially, "We shall be delighted to have a go at the problem."

Clearly, the White Knight on d6, giving check, made the last move. It *could* have moved from e4, discovering a second check, by the Rook on e3. If not, the fallen piece must be on the e-file, blocking the apparent Rook check.

I quickly came to another conclusion, "The fallen piece is Black. The only missing White pieces are the two Bishops. Their home squares are now occupied, and they cannot be on any other square, not moving heterochromatically.

"The missing Black pieces are the Bishop from f8 and the Pawn from e7. But I can't tell which has fallen. Since f8 is not now occupied by any other piece, the Bishop may have just fallen from there, or it may have been captured. The Pawn, too, may have fallen or been captured."

"But not both have been captured," Holmes said. "Count your blessings. If you can prove the capture of one, that paves the way to a solution. It follows that the other is the fallen piece."

WHAT IS THE GAME OF THIS ROOK?

As we arrived at the Chameleon Chess Club, Goodwin, the club secretary, was visibly agitated. My solicitous questioning drew forth the fact that he had been trying to follow play in a chameleon game, but found the players' comments too distracting.

"The players are the Bassets, twins who are absolutely indistinguishable in appearance. But I have unimpeachable evidence that one brother makes only true statements, while the other makes only false statements.

"They tend to sit at the same side of the table, and to stroll frequently for exercise.

36. TRUTH AT THE TABLE
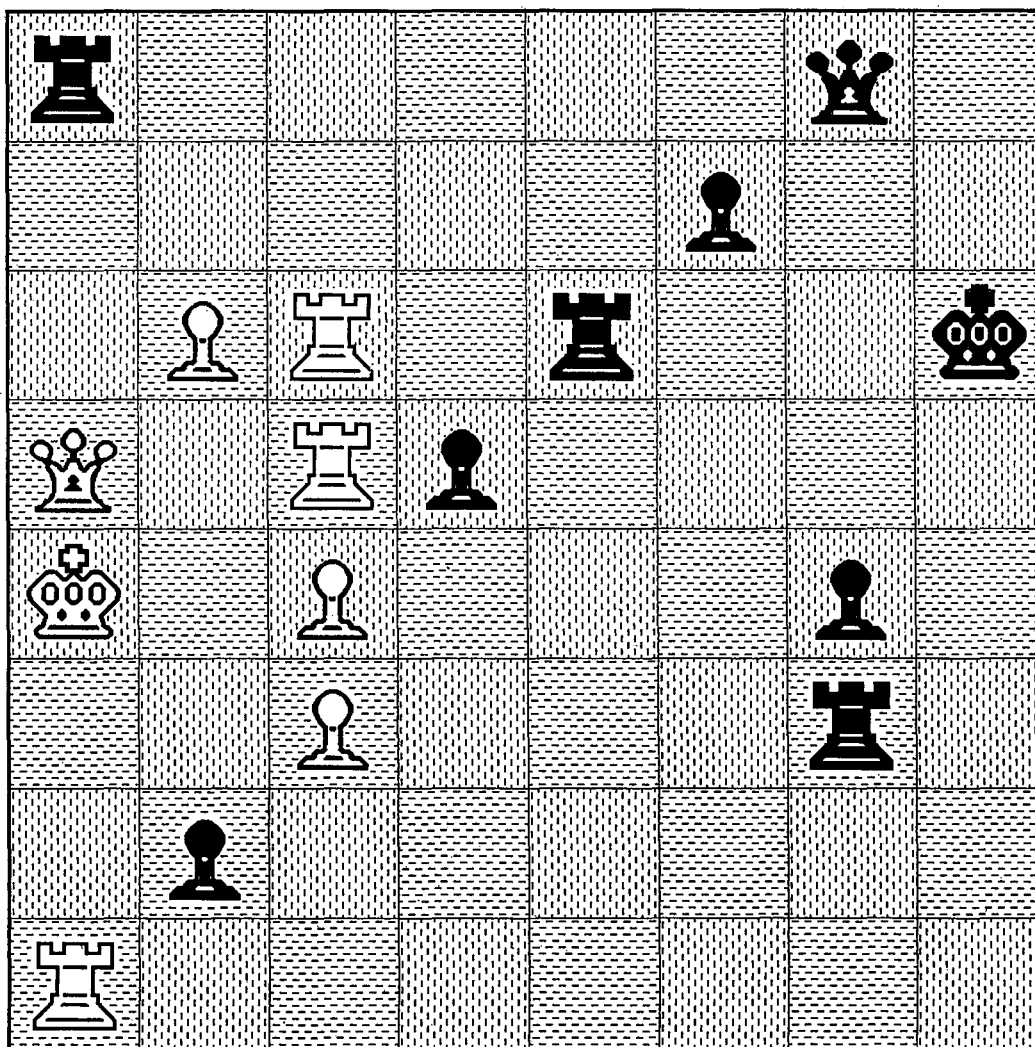
Thus, if one manages to get them sorted out, they don't stay sorted out."

"Their games may get very intricate," said Holmes. "Would it be possible for you to introduce us?"

Goodwin led us to a table, identified the players as the Basset brothers, and me and Holmes by name. One of the Bassets said, "I have been playing Black or the game was neither homochromatic nor heterochromatic." His twin, to my utter astonishment, said, "I have been playing Black or the game was neither homochromatic nor heterochromatic." The liar and the truth-teller had spoken the same words!

Holmes asked, "Aside from that, has anything unusual happened during the game? For example, has a piece fallen from the board?"

One of the twins said, "My brother would say that a piece checking one King has not fallen from the board."

Holmes told me, with obvious self-satisfaction, "Watson, we now know enough to identify the last piece that moved, its location, and the veracity of the player who made the move."

A UNIQUE EXPERIENCE

As Holmes and I approached their table, Lord and Lady Ashley rose to greet us. "This has to be the most unusual homochromatic game we have ever played," Lady Ashley said. "One aspect was unique in our experience."

Holmes immediately riveted his attention on the board, while I chatted about inconsequentialities with the Ashleys. After a few minutes, the sleuth's eyebrows lifted in amazement, and a broad smile lit up his countenance.

"Mr. Holmes," said Lady Ashley, "if you are analyzing our game, I should tell you

37. WHITE'S THIRD MOVE?
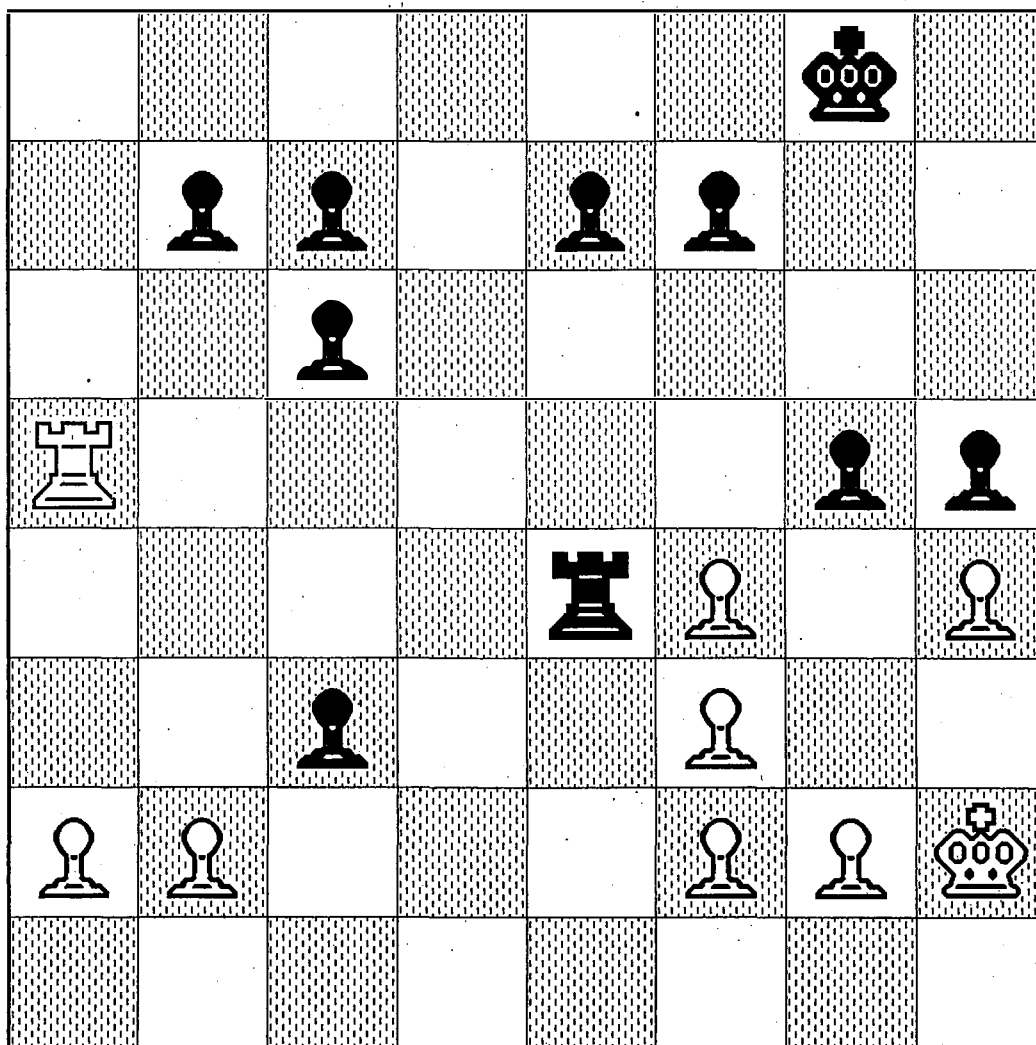
about some circumstances that can't possibly be obvious from the position alone."

"Please don't disclose any further information," Holmes requested. "Much more is obvious than you may imagine. For example, Watson, it is possible to determine White's third move precisely."

Lady Ashley gasped in amazement. "Can you really determine that?"

"Most assuredly," Holmes responded. "Retrograde analysis can sometimes reach extraordinary conclusions, especially with homochromatic play."

"It is far from clear to me what either player's *first* move was," I complained, "let alone the third, when the possibilities would be greatly multiplied."

Holmes argued persuasively that I should make an effort, "Begin with what is obvious to you from the position, and let us see where that leads."

NO POWER TO CASTLE

This game was unattended when we first saw it, abandoned or interrupted. "Can you make anything of the play, Holmes?" I asked.

"Only that it was not homochromatic," he replied, "with five White Pawns on black squares."

"Quite right," roared Briggs, coming up behind us. "Play has been heterochromatic, between me and Stratton."

38. LAST NONCASTLING MOVE?
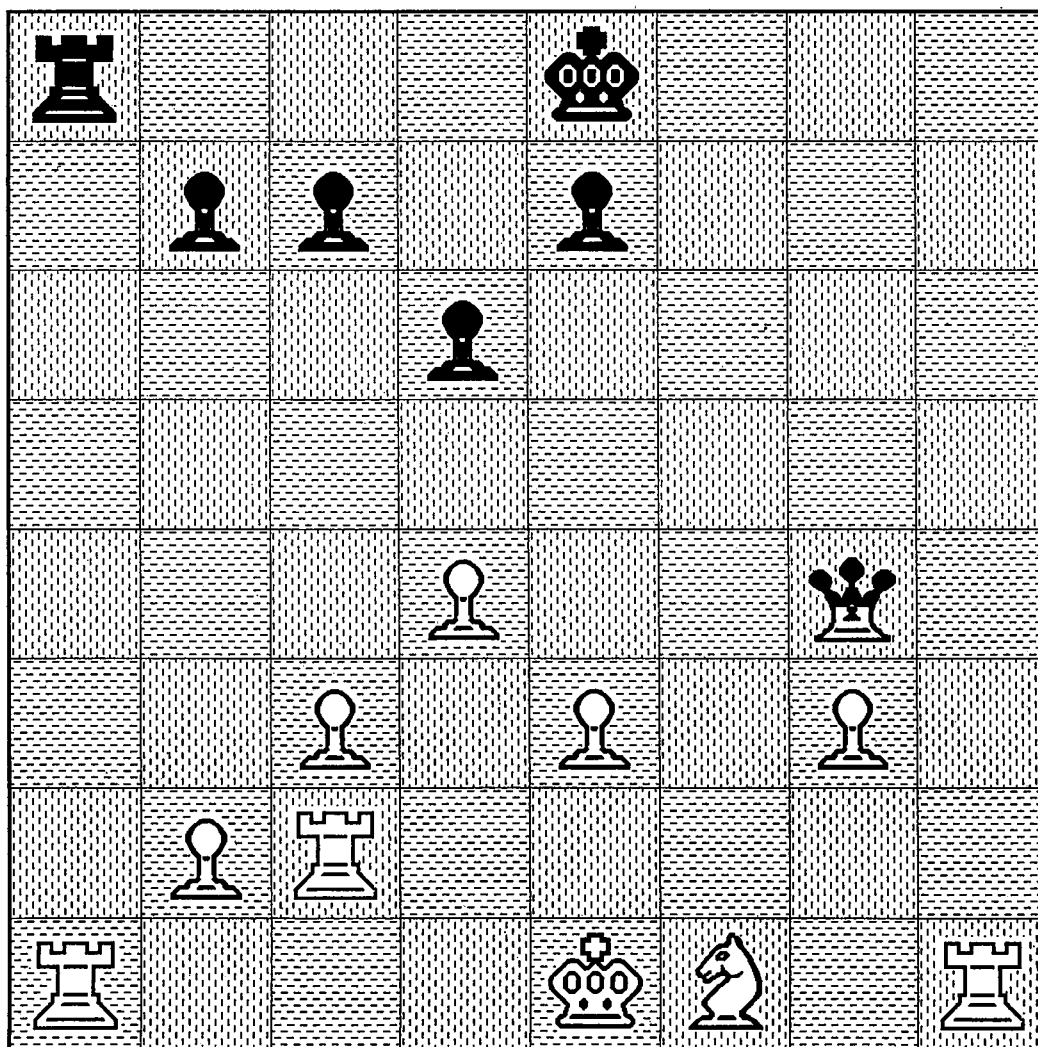
"We haven't seen much castling in heterochromatic play," I said. "But it looks as if one of you may be able to castle from this position."

"I fully intend to castle as soon as Stratton returns," said Briggs. Only my Queen move prevented *him* from castling on his last move."

The only Queen on the board, at g4, was of ambiguous color. I was uncertain whether Briggs was playing White or Black.

Holmes had absorbed every word Briggs had spoken. He looked up from the board and asked, "Watson, what move *did* Stratton make when he was prevented from castling?"

WATSON IS COMMENDED

Holmes and I stopped to chat with Lord and Lady Ashley, who confirmed that this was another of their homochromatic games. I realized immediately that the White Rook on b4 had to originate on the top row in order to reach the fourth row homochromatically, so I remarked to Lady Ashley, who was sitting at the top of the board, "I see that you played White, as usual."

Holmes was astonished. "It is astute of you, Watson, to realize that the Rook cannot be promoted, with White moving up. I had barely reached that conclusion, myself."

39. HOMOCHROMATIC PROMOTION
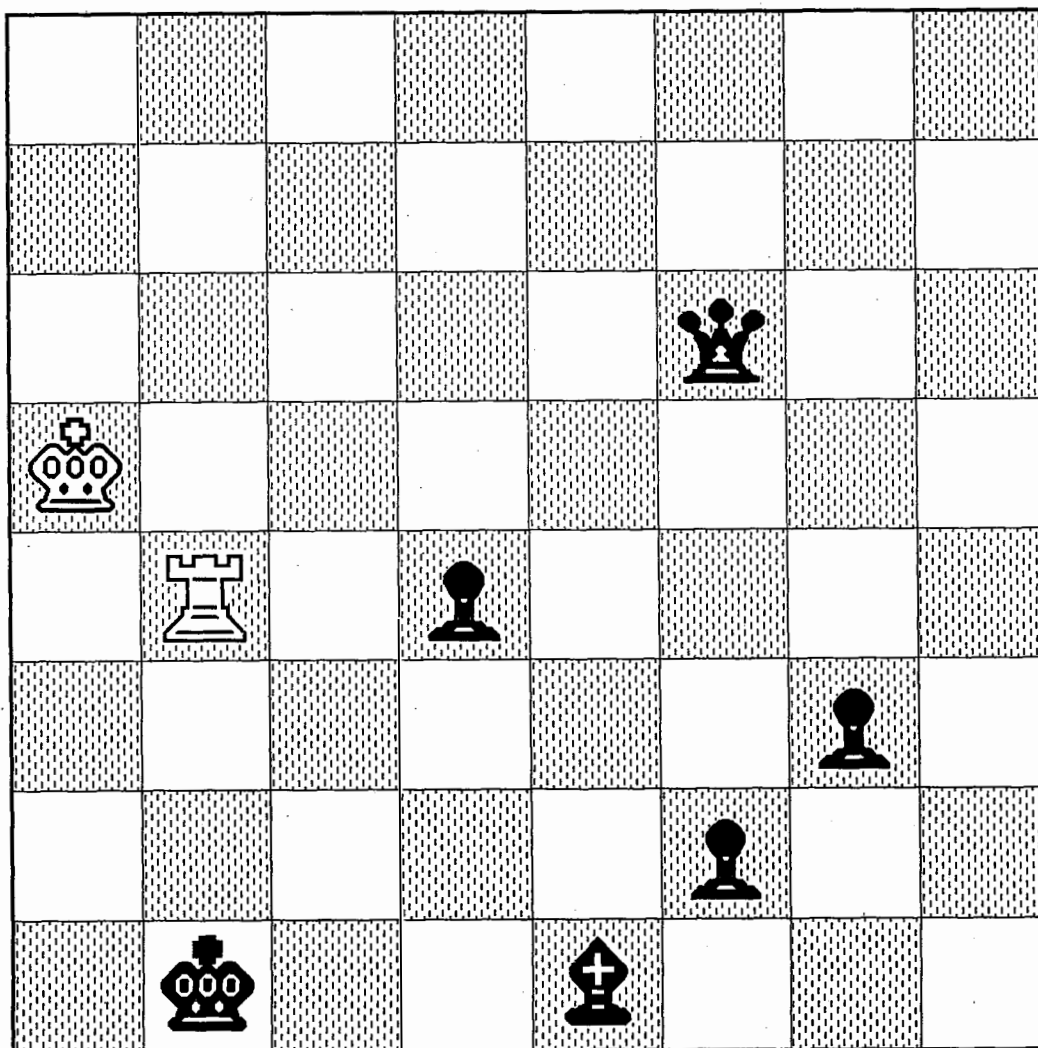
Rather than confess that I had completely overlooked the possibility of such a promotion, I merely shrugged and accepted his rare congratulations with as much modesty as I could feign.

Holmes continued to examine the position, trying, I assumed, to share my unearned limelight by finding some more obscure conclusion. Eventually, he nodded and said, "Although the White Rook is not promoted, there *has* been a promotion. It will truly astonish me, Watson, if you can prove that promotion occurred."

With so many Pawns and pieces of each color missing, it was entirely possible that a Pawn of either color could have promoted. It was also possible that a promoted piece, if there *had* been one, was no longer on the board. With the possibilities so unlimited, I had no confidence in the adequacy of my analytical talents.

Putting the best face possible on the situation, I told Holmes, "I think I will rest on my laurels and let *you* present the logic of the promotion."

DOUBLE TROUBLE

"Here's a table with an improbable-looking position," said Holmes. I was quick to agree. White appeared to be in double check, from the Black Rook on c4 and Black Bishop on c7. Quite clearly, no possible last move of the Bishop could have discovered the Rook check, and no Rook move could have discovered the Bishop check.

"Except when a King is missing," I said, "I've never seen a position so clearly indicating that a piece has fallen from the board."

Goodwin, the club secretary, made inquiries of nearby spectators, and reported, "The

40. UNCHANGING CHAMELEON?
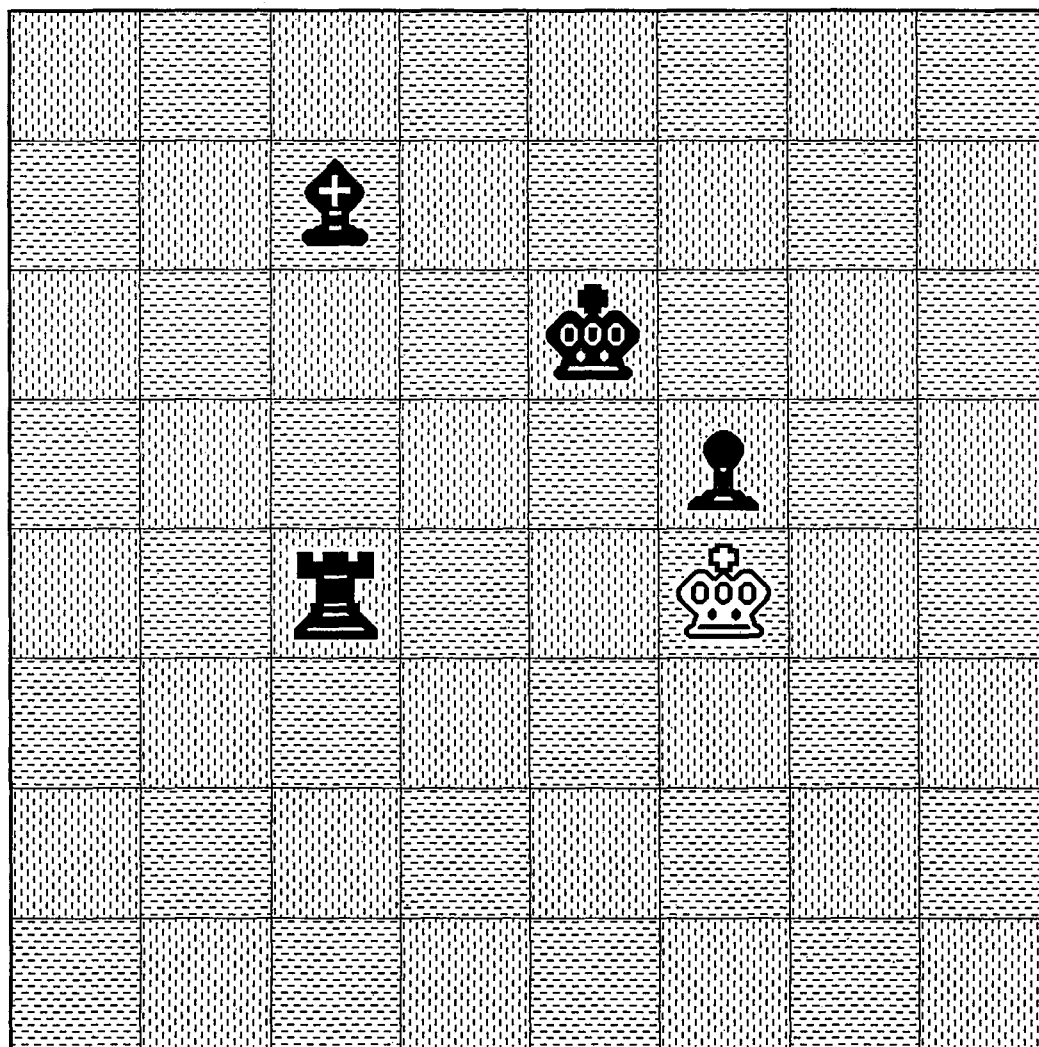
Grimsley brothers played this game. It is an established fact that one of them is invariably truthful, while the other always lies. But they are absolutely indistinguishable, making that information of dubious value. Why, here they are now, returning to the table.''

After Goodwin had murmured, ''The Grimsley brothers,'' and Holmes and I had introduced ourselves, I remarked, ''You certainly do look like identical twins. I had just concluded that a piece has fallen to the floor.''

What I can only call Player 1 answered, ''We *are* identical twins.''

Player 2 equivocated, ''We are identical twins or no piece has fallen to the floor.''

Player 1 cut in again, ''Precisely one piece has fallen to the floor.''

Holmes frowned darkly upon hearing that exchange, his eyes still riveted on the chessboard. Then his face brightened, and he said, ''Extraordinary! Watson, you should be able to deduce whether this game has beem played homochromatically.''

''Let me see,'' I mused. ''If the game was homochromatic, a Black Rook on d6, blocking the Bishop check, would be promoted if Black is moving right. A Black Rook on e5 would be promoted if Black is moving down...''

''No, no!'' laughed Holmes. ''You weren't paying enough attention to the Grimsley brothers.''

''Holmes!'' I protested. ''The same brother who said they were identical twins also said one piece has fallen, and I'm trying to think through the implications of possible blocking pieces.''

Holmes said kindly, ''You overlook the fact that their being identical twins would make *all* of their statements true. Player 2 made a true statement if no piece has fallen; but he also made a true statement if the brothers are identical twins. What lie has either of them told?''

''Then,'' I said slowly, absorbing the implications, ''they are *not* identical twins. Merely extraordinarily similar-looking brothers.''

''More likely *triplets*,'' Holmes replied. ''In either event, the only true statement, made by Player 2, is that no piece has fallen. I repeat my question: was the game homochromatic?''

''I don't even think it was legal!'' I exploded, then felt quite sheepish when Holmes explained.

NOT THE OBVIOUS SUSPECTS

Pitcairn and Johnson were seated at a chameleon board. Holmes, after a quick scan of the position, begged their indulgence of a delay in play to allow retrograde analysis.

"Glad to oblige," said Pitcairn. "We have wanted to observe that process for our own enlightenment. After all, as the poet says, 'No man is an island.'"

While Holmes addressed the board in his fashion, I looked for clues in mine. "The most obvious feature of the position," I noted, "is the White Queen check of the Black King. No move by the King can escape check, and no Black piece can be interposed."

41. FIND THE CHECKMATE
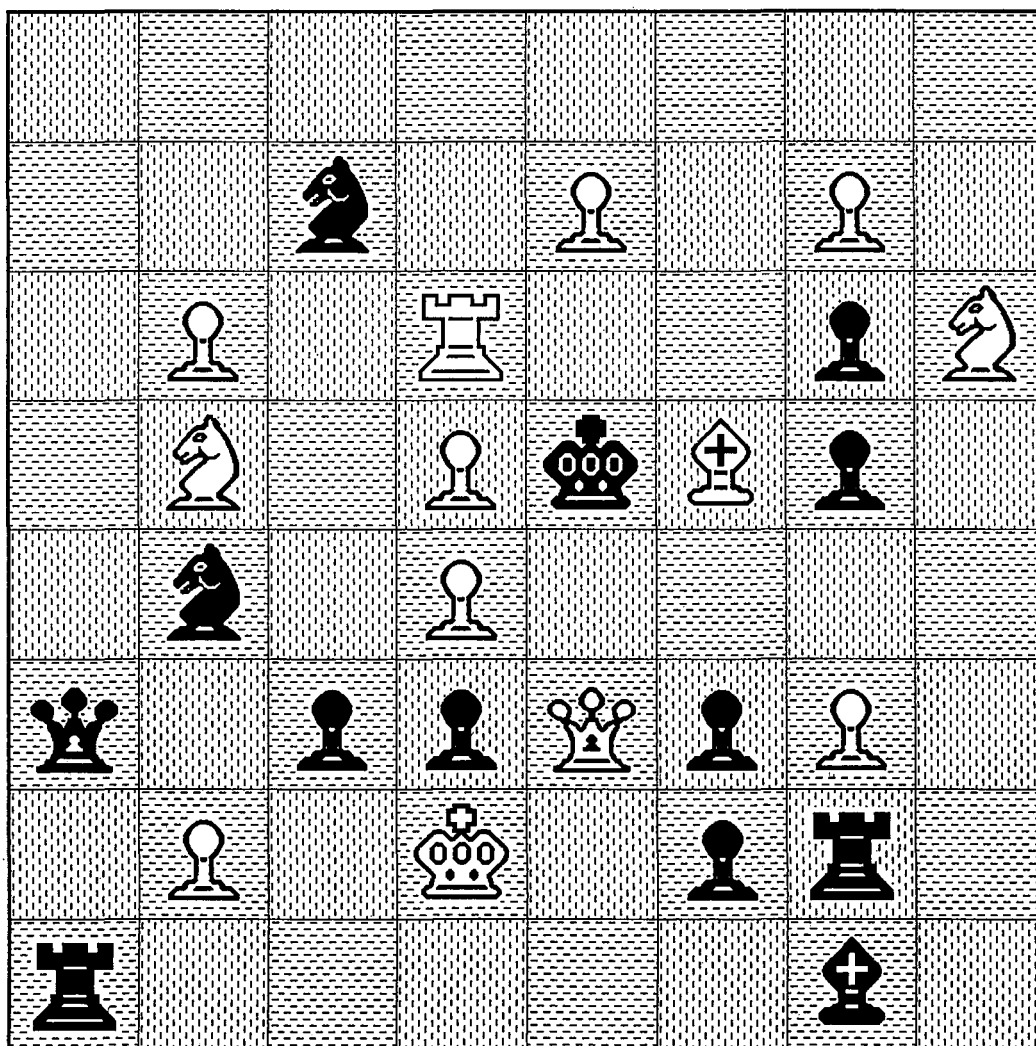
Holmes, his own review of the situation completed, said, "Watson, I wonder if *you* can identify the checkmating move, and the opponent's preceding move." The clear implication was that *he* could.

I systematically mulled over the clues, and was about to announce my findings when Holmes knocked them into a cocked hat.

"The checkmating move, of course," he said, "is *not* a Queen or Bishop move."

I felt almost mutinous, but eventually settled down.

THE RULE OF THREE

Holmes and I had stopped to look over an abandoned game. I could make little of it. White could be moving up or down and, with no check evident, either player might have made the last move. My thoughts drifted back to the table we had just left, and to the unusual conclusion of the game between Evans and Morphy. (*That* game is not detailed here, having no content of interest for retrograde analysis.)

"I was not entirely familiar with the rule that Evans cited," I confessed. "The situation was not a stalemate. In fact, Evans was in check but still claimed a draw, based

42. DEJA VU AGAIN?
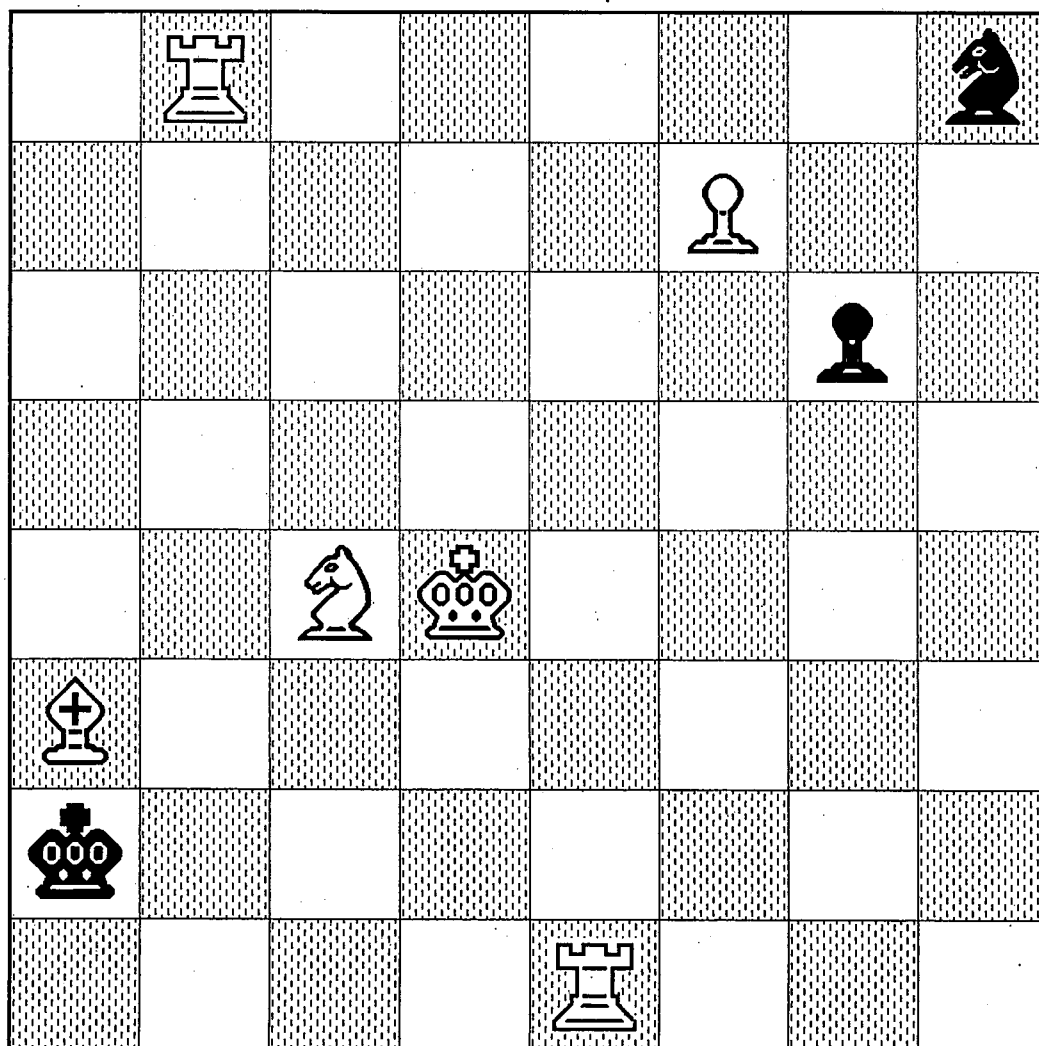
on a move not yet made. It was *not* a draw by agreement; Morphy protested vigorously."

"The rule involved," Holmes told me, "is usually described as the 'threefold repetition' rule. It becomes pertinent when a position has occurred twice previously in the game, with the same player moving to create that position each time. If *that* player has a move available that would create the same position a third time, he can claim a draw, but must do so before making the move. That is precisely what Evans did."

"Except in tournament play," I observed, "it is rare to have the meticulous recording needed to substantiate such a claim."

Holmes agreed. "Ironically, Evans and Morphy arranged for their games to be recorded in the hope of finding an interesting challenge to retrograde chess analysis."

I pondered, "The situation of a triply repeated position must arise fairly often, with the players usually unaware of the repetition. *This* otherwise unappealing position could, perhaps, be such a third repetition."

At that suggestion, Holmes scanned the board afresh, then said, *"If* this position qualifies as a third repetition, and *if* the players were aware of that fact, then Black *should* have claimed a draw, expecting otherwise to be defeated."

Astonished, I blurted, "Good Lord, Holmes! Are you saying you know that Black made the last move?"

"I sincerely hope he *isn't*," said Rathbone, approaching the table. *"I* moved last in this game, with the White Bishop."

I expected Holmes to look mortified. Instead, he said smugly, "Watson, you should be able to determine how many times this position has occurred, with Black having the move."

SHAGGY-DOG STORY?

We recognized the man standing beside this chess table as being one of the Basset brothers. Of course, it was impossible to distinguish whether he was the invariable liar or the always-truthful brother.

Holmes asked, stupidly, I thought, "Did you and your brother play this game?"

Basset responded, "Lord and Lady Ashley were the players. The game was homochromatic."

43. TRUTH BE TOLD
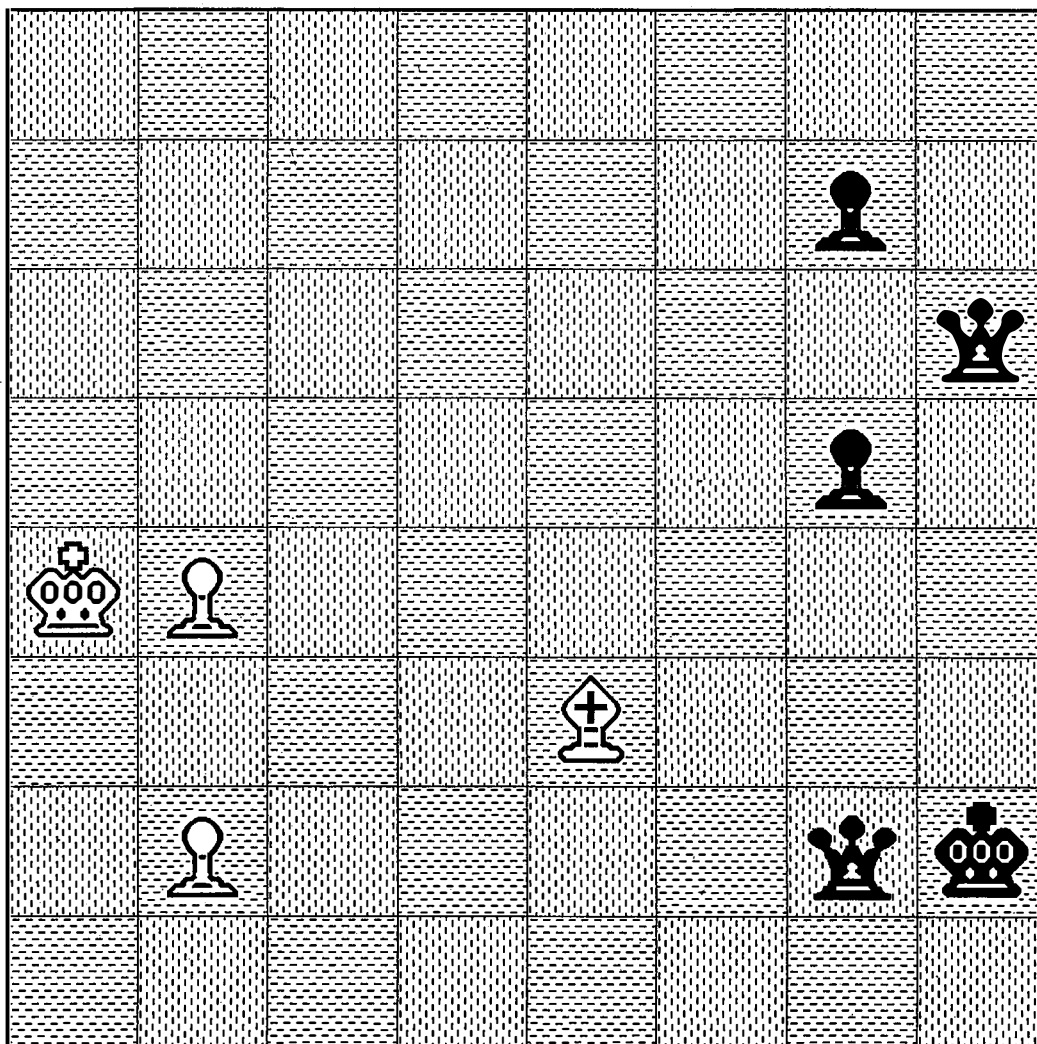
Holmes was careful to phrase his next question in the "What would your brother say?" form, and learned that White had given odds of the Queen.

I wondered which of the Basset brothers we were facing, and sought an independent answer to the question of homochromatic play. One insight was almost immediate.

"Holmes," I announced, "play could not have been homochromatic with up/down movement. That would make the vertical gray squares white, and the Kings would be on the wrong colors."

"That is quite true," said Holmes. "And what do you deduce about left/right movement?"

That question required more study. Finally I said, "I can rule out White movement to the left, but White movement to the right remains a possibility."

Holmes said, "Congratulations, Watson, on carrying the analysis that far. Ruling out White movement to the left shows you have learned some past lessons well.

"A quite different insight is required to address whether White moved to the right homochromatically.

"When you have answered that question, you will know whether Basset's statements are the truth, or merely a shaggy-dog story."

CAPTURE CHRONOLOGY

We decided to see whether the Ashleys, at a corner table, had an interesting game in progress. As we approached the table, Lord Ashley, playing Black, completed a Pawn capture on his right side (closer approach showed that the capture was at a6). I could not see what piece he had captured, and asked Holmes. He was of no help, having been distracted by a passing acquaintance.

The Ashleys stood to greet us, Lord Ashley waxing enthusiastic about Holmes's recent solution of a problem with three fallen pieces. "I have observed that you are somewhat

44. AT WHOSE HANDS?
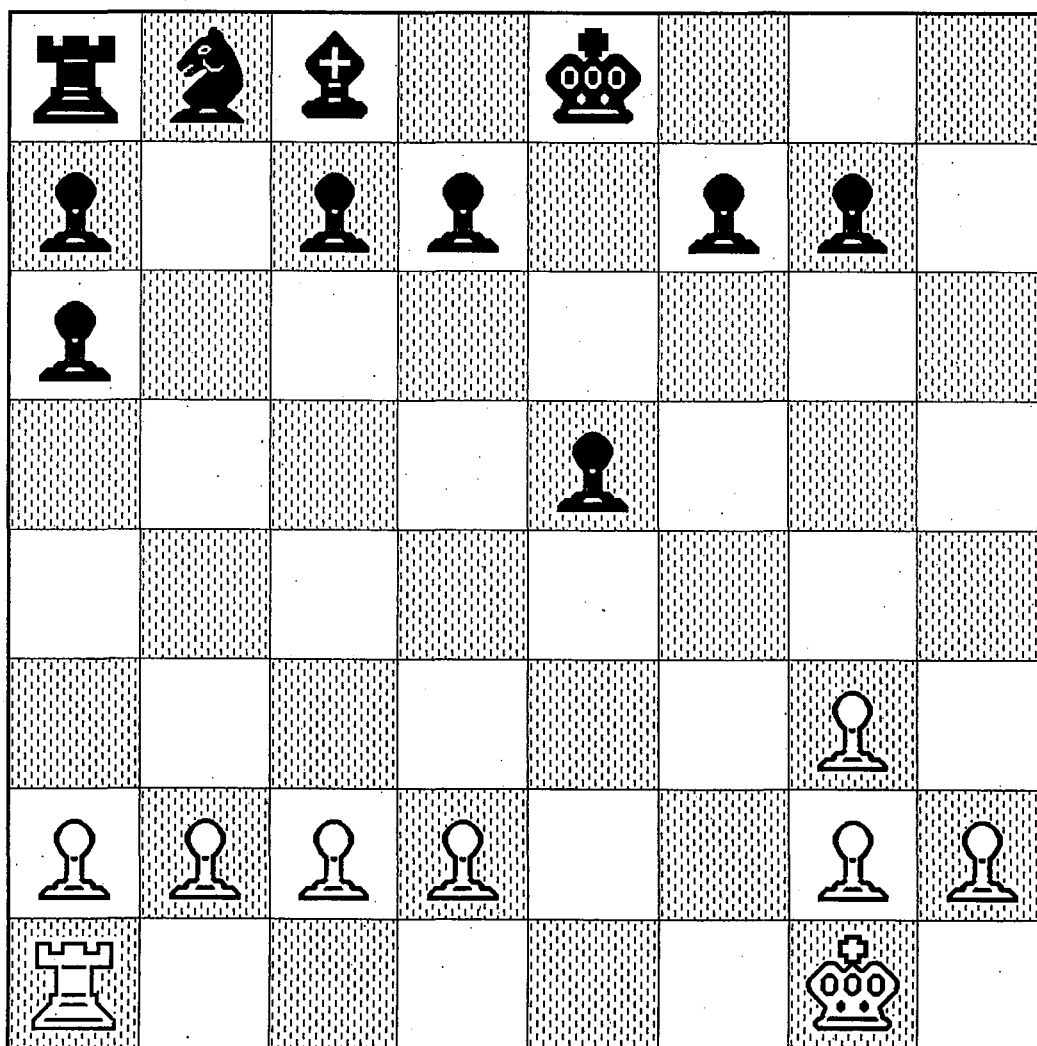
prone to jostling the board. Have you ever continued play with that many fallen pieces?" I inquired.

"Never with more than two fallen pieces," was his response. "It becomes too taxing to the memory."

"May we assume that this is another of your homochromatic games?" asked Holmes. "And has there been anything unusual about it?"

Lady Ashley replied this time, "It has been a routine homochromatic game, with no odds given. I did notice that no two captures were on the same square. And for the first time in several games, the first piece I captured was not a Bishop."

"One other aspect was a bit unusual," added Lord Ashley. "No Rook ever retreated from the most advanced rank it reached."

"Holmes," I interjected, "with twelve pieces missing, I don't suppose you can possibly identify the piece captured on the last move?"

"I can do better than that, Watson. I can demonstrate the complete order in which pieces were captured, where each piece was captured, and by what opposing piece."

I drew a chuckle from Holmes by commenting, "Since you don't seem to need my help, I will just take notes."

DEATH SITE FOR A ROOK

"Say, good Knight," graciously conceded Burns as his opponent checked the Black King.

"Thank you," said Allen. "You might say the check is in mail."

"At least there is no question of castling," I remarked to Holmes. We were looking over another position on a conventional board.

"If you had arrived a little sooner, there could have been such a question," said

45. AN OPEN FILE
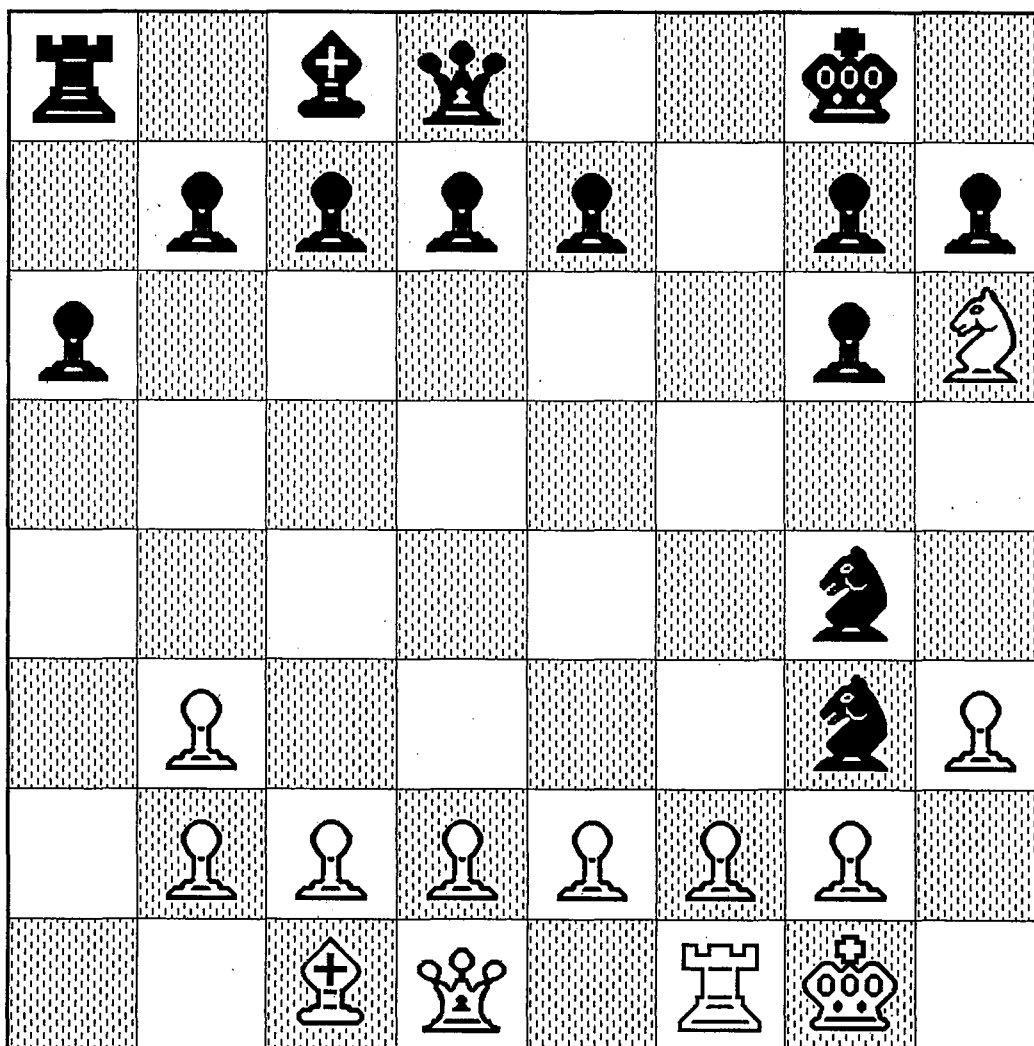
Burns. "We have both castled within the last four moves."

A gleam of the eyes suggested that the comment conveyed something to Holmes, who inquired casually, "I assume we *can* rule out the giving of any odds?"

Allen confirmed that no odds had been given. "We play as strict equals, though Burns is stricter than I am. All of the missing pieces have been captured. I imagine you can determine where some of them were captured, from clues or evidence, though we wiped up the bloodstains."

I realized quickly that each side had lost a Bishop, captured on its home square. The other missing Black piece, a Rook, was the only possible capture by the White Pawn at b3. In addition to a Bishop, White was missing one Rook and one Knight; a Black Pawn had captured one of those pieces at g6, but no other capture site was obvious.

"The piece captured at g6 was the White Knight," I stated confidently. "That capture had to occur before the Black Rook could reach the board. And, until after the Black Rook had been captured at b3, the White Rook could not reach the board to be captured. But there is no clue as to where the White Rook *was* eventually captured."

"I have two quarrels with your last statement," Holmes told me. "The White Rook's death site is completely determinable. And *'eventually'* has an objectionable connotation. To make amends, I require you to identify Black's last move as well as the Rook capture location."

PARITY OF FOUR

Cameron, playing Black on the chameleon board, castled on his Queen side as we approached. His opponent, Lyons, moved a hand toward his King, then stopped to greet us.

"It looked as if you might have planned to castle also," I remarked. "I see no obstacle, of course."

"Quite right," Lyons answered. "I do have the option of castling on either side."

46. MORE THAN ONE WAY
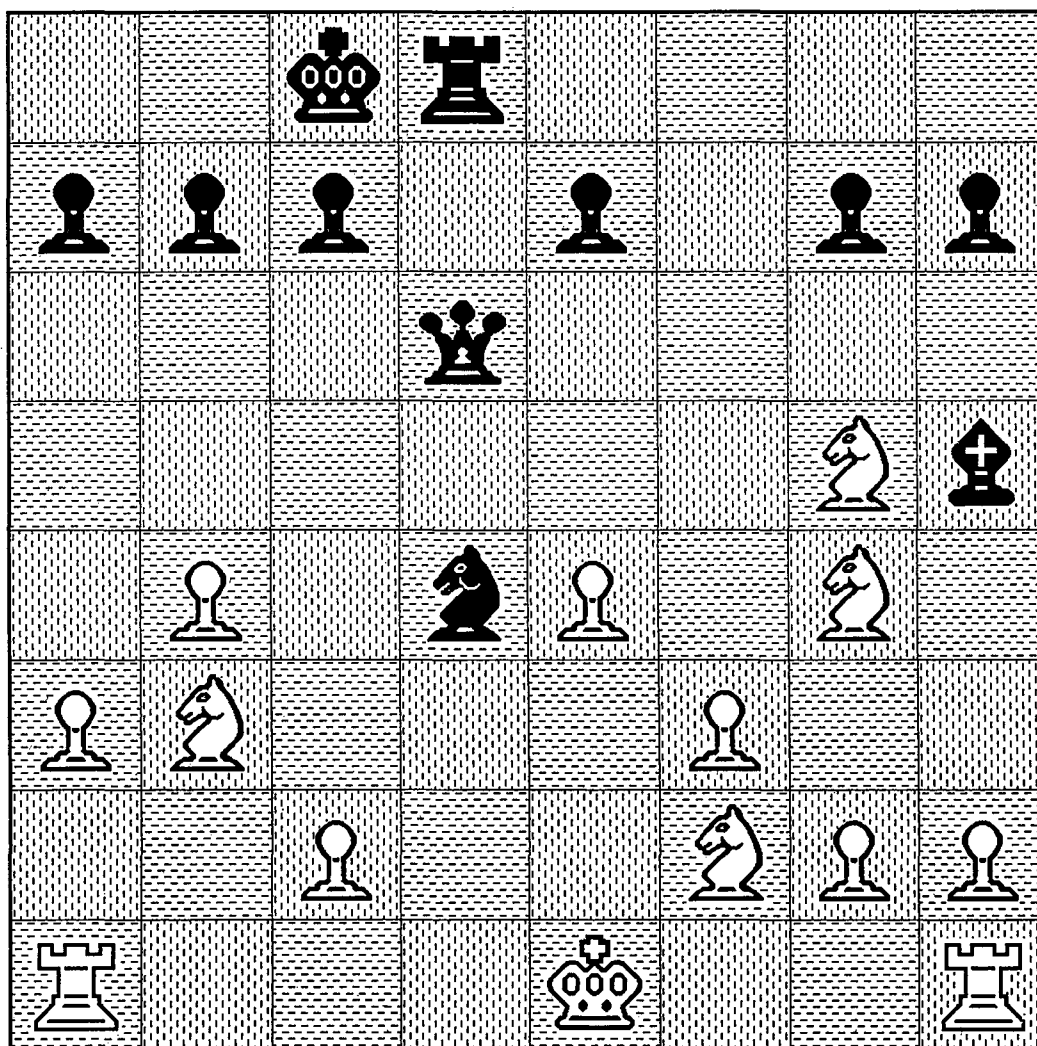
Holmes had been scrutinizing the position, and smilingly suggested, "Watson, with the suspense removed from the castling question, the only retrograde analysis problem seems to be whether White Knights have moved an odd or even number of times."

I shrugged, "Once the number of Knights on one side reaches three, continuing to add Knights does not greatly complicate the analysis of move parity. Of course, with reverse promotion possible, I must ensure that there have not been more than the visible *four* White Knights. And the promotion square colors may depend on which side has promoted."

AMBIGUOUS HETEROCHROMATIC

Holmes and I perused this deserted conventional board, noting especially the Knight of ambiguous color on d3 and a marker on g7 that might be any piece of either color, except a King, of course.

"Hello, gentlemen." The voice was that of Kingston, a step ahead of Huddler. "We left this game to seek refreshment, and now are embarrassed to admit we are not sure whose move it is. I don't suppose your vaunted skill at retrograde analysis could answer that question?"

47. ONE WITHOUT THE OTHER
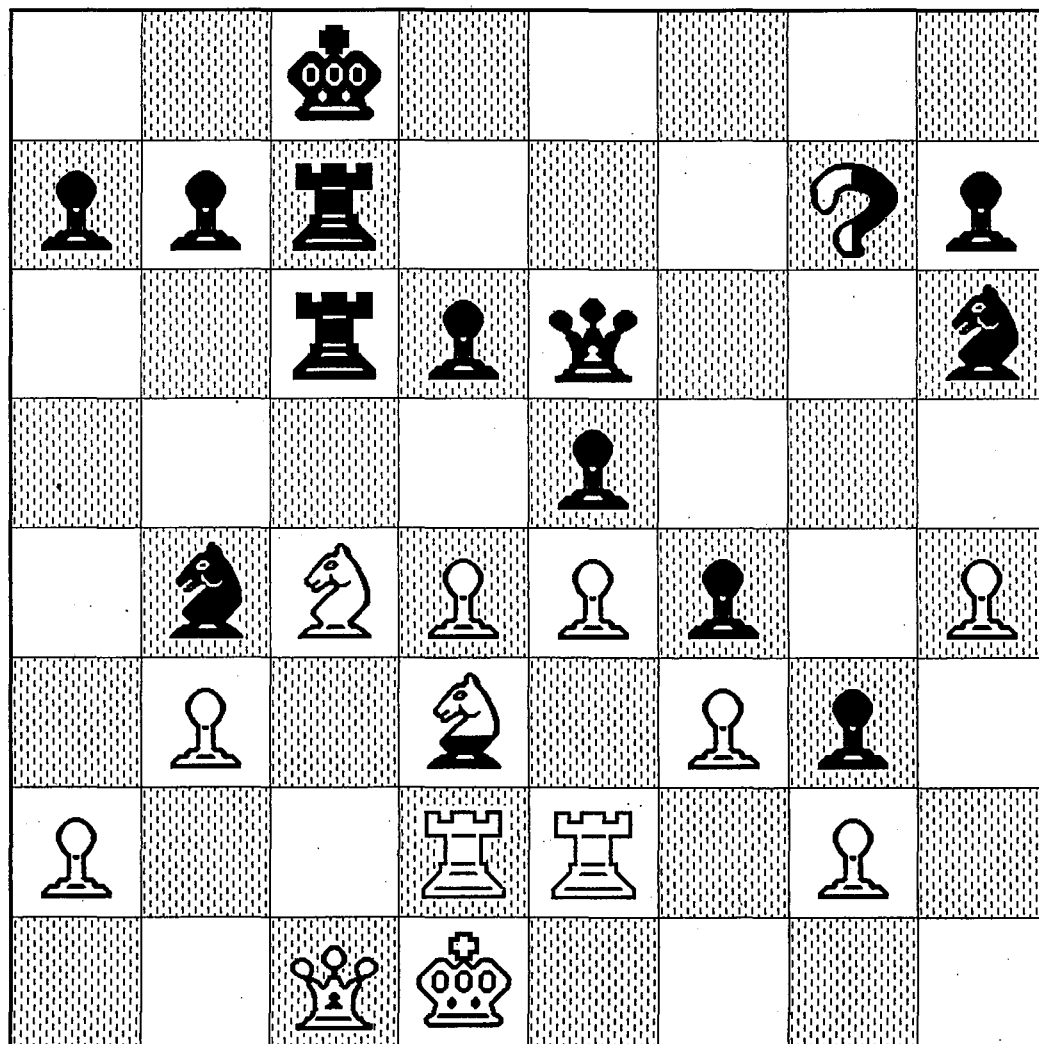
"Not without more information than the position affords," said Holmes. "Perhaps you were playing with a restrictive agreement?"

"By Jove, yes, that might help," said Huddler. "Our play has been heterochromatic. And, as White, I gave odds of the Queen's Bishop's Pawn."

"That helps enormously," Holmes assured him. "We now know the final locations of all 32 pieces, and a count of move parity can tell who has the move."

I protested mildly. "We can locate only 15 pieces of each color precisely. One of the two ambiguous pieces is White, necessarily the second White Knight, since no White Pawn has promoted, and that Knight may be on either d3 or g7. The missing Black Pawn clearly has promoted, to a third Black Knight if that on d3 is Black, or to a Black Knight, Rook, or Queen if the Knight on d3 is White.

"If we knew whether the piece on d3 or that on g7 is White, we could establish move parity separately for White and Black. If those parities are the same, Black moved last; if they differ, White moved last. If the two ambiguous pieces were on squares of the same color, we could solve the problem without knowing which piece is White. But with the ambiguous pieces on opposite square colors, White's move parity may be odd or even, and so may Black's."

"Yes, Watson," said Holmes, "but *not independently* of one another. Approach the problem with that outlook."

KNIGHTS ERRANT

"You must see the heterochromatic game being played by Russell and Whitehead," said Lord Ashley. "Lady Ashley is waiting there and prevailing on them not to make another move until you have seen the position."

Curiosity piqued, Holmes and I made haste to follow Lord Ashley. Lady Ashley introduced us to the two scholarly-looking players, now lounging over a chameleon board.

"I have watched the play from the beginning," said Lady Ashley. "White gave odds of the Queen's Bishop's Pawn. As you can see, two Knights that fell from the board have

48. TO HIDE A HIPPOPOTAMUS
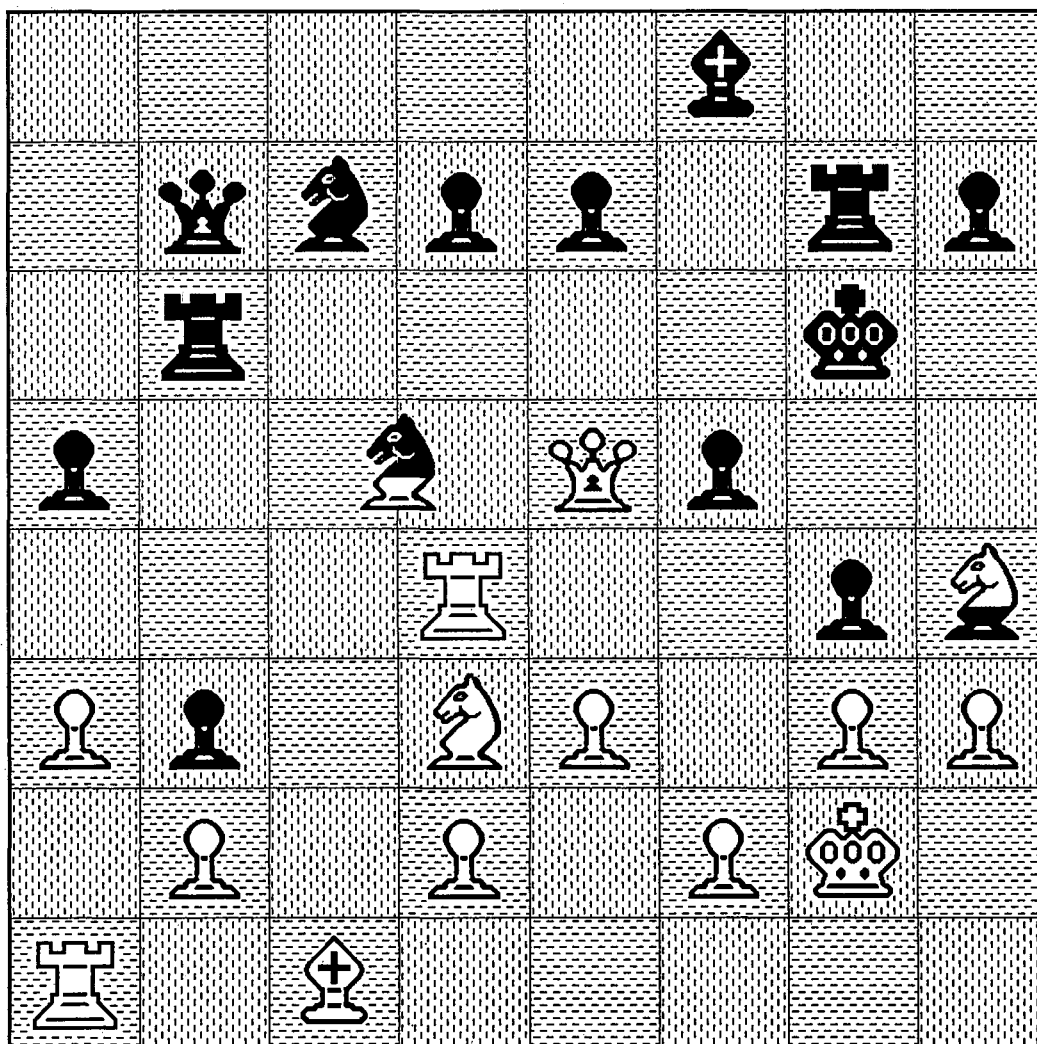
been replaced with ambiguously colored pieces, one of them a bit carelessly placed. I wondered whether retrograde analysis could determine absolutely which player made the last move."

I believe it was Whitehead who murmured, "That *would* be logical positivism."

Holmes and I surveyed the board. "Clearly," I said, "the Knight on h4 made the last move since, depending on its color, it is checking either the White King on g2 or the Black King on g6.

"Moreover, if the ambiguously placed Knight belongs on c5, White is in check from the Black Queen. In that case, to prevent mutual check, the Knight on h4 has to be Black, and made the last move from f3, checking and discovering the Queen check. On the other hand, if the ambiguously placed Knight belongs on d5, the only check is by the Knight on h4, which may be of either color."

"Watson," asked Holmes, "can heterochromatic move parity resolve the issue of who moved last?"

"It might if I could account for all pieces on both sides," I replied. "Including the Pawn given as odds and the Bishop captured on f1 without moving, the disposition of 15 White pieces is known completely; one of the ambiguous Knights *may* be a sixteenth White piece. Dispositions of 14 Black pieces are clear, with one ambiguous Knight a fifteenth. The problem is the Black Pawn from c7. With the White Bishop on c1, that Pawn did not promote to a *third* Black Knight. So one of the ambiguous Knights definitely *is* the sixteenth White piece. The other is the fifteenth Black piece. But we don't know where the missing Black Pawn was captured on its file; without that information, a count of move parity must remain incomplete."

"The position provides all of the information needed to determine the last square occupied by the missing Pawn," said Holmes. "Think of the classic problem of hiding a hippopotamus."

"I'm not familiar with that problem," said Russell. "I'm still working on the Spanish barber."

NUMBERS INSUFFICIENT

This abandoned game aroused my interest because of the Pawn of ambiguous color on g5. When I pointed it out to Holmes, he looked over the position quickly, and commented, "That isn't the sole unusual feature. With only five Pawns missing, there have been at least four promotions."

Verifying that assertion was not at all difficult. A third White Knight clearly proved one promotion; two Black Bishops on black squares proved a second. It took me a moment to notice that Bishops on f1 and f8 had been unable to move; they had been

49. TO RESOLVE A PAWN
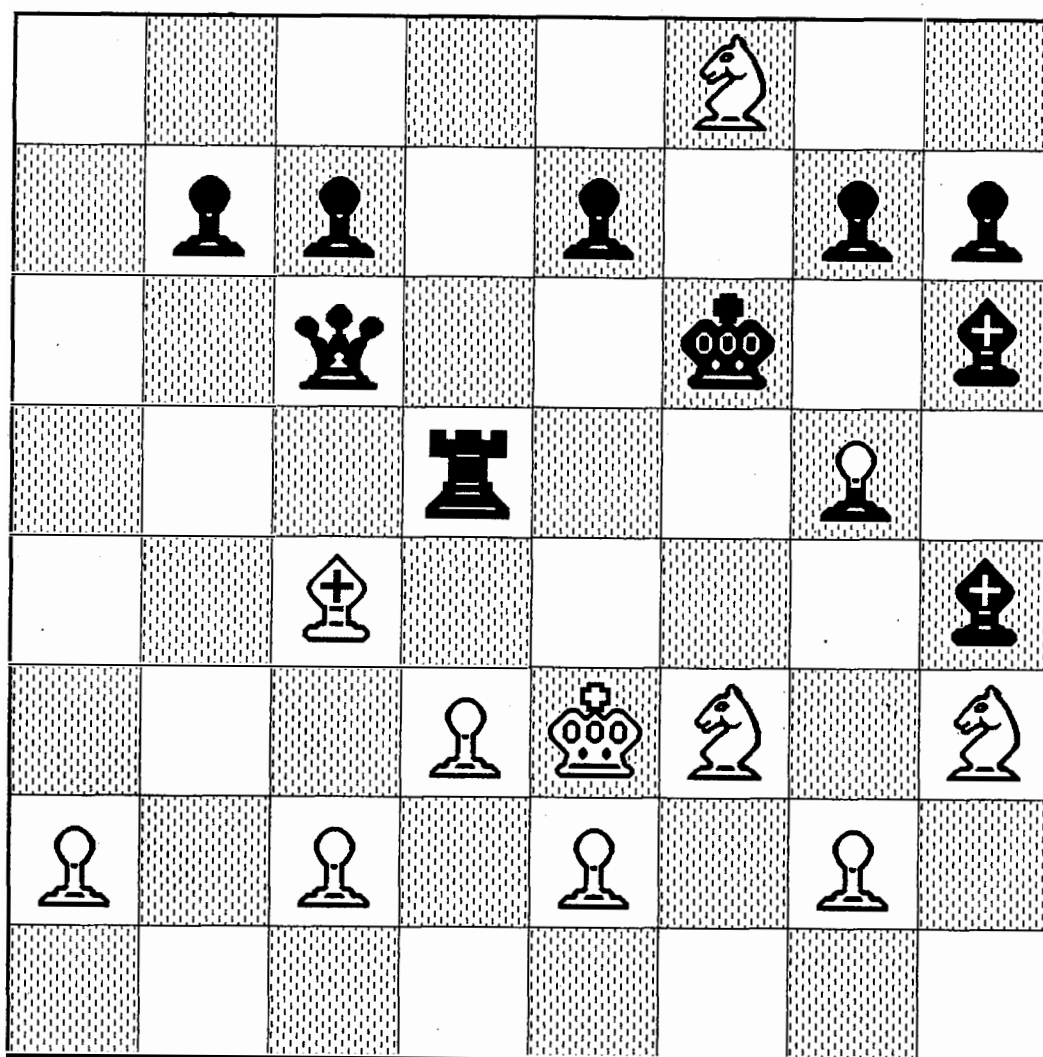
captured at home, and all three Bishops on the board were promoted.

"Depending on the color of the ambiguous Pawn, each side has either two or three missing Pawns, and two obvious promotions," I observed. "The count of promotions does not clearly resolve the ambiguity."

"No, it doesn't" Holmes agreed. "The *numbers* of promotions do not provide a solution. As someone has said, 'The devil is in the details.' If you consider the how and where of each promotion, those details will define the color of the ambiguous Pawn, and each player's last move."

CORRELATED CAPTURES

Lady Ashley called to us, and we hastened to join her and Lord Ashley. They stood by a table with a somewhat disordered board. A Rook of the ambiguous White-Left pattern stood on d7, and a White Pawn might belong on any of four squares.

Lord Ashley commented, "The Pawn location may offer some interest to you as a problem. At least, all fallen pieces have been replaced, and I would not expect the Rook color to cause you much difficulty."

Lady Ashley confirmed that this had been a homochromatic game, with no odds given.

50. HOMOCHROMATIC AMBIGUITIES
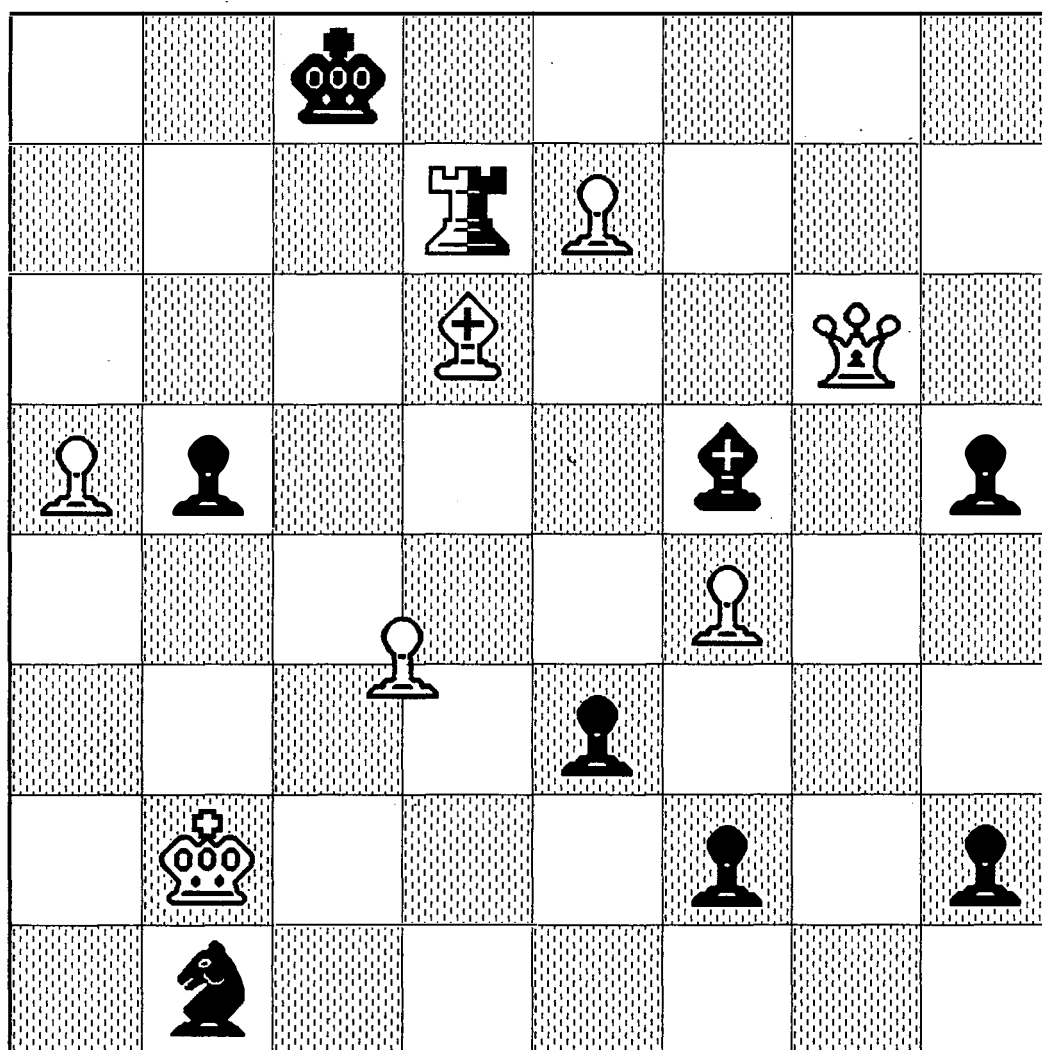
I was able to establish the move direction rather quickly, "If Black were moving down the board, the Knight on b1 would be promoted, but that is not a problem. The promoting Pawn could have captured three White Pawns, one of them en passant, with the fourth capture the White Knight originally on b1.

"But the Black Pawns on black squares e3, f2, and h2 would have required eight captures to reach those positions, with no more than three of those captures en passant. And no more than four White pieces that moved on black squares are missing -- at most one Pawn, a Bishop, a Rook, and an immobile Knight. So White is moving down."

"Quite so, Watson," agreed Holmes, who had given the position a thorough scrutiny before relaxing in satisfaction. "And what do you see as the next step in resolving the ambiguous Rook and Pawn?"

I was relieved at having a reasonable answer, "I would question the number of captures available to White Pawns on white squares. If the Rook on d7 is White, it is promoted and required four such captures. Those included three Pawns, one of those en passant, and the Black Rook on white. That would leave no further captures on white squares for the ambiguous Pawn, so it could only be on a black square, c3 or d4."

"Next," Holmes suggested, "you might examine the relationship between two captures – those on e3 and g1." He smiled in pleased self-satisfaction when Lady Ashley gasped in astonishment.

SIX DEATH SCENES

This was going to be a slow game. Morris, playing Black, was taking an inordinate time in deciding how to respond to check by Desmond's Knight. While waiting, I examined other aspects of the position, having just arrived on the scene.

"I say, Holmes," I commented. "This *is* unusual. White has castled, quite clearly. Since then, he has moved his King to the corner square and moved his Rook and Queen close behind it."

"Unusual, yes," Holmes replied. "But, together with the Knight check, it tells us only

51. AFTER CONSECUTIVE CASTLING
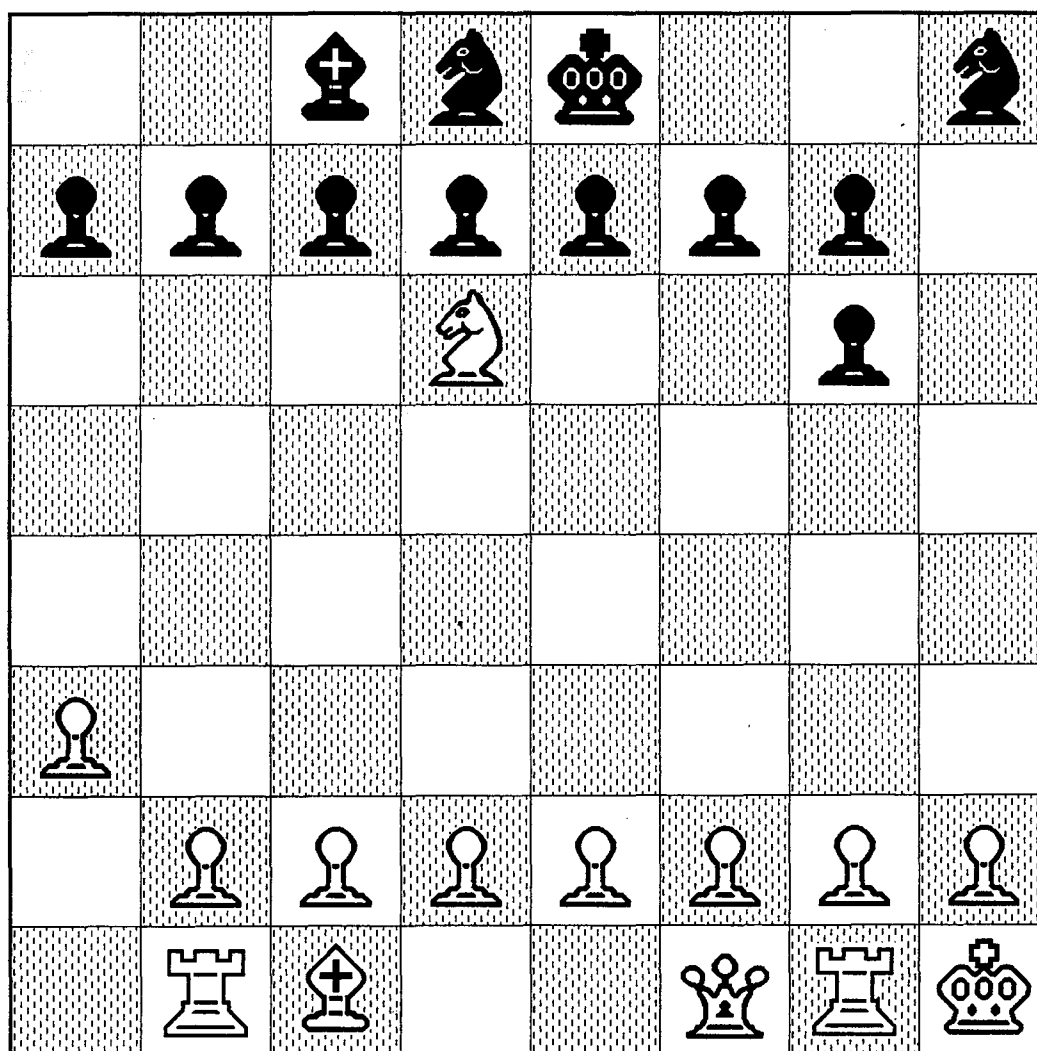
that White castled at least four moves ago."

Desmond looked up, and seemed to choose his words carefully, "And I have not made more than seven moves since I castled."

"Black seems not to have castled," I observed. "And he has allowed his Knight on h8 to be immobilized."

Morris responded to my possibly cavalier statement like an animal protecting its territory: "Watson, I *did* castle, immediately after Desmond castled. Obviously, I have moved several times subsequently."

That information seemed to inspire Holmes. His eyes darted about the board. Nodding with satisfaction, he said, "Watson, it is possible to determine where each of the six missing pieces died. Care to have a go at the problem?"

CASTLING AFTER PROMOTIONS

As we drew near, Oliver, playing White, castled. Wendell then castled on his Queen's side. Play paused when they noticed our presence. Oliver, always courtly, asked, "Holmes, would you like to pass judgment on our play?"

Holmes answered judiciously, "Our concern is only how the past has shaped the present. We don't question motives, but only means and opportunity. And our burden of proof requires not only ruling out reasonable doubt, but excluding even the most farfetched shadow of a doubt."

52. TWO-COLOR COORDINATES?
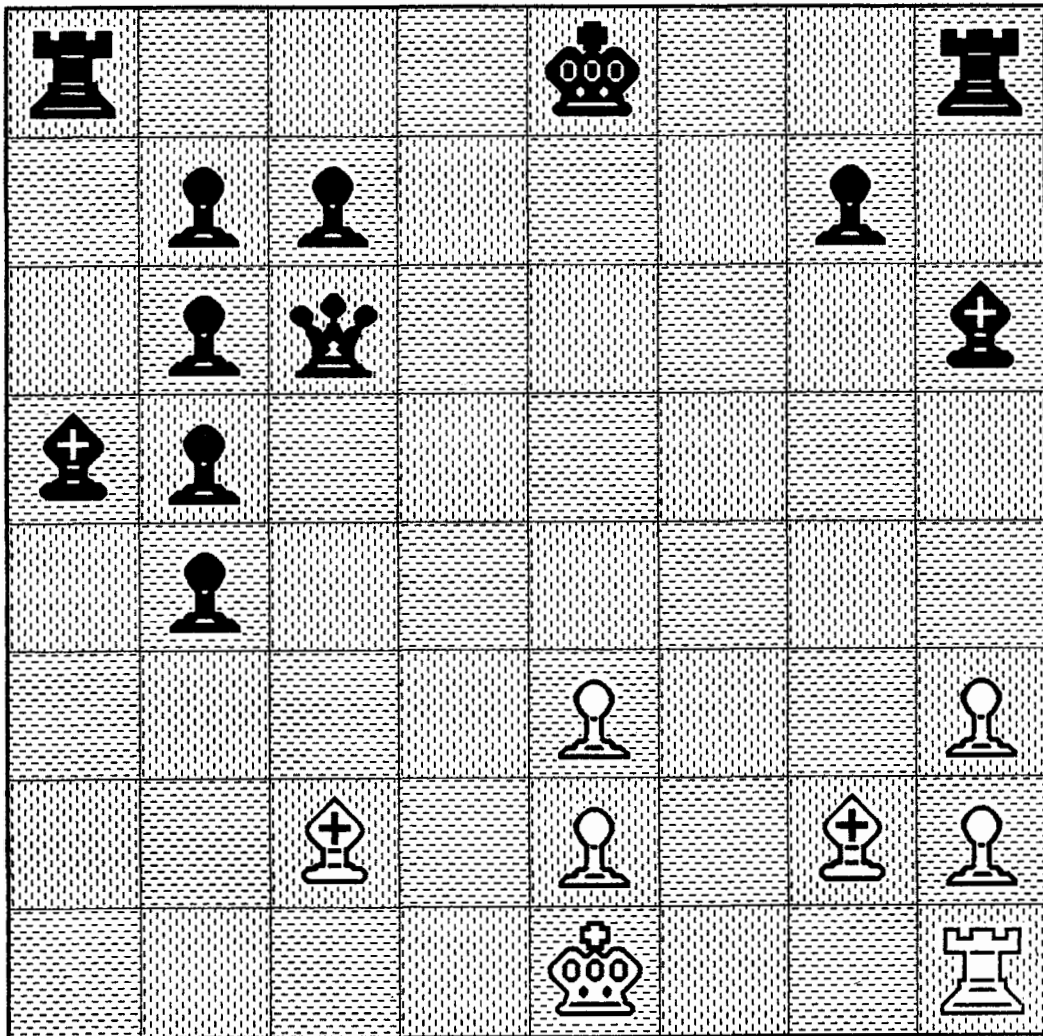
"Is the evidence sufficiently clear to permit a verdict?" asked Wendell, "or would you like to interrogate us?"

I interjected, "It is beyond dispute that two promoted pieces are on the board, one Bishop of each color."

Holmes added, "And I find that there has been a color-changing promotion. Watson, I appeal to you to settle the issue of where the obvious promotions occurred."

CASTLING COULD KEEP

Tinker leaned to his left and nimbly scooped up a piece, almost before it hit the carpet. "Got it on the short hop," he said, and lobbed it to Evers, the second player. The latter caught it with sure hands and touched it down on b2 -- a White Pawn.

"Only the one down?" I inquired. "Yes," answered Evers. "We didn't have a chance for another."

Tinker, playing Black, returned to cogitating over his next move. Evers confided, "I imagine Tinker is deciding whether and where to castle, with both options available."

53. PROMOTION ORDER
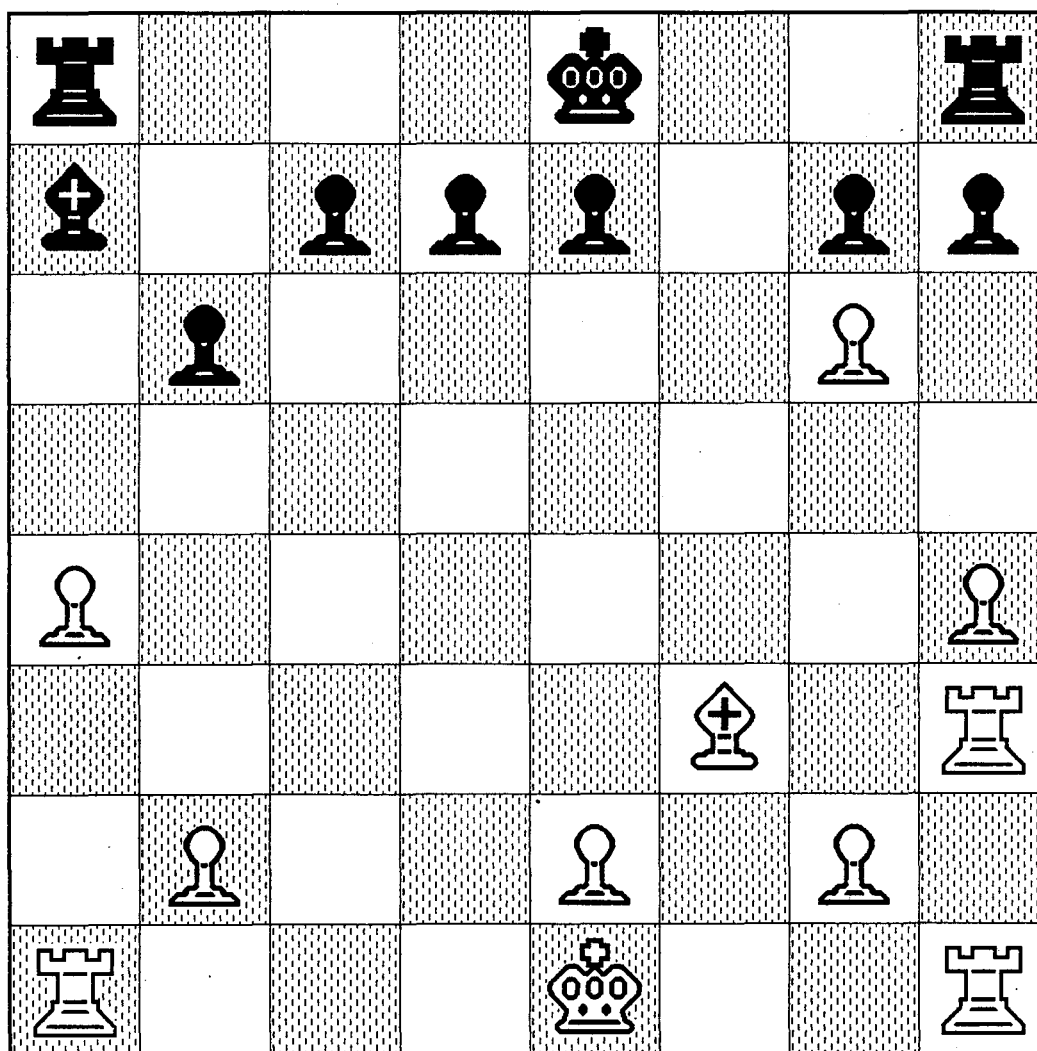
"As wide open as your White formation is," I said, "I can't imagine you have the same option."

"Surprisingly," he said, "I could have castled on either side on my last move. But I captured a Black piece, instead."

He continued, "Our play may have created a problem worth a few minutes of your time. I am sure you see the evidence of multiple promotions."

A third White Rook spoke clearly of a promotion. And a fairly brief perusal revealed that both Bishops were promoted, the original Bishops hemmed in by unmoved Pawns, and necessarily captured without having moved. "Yes," I said. "Three promotions are obvious."

"I am a mere cub in retrograde chess analysis," said Evers. "But I believe it should be possible for you to determine where each promotion occurred."

Holmes had remained silent, but heard every word while scanning the board. "That *is* possible," he ruled, "and it is possible further to identify the *order* in which the promotions occurred. As a byproduct, one can identify the piece you captured on your last move, and the capture location."

"I can see clearly where the two White promotions occurred," I said, "but find two possibilities for the Black promotion. I know which promotion occurred first, but not the order of the other two."

"Take it as far as you can, Watson," Holmes urged. "Perhaps then an assist will let you field the rest of the problem."

A PREMATURE CONCESSION?

Smathers gave us a shamefaced greeting as we approached a table that he and Ayres were just leaving. "I have been forced to resign," said Smathers, "confronted with an inevitable checkmate."

After commiserating, we looked over the now-deserted board. Black was in check from a White Rook on c8. It appeared that his only move was to interpose his Queen at b8. White could checkmate on the next move by a Pawn capture of the Black Queen and promotion to White Queen.

54. SMATHERS RESIGNS
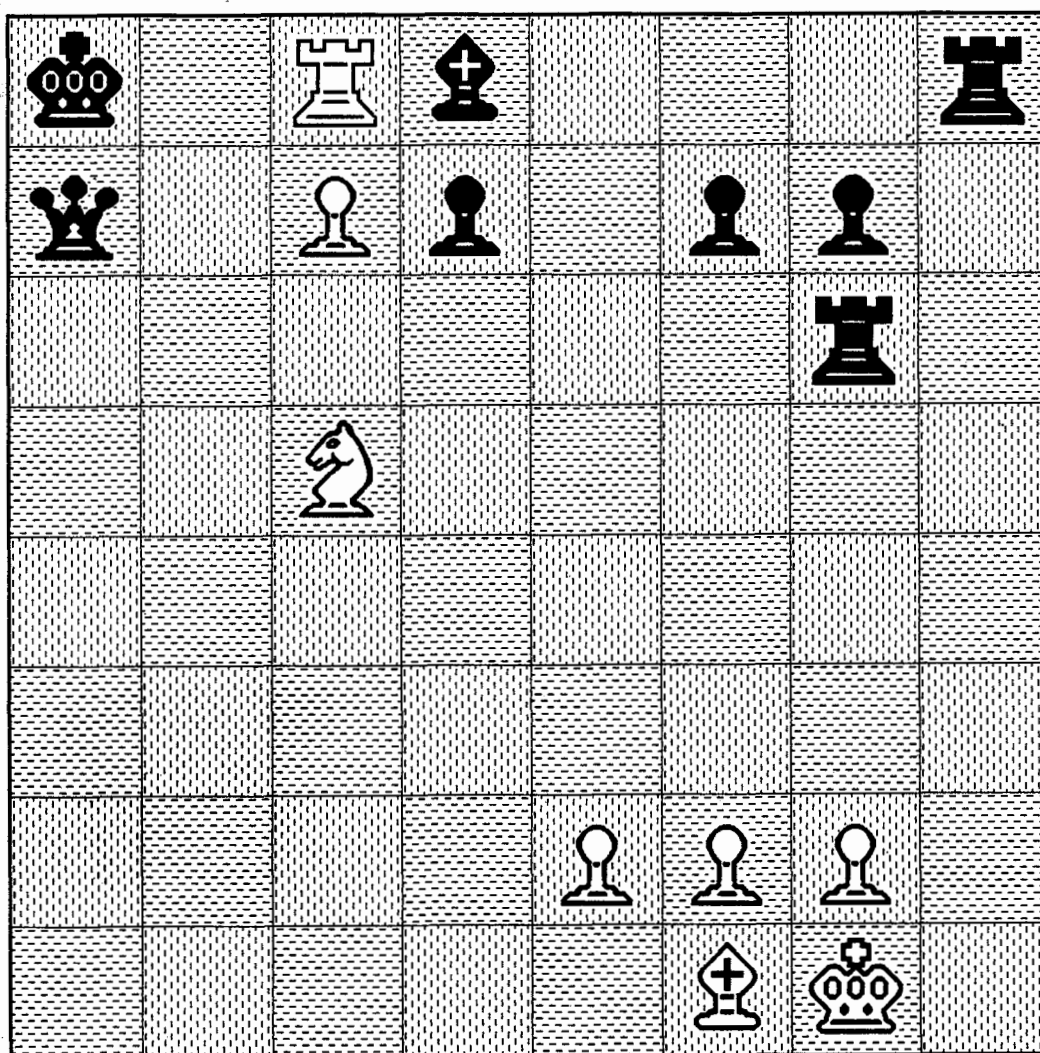
"Well," I said to Holmes, I am satisfied that Smathers resigned only at the last possible moment." I was astonished to hear that Holmes disagreed.

"A forced checkmate will require at least five moves," he said. "And Ayres will have to play very cautiously, indeed; his victory is by no means assured."

A light dawned. I said, "Of course, if Black is moving up the board, the Pawn on d7 can capture the checking Rook and promote. Material superiority would favor Black, but a checkmate would require several moves."

"I can assure you, Watson," said Holmes, "that Black is *not* moving up the board."

With Black moving down the board, I could see no move that would stave off immediate checkmate, and said so.

Holmes, in his exasperating fashion, said, "The next move will become obvious once you know what the last move was."

DISPLACED AND PROMOTED

Lady Ashley had informed us that Farquhart and Loman often played heterochromatic games, so we stopped at a table where they were playing. Both stood to greet us, without having made a move since we first saw them. We were left in the dark as to which was playing White.

"Our play has slowed considerably," said Farquhart, "since we have taken up compatible hobbies. We spend much time bringing one another up to date on those activities."

55. CORNERED PAWN
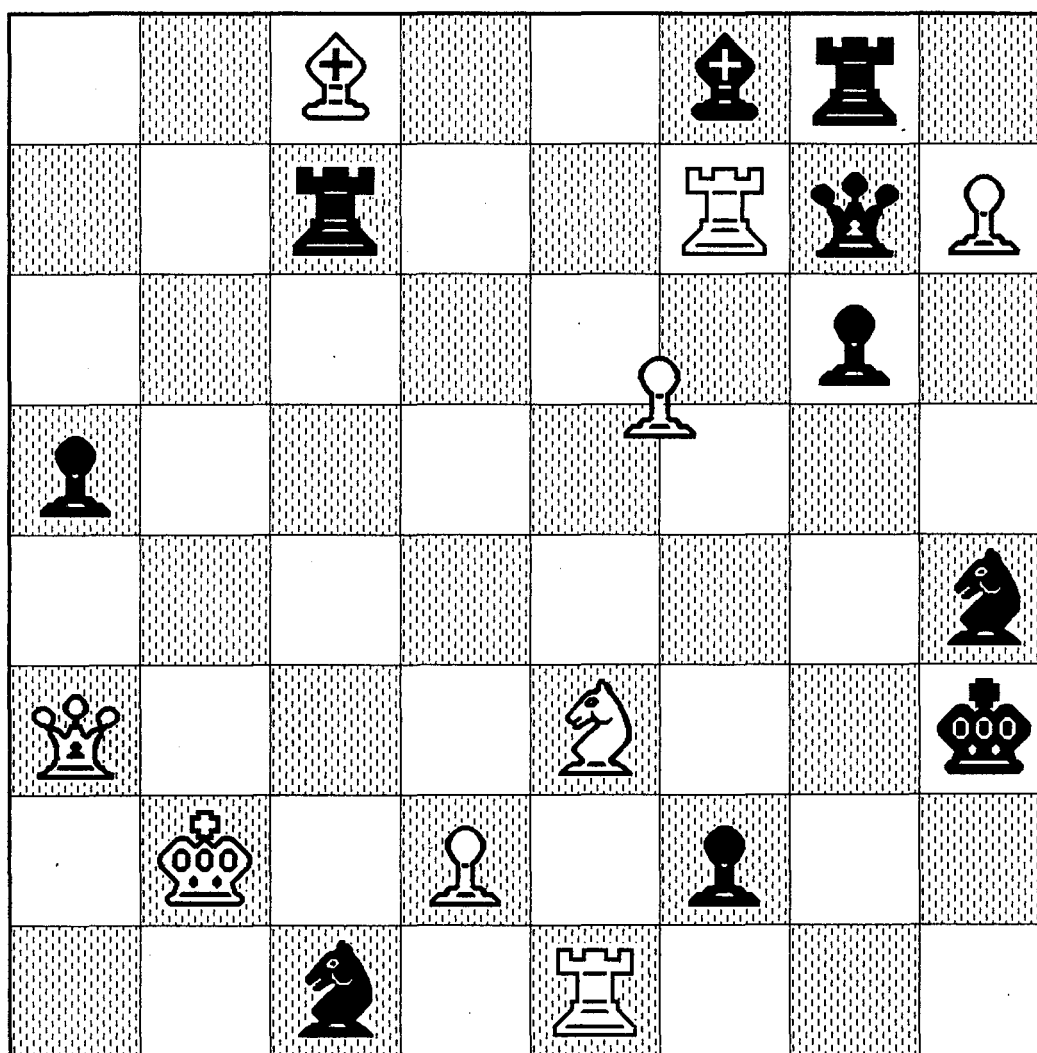
Loman added, "Farquhart has a son old enough to be interested in constructing ship models, but young enough to need much help. I have just been informed that the Farquharts made a galleon."

Though far more interested in the chess game, I inquired courteously, "And do you also construct model ships?"

"No," Loman answered. "I have been trying to recreate in miniature some artifacts carved by American Indians."

Farquhart nodded in confirmation, "Only recently I provided some carving assistance to Loman on a totem pole."

Feeling a bit like Alice at the Mad Tea Party, I tried to steer them back to discussion of chess, by referring to the most obvious feature of the board - the careless placement of a White Pawn, straddling the corner of four squares. "Was the Pawn moved by a rolling galleon or cornered by Indians?"

"And is this one of your heterochromatic games?" asked Holmes, possibly to fill the awkward silence at my poor jest.

"Yes," replied Loman. "Are you both becoming comfortable with analysis of that game form?"

Holmes deferred to me, and I volunteered, "Some aspects of the heterochromatic game are fairly familiar. For example, it is obvious that there has been at least one promotion. But I have not yet determined where the ambiguously located Pawn belongs."

"Once you know where the Pawn belongs," said Holmes, "you will realize that there have been precisely *two* promotions."

CASTLING WITH AMBIGUITIES

Carroll and Greensleeves, the latter seated behind the Black King, glanced up as we approached. Greensleeves announced, "It is my move, and I intend to castle. Would you like me to hold off until you have examined the position?"

Intrigued by three displaced pieces (Black Pawns on c7/d7 and c6/d6, and Carroll's White King on the intersection of four squares), I said, "Please do delay castling."

I continued, "Clearly, one of the displaced Black Pawns has to be on a white square, to block the check by the Bishop on a4 that would prevent castling. And the White King

56. ON WHICH SIDE?
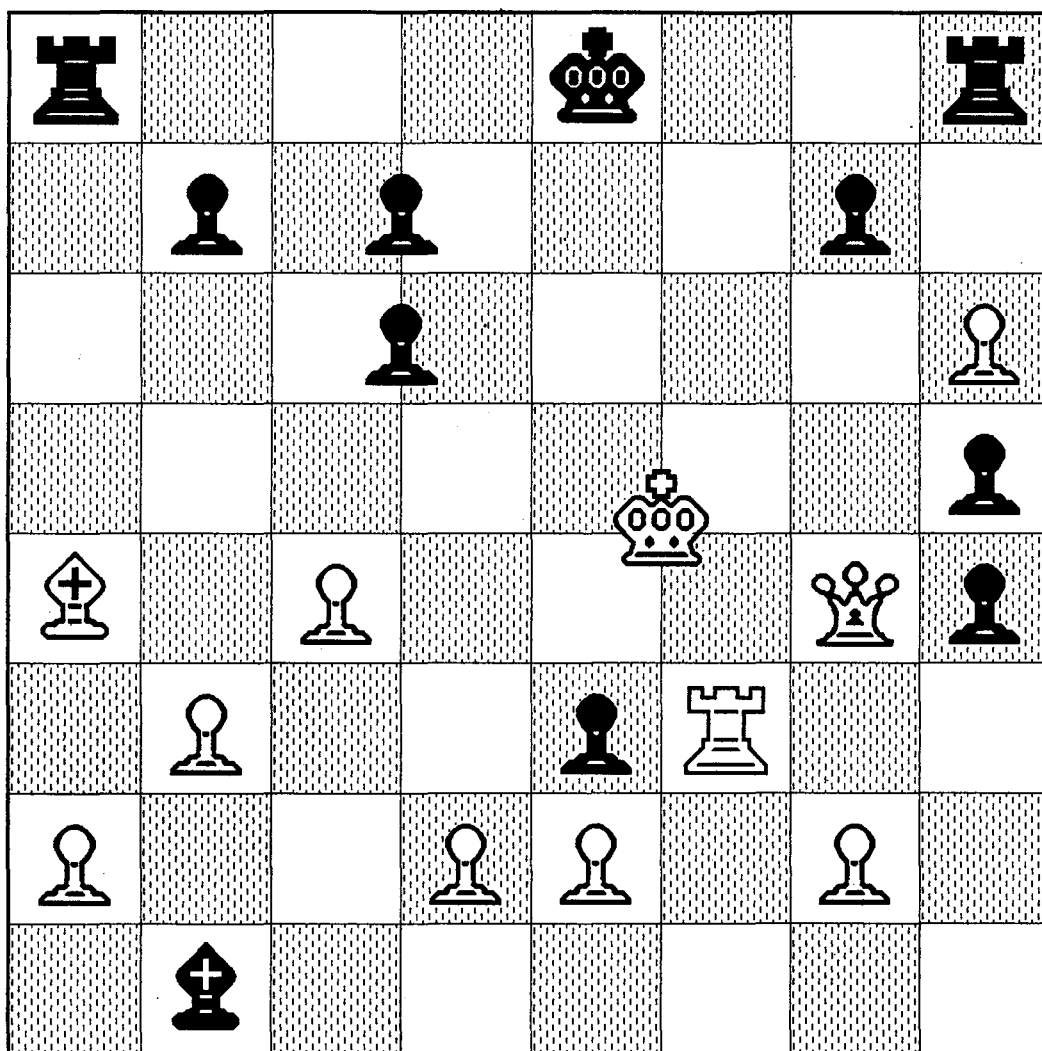
must be on one of the two black squares, or check by the Black Bishop on b1 would prevent castling or any other Black move.

"If we knew on which side Black will castle, we could go further in resolving the location ambiguity."

"And vice versa, Watson," said Holmes. "Resolving the location ambiguities clearly determines the side on which Black will castle. Which side is that?"

FEVEREL'S ORDEAL

Feverel sat alone, staring at the chess table so disconsolately that I felt obliged to cheer him up. I communicated my intention with a gesture to Holmes, and we approached the table. It was easy to see the reason for the melancholia. On the chameleon board, White had an overwhelming force arrayed against the lone Black King. "Feverel," I said with all the heartiness I could muster up, "join us for a whiskey. You can't win every game."

"But what a way for it to end," he complained. "See how one-sided it was. Meredith announced smugly that my next move would be my last, made his move, and left without

57. FORCED FINALE
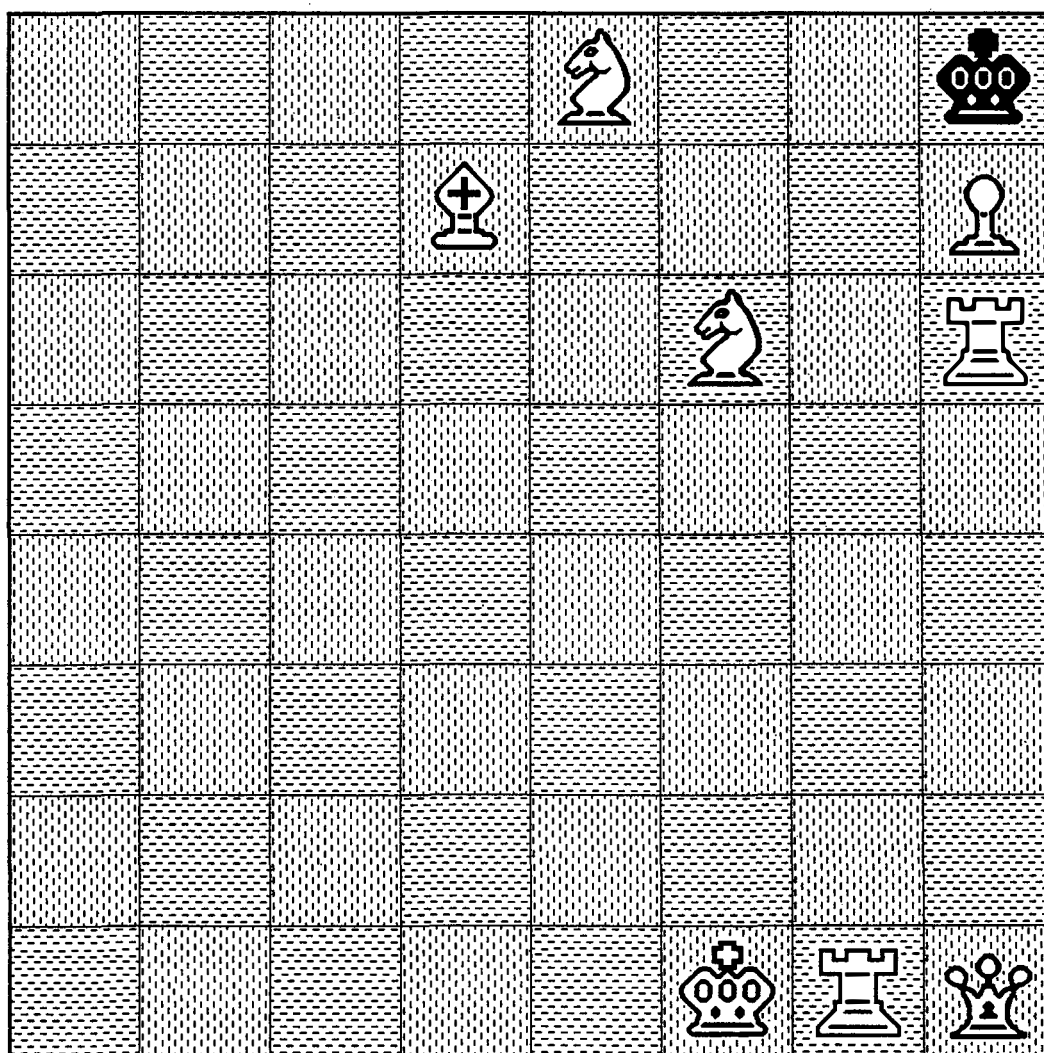
even waiting for my move or my concurrence.''

Holmes, who had been examining the board, chimed in, "Clearly, you did move after Meredith left. I assume that you did not capture on that move?''

"No," said Feverel, "neither of us captured in the last three moves.''

Trying for a bit of levity, I said, "At least it is clear which piece you moved last, and from where.''

"I'm amazed that you can determine my last move, Watson," said Feverel. "Frankly, for a moment I was convinced you thought I had been playing Black.''

"I have been careful," Holmes chuckled, "to keep my comments from revealing that the situation is a stalemate rather than an imminent checkmate. I can imagine your frustration at being confronted with a forced stalemate despite your impressive advantage in material.''

"Now, Holmes," my voice somewhat cross, "You cannot know certainly that Feverel was not playing Black, his last move a King move. I concede that he cannot have moved from g7; there, he would have been in double check, from the White Rook and the Knight on e8, with no Knight move that could have discovered the Rook check.

"If White were moving up, the Black King could not have made the final move from g8, where it would have been in an impossible triple check, from Pawn, Knight, and Rook. But, with White moving down, the Black King *could* have made the final move from g8. There, it would have been in double check by the Rook and the Knight on f6, but a Knight move from g4 could have produced that double check.''

Holmes explained, "But Black would not have been *forced* to move from g8 to h8. He could have staved off immediate checkmate by moving from g8 to f7 or f8.

"With White moving down, it is also possible that the Black King was already on h8, and that Black's last move was a Pawn from e7 to e8, promoting to the White Knight. But that move also would not have been forced. Only the promoting move would have been available to Black, but the promotion could have been to a Black Queen or Rook, either of which would have prevented a next-move White checkmate with the White Rook to g8.

"It is also possible that White was moving up, with Black's last move a reverse promotion on row 1. With capture ruled out, the promotion was not to the White Rook; the promoting Pawn could have moved only from g2, where it would have given imaginary check. The move had to be from h2, with promotion to the White Queen. Note, again, that Black would have had an alternative. Although Black did not capture on the final move, he *could* have captured the Rook on g1, promoting to a Black Queen or Rook. The White King would have been in check, and a King move or capture of the checking piece would deny White an immediate checkmate.

"White had no *forced* one-move checkmate. A forced one-move conclusion had to be a stalemate forced by Black. Feverel, who moved last, was playing White.''

NO LATE WHITE PAWN MOVES

Payson, seated as Black, and Terhune as White were chatting with Collier, their game temporarily at a standstill. We overheard Collier say, "That *is* quite a long time."

"Oh, hello Holmes, Watson," said Terhune. "I had just mentioned to Collier that my last Pawn move occurred before the twentieth move of the game. I believe you heard his comment."

Holmes pricked up his ears, like a hound on the scent, and his eyes darted all over the board. To make up for his inattention, I offered a comment, "A position like this one

58. A CASTLING MATRIX
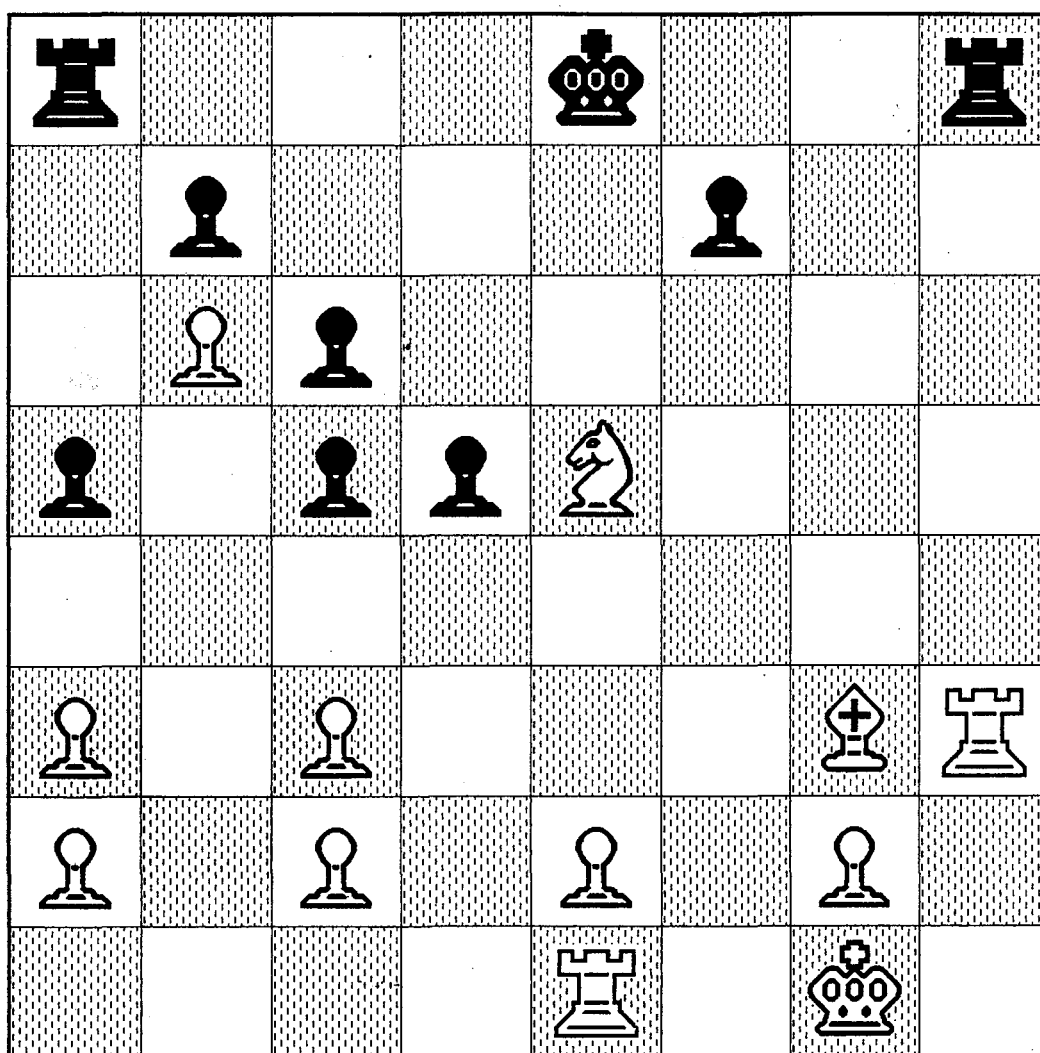
always raises the same question in the mind of a retrograde chess analyst: can Black castle? I imagine that is the question that Holmes is pursuing so intently. I must confess that a quick perusal leaves me without a clue as to the answer.

"There is no evidence that a White Pawn has promoted, and only one could have. The five Pawns on the a-, b-, and c-files have captured six of the seven missing Black pieces. If the Pawn from h2 promoted, with only one remaining capture available, it could promote on either g8 or h8. A White Pawn never came close to the Black King or its Queen's Rook, to force a move that would deny castling. So, although the ability to castle can almost never be proved in the affirmative, there is no reason to reject the possibility of Black castling."

Holmes glanced up and asked Terhune, "Can you assure me that you were, in fact, playing White?"

"Yes, old chap," Terhune replied. "We have not moved from our seats since the beginning of the game."

Holmes appeared to be under considerable stress. "Watson," he asked, "do you have a pencil and paper with you?"

I pulled pencil and dog-eared notebook from an inner pocket, opened the notebook to an empty page, and proffered it to Holmes. He began to draw up a table, his eyes flitting from place to place on the board before every tabular entry. All of us were staring at this unaccustomed activity, and silence reigned.

After a somewhat embarrassing interval, Holmes turned his attention to us and said, "Initially, I was not certain the question of castling could be answered conclusively. I have now determined that it can. I confess I could not have solved the problem without pencil and paper, and doubt that anyone could."

"What kind of information have you been tabulating, Holmes?" asked Payson.

Holmes answered, "The minimum number of moves required for each Black piece to reach each square on which a White Pawn captured."

I began to see an inkling of his approach.

INSTANT REPLAY

Holmes excused himself, obviously embarrassed by our flattery, while Payson, Terhune, Collier and I voiced our amazement over the fantastic application of move counting we had just witnessed. When Holmes returned, he said, "You are not going to believe the coincidence I am about to reveal." He led us to a nearby table where, to our astonishment, the position was identical to that just analyzed.

Greer was playing White. "Don't tell me," I said, "that your last Pawn move occurred before the 20th move of the game."

59. SHADES OF GREY
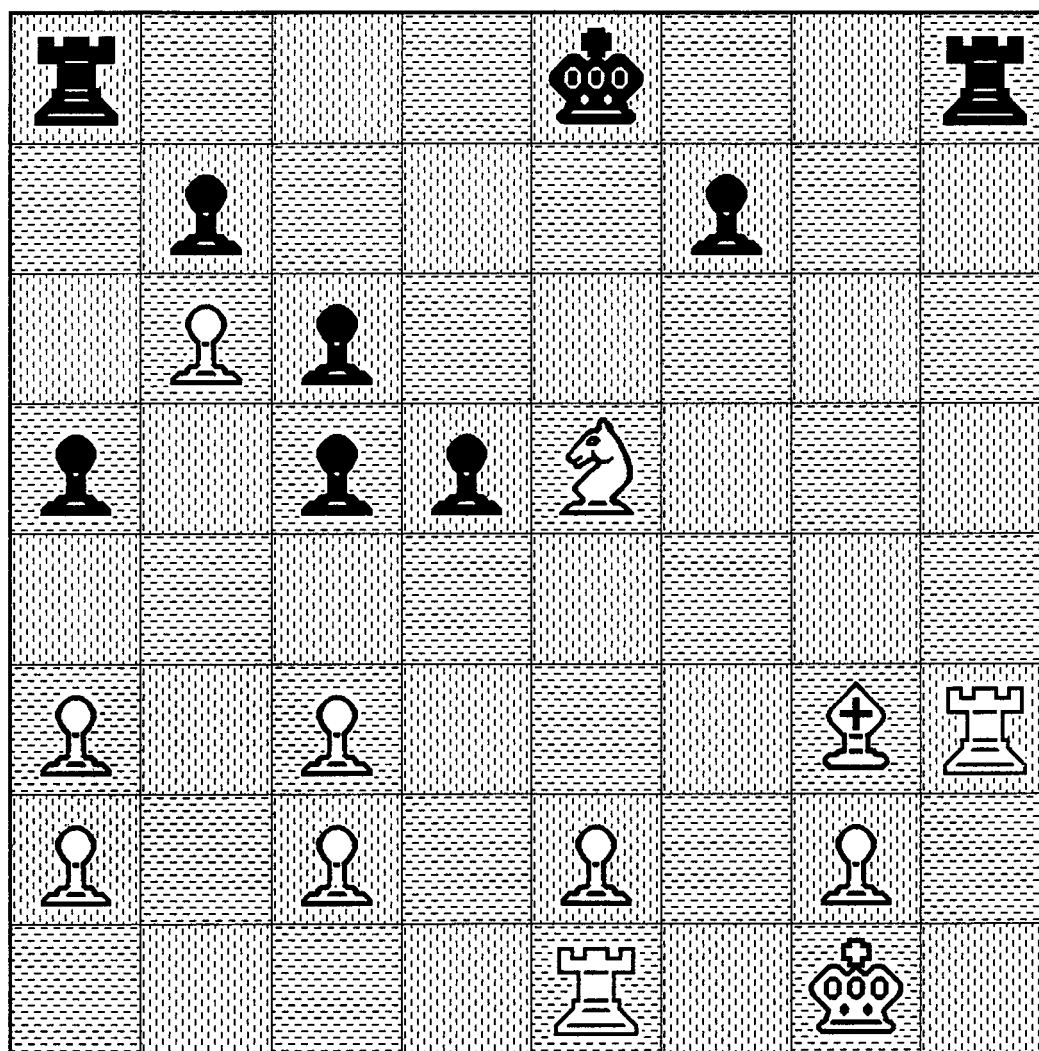
"Good lord, Watson!" Greer exclaimed. "It certainly did, but I am amazed that you could deduce that fact."

Not troubling to explain, I turned to Garson, playing Black, and asserted, "It is quite clear that you can't castle."

"I certainly can!" Garson insisted indignantly. "And I *will.* With both options open, I am trying to decide on which side to castle."

I appealed to Holmes, "How can this be?"

Holmes chuckled, "The two situations are *almost* identical. But there *are* shades of grey."

I realized the difference to which he was alluding. *This* game had been played on the chameleon board.

After we had analyzed the difference the board made, Holmes confessed that he was pulling my leg. He had conspired with Greer and Garson to set up the position, and coached them in their lines. I must say their acting deserved acclamation.

WHO IS IN CHECK?

Langley and Wright were concentrating on their game. The only unusual feature I noticed was a Queen of ambiguous color on square c5. It was on the same file as the White King, and on the same diagonal as the Black King, checking one King, but which one?

"I see no indication of any captures by Pawns," I remarked. "In fact, few pieces seem to have gone down at all."

Langley said, "No, but the last move *was* a capture."

60. INDETERMINATE QUEEN
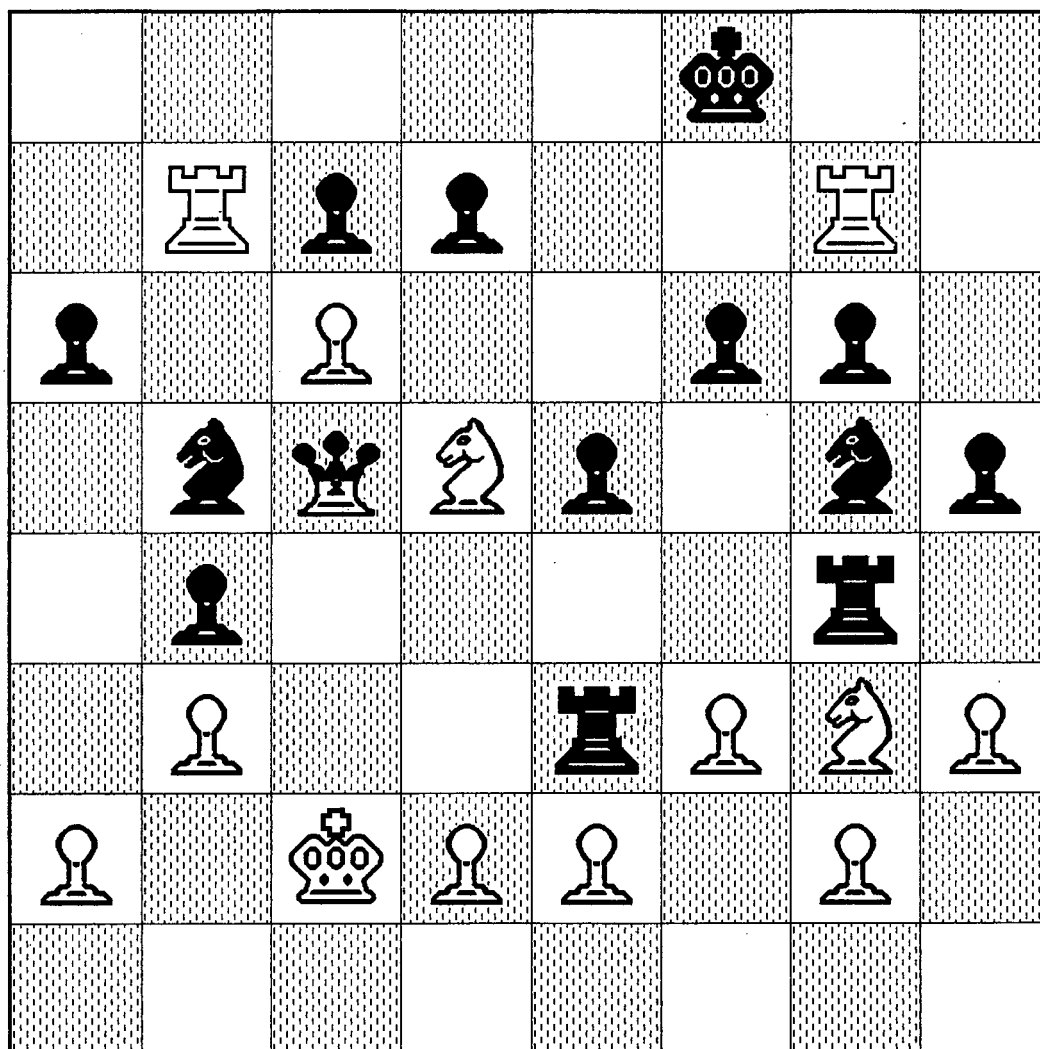
That told very little to me and, from the expression on his face, equally little to Holmes. I spent some time in convincing myself that the Rooks could have navigated the narrow lanes left by Pawns; they could.

After Holmes had scanned the board, a thought seemed to strike him, and he asked, "Is it possible that your game has been heterochromatic?"

"Yes, we *have* been playing heterochromatically," said Wright. "That game form gives both of us a lift. A good plan is essential, of course."

Langley concurred, "Not nearly so draggy as a cross-section of conventional games we have profiled."

"Then, gentlemen, if you will indulge us for a few minutes, the game does present a problem in retrograde analysis. I have solved the problem, but would appreciate your letting Watson work through it."

"What problem do you have in mind?" asked Langley.

Holmes answered, "Determining whose move it is, and what the last move was."

Wright and Langley glanced at each other, and nodded in common consent. Wright said, "It might be interesting to observe Watson's route to a solution." And both sat back in their chairs.

"Carry on, Watson," Holmes invited. "Preferably without a lot of excess baggage."

MATING NEAR

With the apparent move direction, Chatterley was close to being mated by Mellors on this chameleon board. White had an apparent triple check, involving a Rook on e5 and Queens on both a4 and h5.

"Obviously, a piece has fallen from the board," I said.

"Correction, Watson," said Holmes. "It is obvious that at least *two* pieces have fallen."

61. THREE CHECKING?
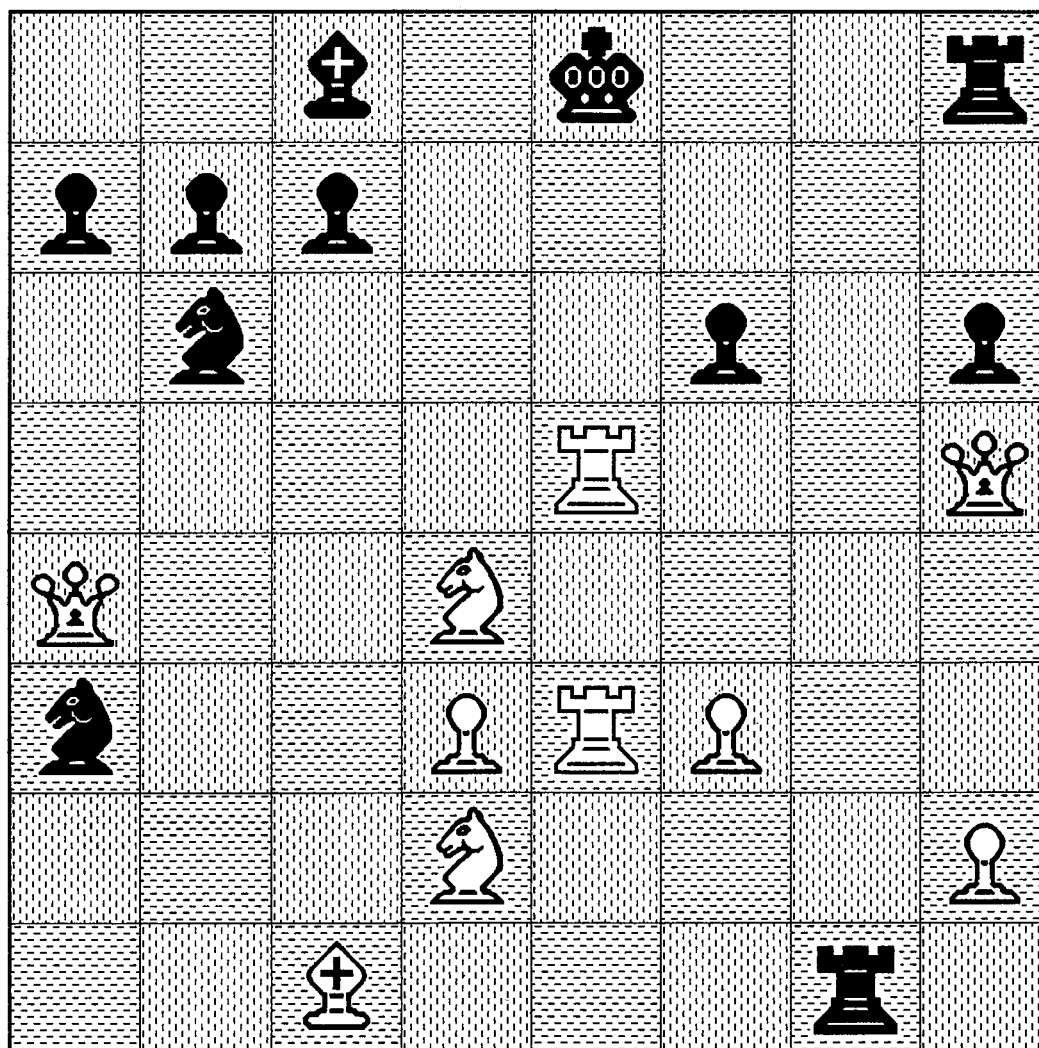
"Yes," said Chatterley. "Although he knows his game, Mellors *is* occasionally awkward. This time he has knocked off two pieces."

"It's partly Chatterley's fault, insisting on fancy variations," said Mellors. "This game has been heterochromatic."

"You can spare us further details," said Holmes. "I believe that Watson can delineate the position and your last move."

"I'll see if I can put things together," I said. "There doesn't seem to be anything novel here."

ONE PLAYER TWO SHORT

Greystoke and Burroughs were on their knees beside the table. When we greeted them, both resumed their feet, but did not seat themselves.

Burroughs explained, "Two of my pieces have fallen from the board. We shall have to continue without them."

Greystoke added, "Before the last move, Black could have castled."

"That information *does* completely identify the last move," said Holmes.

62. CANCELED CHECKS
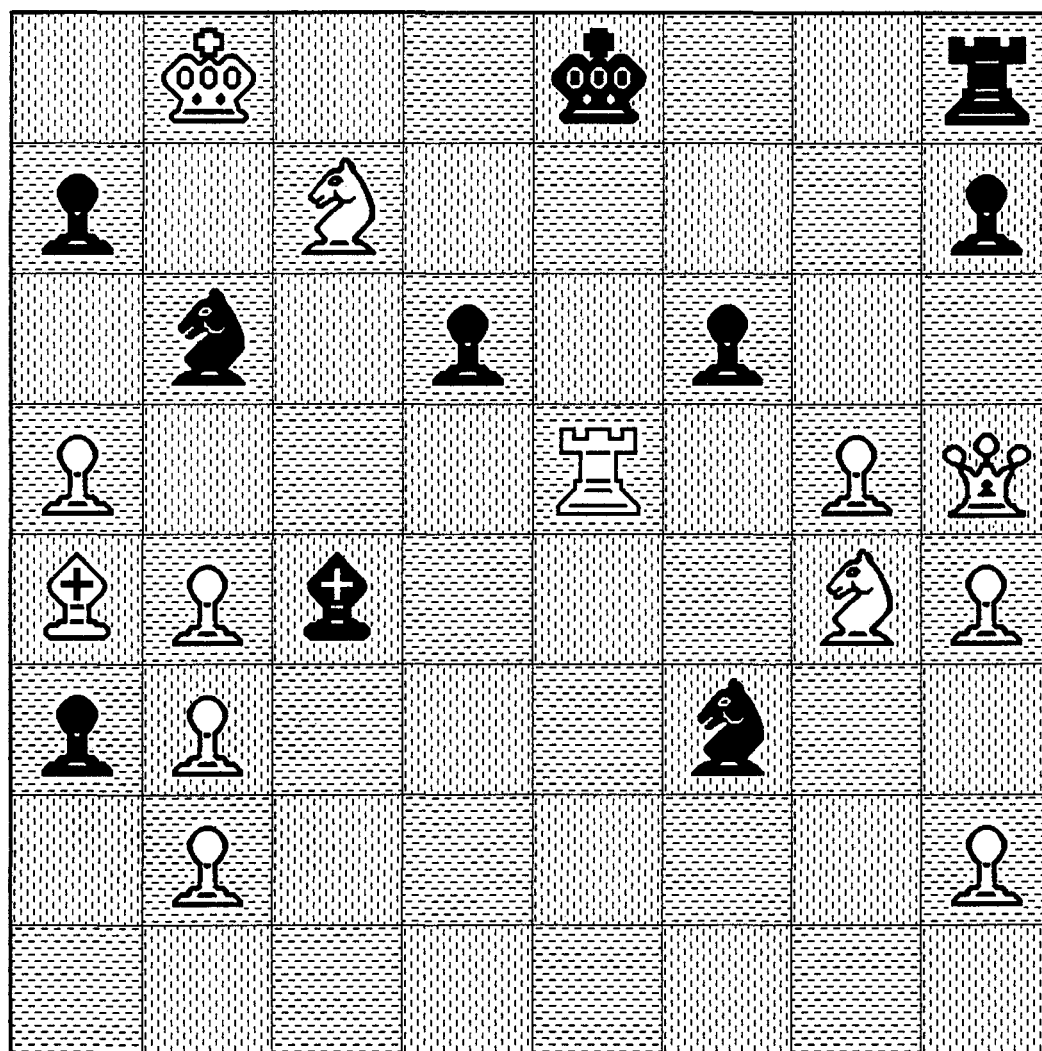
"Holmes," I protested, "it was already obvious that the last move was by the White Knight, to c7, since it is giving check."

"True as far as that goes, Watson," said Holmes. "But before Burroughs limited the fallen pieces, the Knight might have moved from any of five squares. Knowledge that there are two fallen pieces of the same color reduces the possibilities to two specific Knight moves. And only one of those two moves is consistent with Black's freedom to castle before the Knight move."

I silently worked through the possibilities, then nodded affirmatively. "Yes, I see the only possible last Knight move. In fact, if chameleon promotion were ruled out, it would be possible to identify and position the two fallen pieces."

"That *is* possible," Holmes asserted, "even *with* chameleon promotion. Suppose you begin the analysis, Watson, and I will chime in if necessary."

WATSON THINKS HE CAN

Holmes and I stopped, both riveted by the first game we had observed that used only the pieces of ambiguous colors together with the chameleon board. Holmes introduced me to Masters, who in turn introduced his opponent, Miss Johnson.

"We have been playing a heterochromatic game," said Miss Johnson. "Can you make any sense of it?"

I volunteered, "It has become second nature for me to look for some specific clues in heterochromatic play. The ambiguous piece colors *do* require greater concentration to see

63. BOTH LAST MOVES
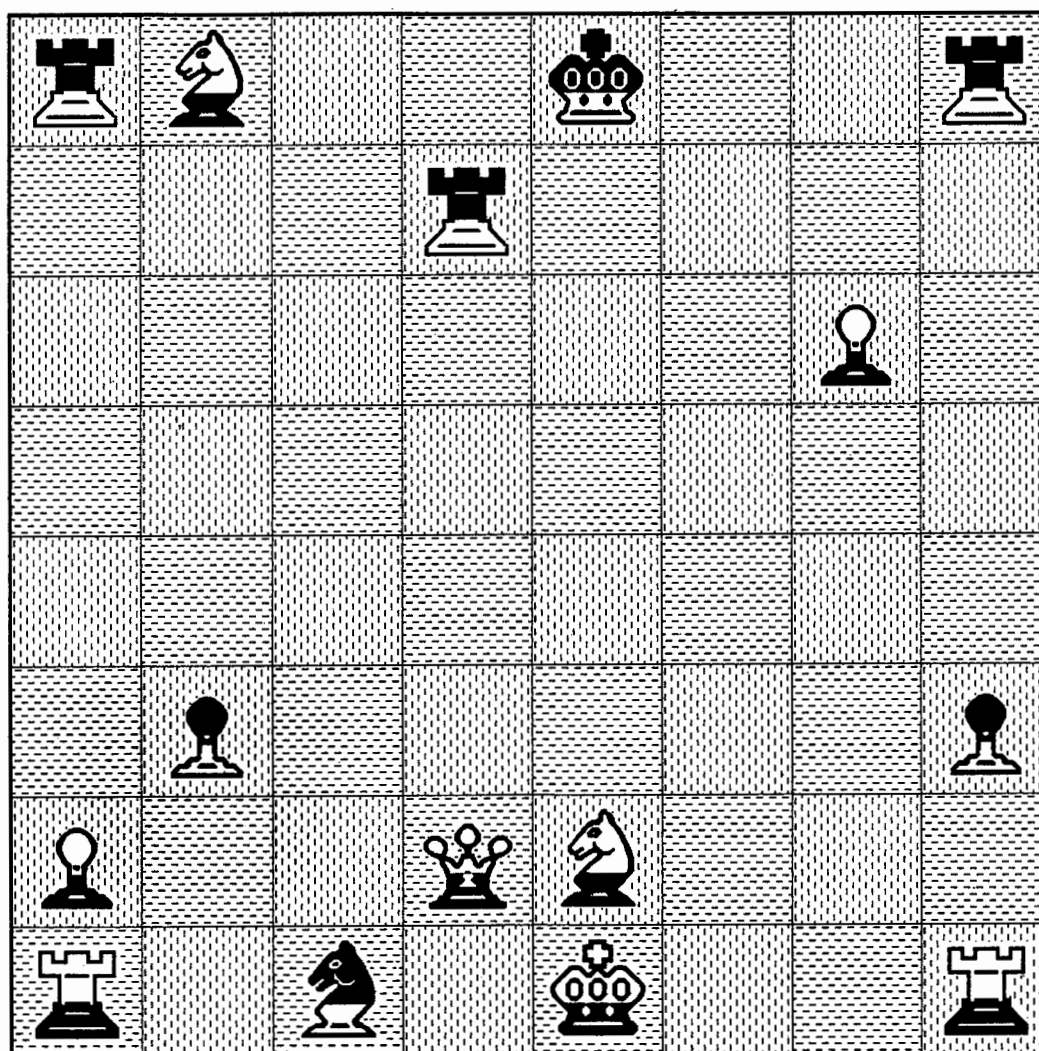
patterns. Two Pawns of the same color on the third row prove that rows cannot be files; movement is up and down, as your seating suggests. A third Rook of one color proves at least one promotion; and, with no Pawns on four of the files, further promotions are a possibility.''

Holmes spoke approvingly, ''Very good, Watson. You have covered everything I can glean from the position, except the trivially obvious fact that neither player can castle.''

''No,'' agreed Masters. ''Too bad you didn't arrive a little sooner. I could have castled on my last move if I had chosen to do so.''

''And I, as well,'' Miss Johnson contributed. ''Does that information help you to determine which way White is moving?''

At that exchange, Holmes increased the intensity with which he had been scanning the board. In response to the question, he nodded. ''It defines not only the move direction, but also each player's last move. Watson, why don't you have a go at it? I'll just see that you don't go astray.''

There seemed little chance of *that* happening, with so obvious a position.

MATERIAL EQUALITY

Holmes and I stopped to chat with the Ashleys, who were not absorbed in this game. I soon realized why; although the two sides were identical in material strength, the situation was a checkmate by Black.

"I suppose this has been your usual homochromatic game," I remarked.

Holmes leaped in before either player could respond. "No, Watson. The number of Pawn captures obvious here would be impossible in homochromatic play."

64. DISPLACED TRIO
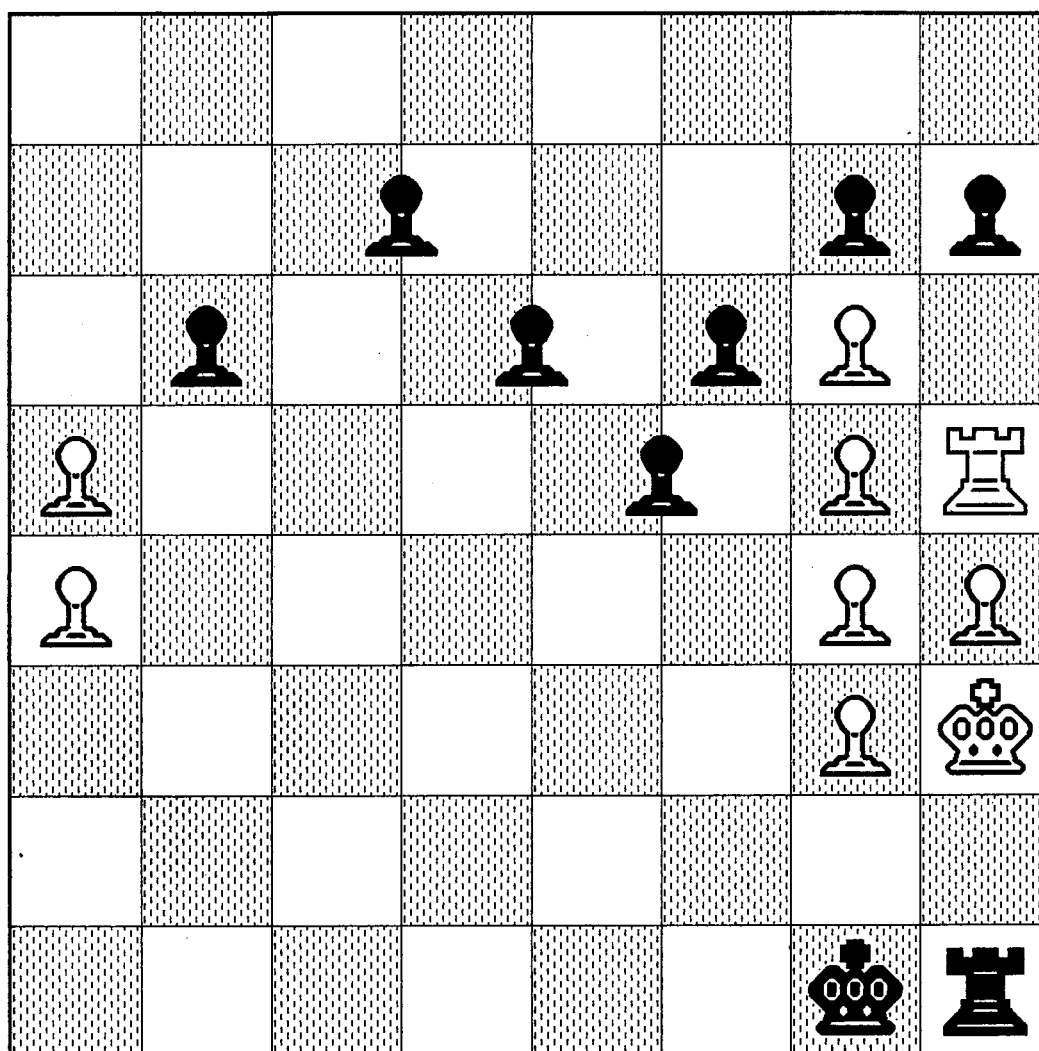
It is said that doctors make poor patients, but I made a quick recovery. "You are right, Holmes. White Pawns have made seven captures, all on the central four ranks. Although seven Black pieces are missing, two are Knights that would have remained on their home squares, and home rank, in a homochromatic game."

"We *do* occasionally play a more conventional game, just so we won't forget how," laughed Lady Ashley. "Oh, dear! I just noticed the displaced Pawns, but don't suppose there's any point in adjusting them now."

"Why don't we let Watson make the fairly simple Pawn adjustments?" Holmes suggested. "He might also account for the fates of the two missing Pawns."

CONSECUTIVE BISHOP CHECKS

Micawber stood, disconsolate, by the chameleon board. "I really thought I was better than Chuzzlewit," he said mournfully.

It was readily apparent that Black was checkmated. A Bishop on g8 gave check. Each square to which the Black King might move was checked by the White King on b1, the Rook on a8, or the Queen on d4. The move direction was not obvious. But no White piece could capture the checking Bishop, and no piece could be interposed between the Black King and White Bishop, regardless of move direction.

65. PLAY BEFORE MATING
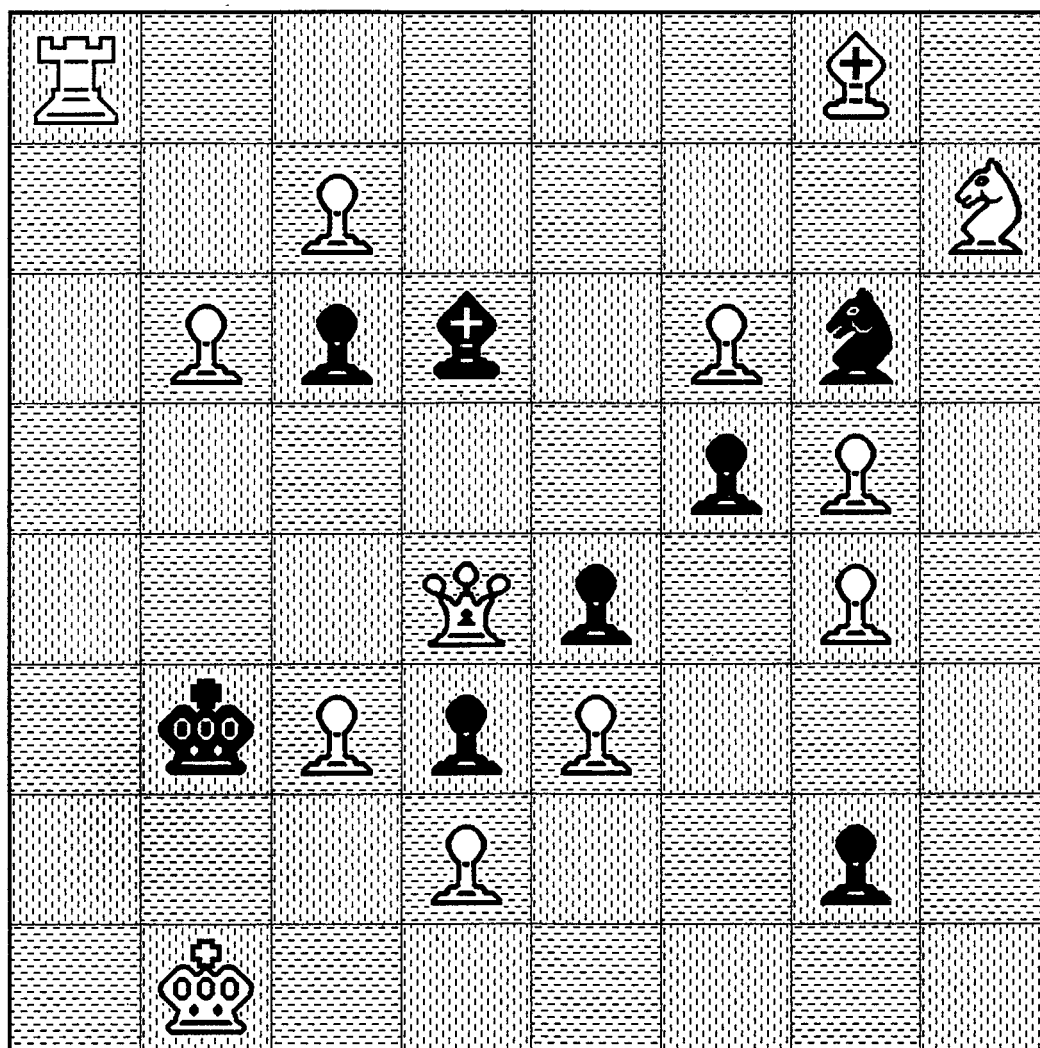
"An experienced retrograde chess analyst," said Holmes, "cannot uniquely identify White's last move."

I turned my attention to the White checking move, and said, "I see what you mean, Holmes. There are two possible last moves by White, with different move directions." But I was not prepared for my friend's next comment.

"I see *three* possible last moves," said Holmes. "Can it be that White's last two moves were *both* Bishop checks?"

"Yes," answered Micawber. "The checkmate *is* the second of two consecutive Bishop checks."

Holmes nodded in satisfaction, "Then the uncertainty about the last move vanishes. And only one preceding White move allows the unique last move."

ALL IN MOTLEY

Washington and one of the indistinguishable Basset brothers stood beside a table with this unusual array of pieces. Most were of the ambiguous White-Up and White-Down patterns, but two Rooks were of the White-Up-Left and White-Up-Right varieties. I thought of the latter patterns as *doubly ambiguous*, since each stood for one of the already ambiguous majority patterns.

I knew better than to question Basset without knowing if he was the invariable liar or the always-truthful brother. After I confirmed that the reliable Washington had observed

66. TRUE COLOR
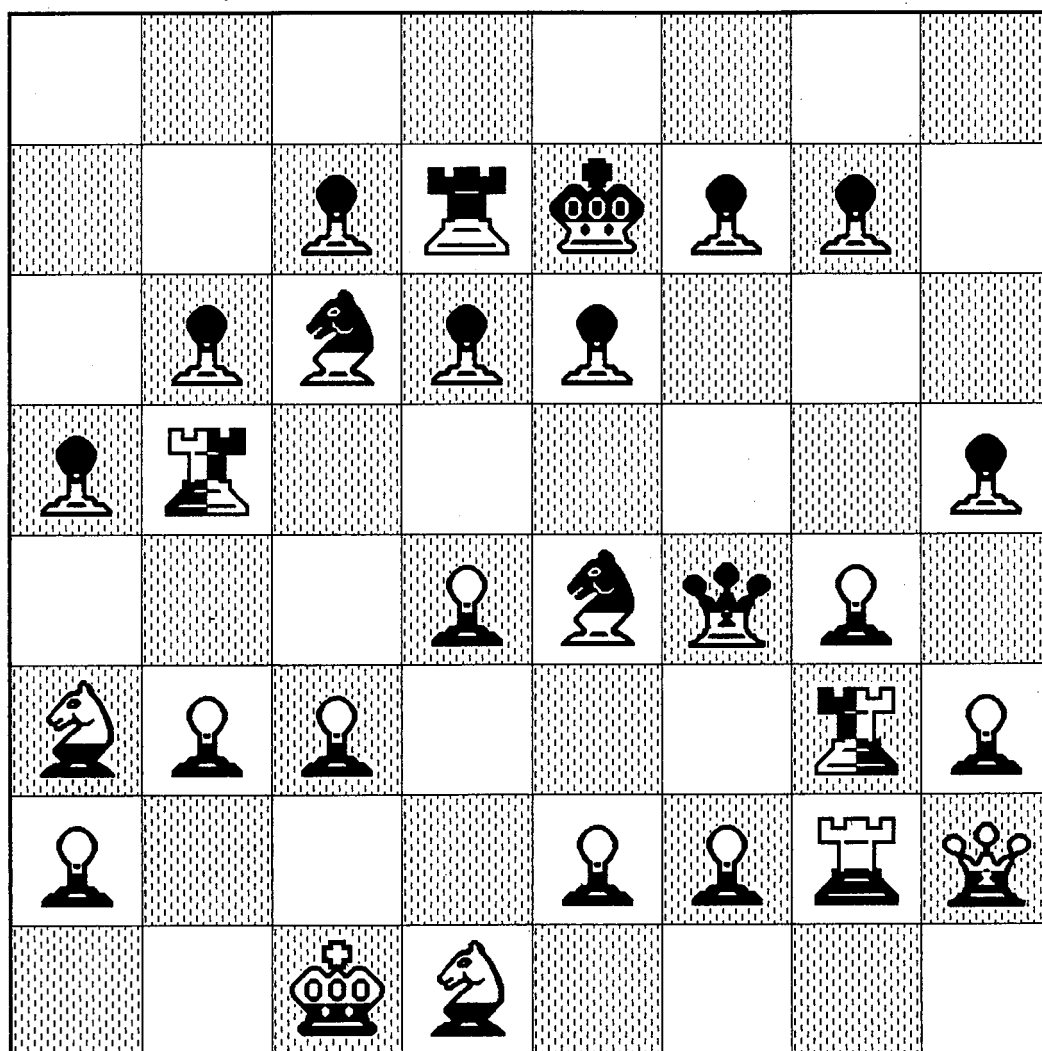
the entire game, I asked Washington, "Do all of the pieces of each ambiguous pattern belong to one player?"

"Yes," he replied. "The Basset brothers played the game, which was heterochromatic. Recognizing your enthusiasm for retrograde analysis, I won't volunteer anything further."

Holmes studied the position intently for a few minutes, then nodded in self-satisfaction and started to turn away. He paused when Basset volunteered, "The last move was a Queen move. I made the last move, and made three more moves with the combination of Queen and Rooks than my brother did with his corresponding pieces."

That statement led Holmes to spend another minute perusing the board. He then smiled and said, "Watson, you should be able to deduce whether the Basset brother who just spoke played White or Black."

BESTING A FOE

As Holmes and I relaxed over coffee, Goodwin, the club secretary, led a stranger to us and introduced him as Professor Adler, from Vienna. Speaking with a slight accent, the professor addressed Holmes.

"I understand you are an authority on retrograde chess analysis," he said. "In Vienna, I once saw a man, Professor Moriarty, who was said to be unmatched in that field of expertise, solve a very difficult problem. I was so impressed that I recorded the position and circumstances. Would you care to see the diagram?"

67. MORIARTY'S QUESTION
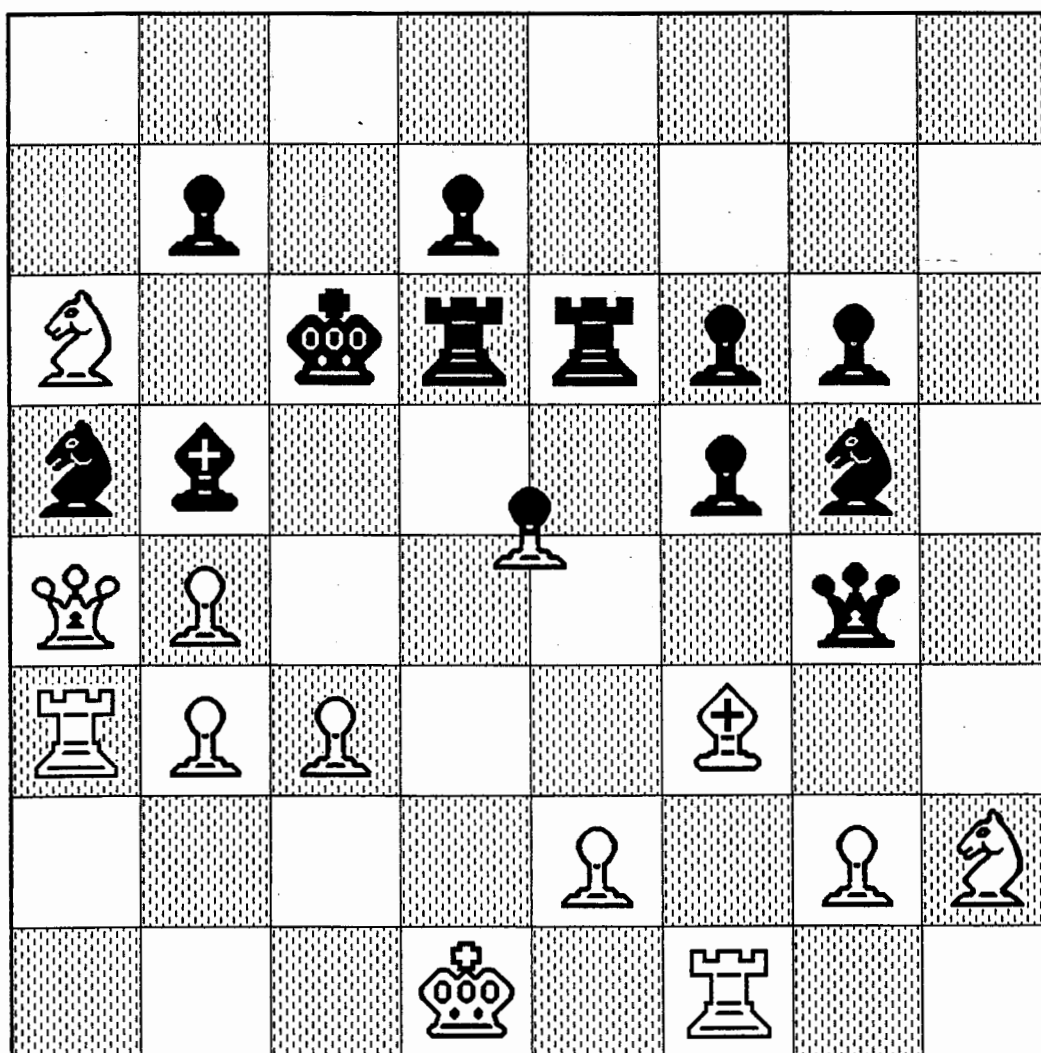
"Indeed I would," Holmes replied. "Any example of Professor Moriarty's analysis is of extreme interest to me."

Professor Adler pulled out and unfolded a well-worn diagram, apologizing for the creases. "Please note," he said, "that a Pawn of uncertain color is ambiguously located, occupying one of four squares: d4, d5, e4, or e5. The problem was to determine the color and location of that Pawn."

"I observe that both Bishops on the board are promoted," said Holmes. "Other elements of the position would require some time to work through. Would it be possible for me to copy your diagram, or borrow it overnight, and discuss my findings tomorrow?"

"I will entrust it to you," said Professor Adler. "If you cannot solve the problem, there is no point in my continuing to carry it around."

"Was Moriarty given any information beyond the position, itself?" asked Holmes.

Yes," replied Adler. "He spent much time over the position without proclaiming a solution, and appeared on the verge of abandoning the problem and walking away. Instead, he asked an unusual question: 'Did Black and White promote equal numbers of Pawns?' The answer permitted him to identify both Pawn color and location."

"And what was the answer?" I inquired.

Obviously embarrassed, Adler answered, "Alas, I do not know. I heard his question, but not the answer. I also did not record his solution.

"I should not have piqued your interest when I was unable to provide the information that Professor Moriarty needed in order to solve the problem."

Holmes said, "I believe I may be able to solve the problem with only the information at hand. Let us return tomorrow evening, and I will divulge my logic or confess my failure."

I was extremely skeptical. Holmes had never before expressed any belief that his retrograde chess analysis skills were superior to Moriarty's. But his recital the following evening vindicated him.

LOOK-AHEAD REQUIRED

Crenshaw and Mellon were the players of this chameleon game. I commented on the marker at d6: "You seem to have dropped a piece from the board."

"Worse than that," replied Mellon. "Actually, three pieces have fallen to the floor, but we have had time to mark only one. If you are interested in analyzing the game, you may want us to mark the other fallen pieces."

"It may come to that," said Holmes. "But please hold off for a moment while we take stock of what is visible."

68. THE ROOK ON H7
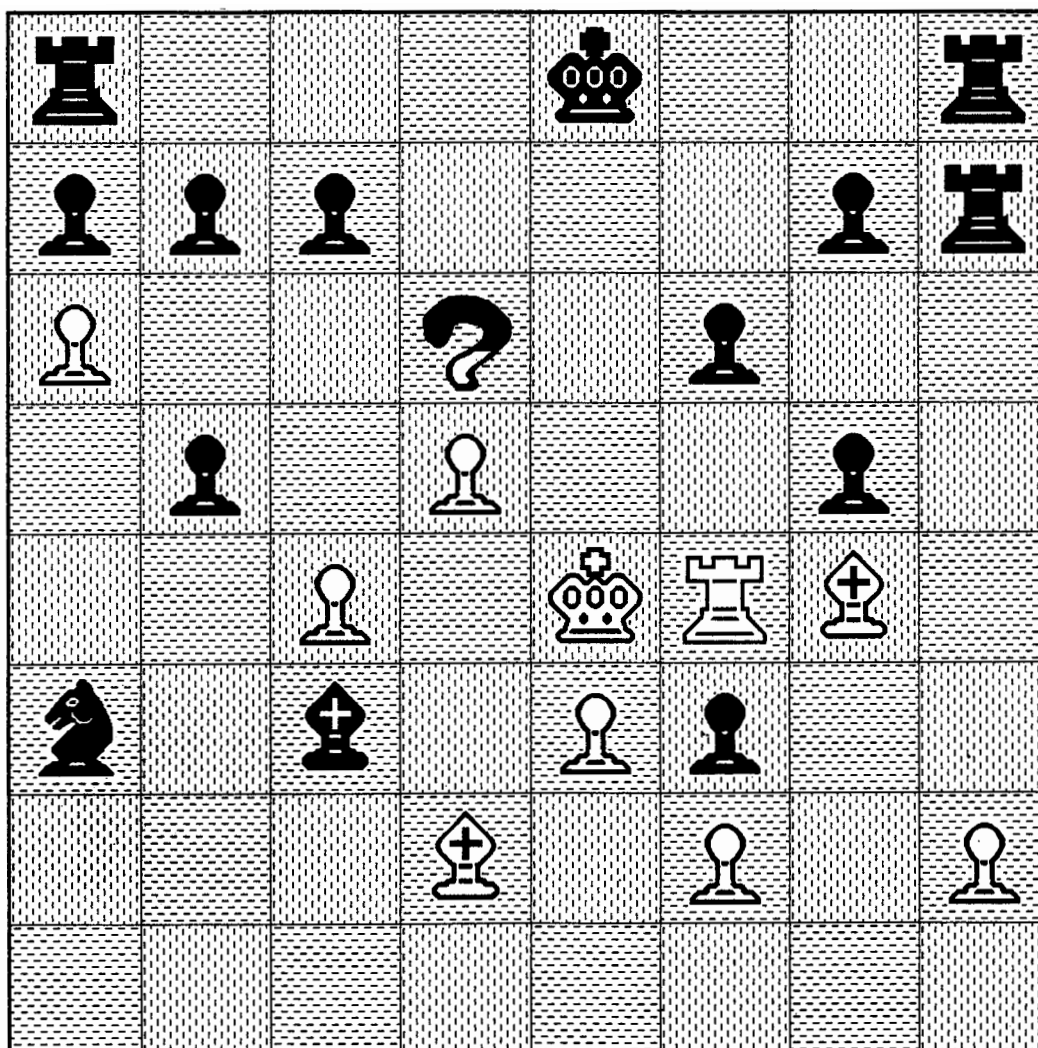
"The most obvious feature," I commented, "would seem to be the potential for Black castling."

"There is no way you could tell," said Crenshaw, "but immediate castling is not in the cards. Playing Black, I am in check. However, I *can* castle in no more than three moves."

That comment provided food for thought, but I could not digest its significance. Mellon added another such morsel, "And I can prevent him from castling in *fewer* than three moves."

"Delicious," Holmes murmured as he continued his scan of the position. Then his face lit up, and he proclaimed, "There is only one possible combination of fallen pieces and their locations. The proof *is* somewhat lengthy. And it requires examining the future to comprehend the past."

"I would not know where to begin," I confessed.

"Start with the Black Rook on h7," Holmes suggested.

ORIENTATION EXPRESS

Something about an abandoned game caught Holmes's eye. He motioned me into remaining silent while his eyes darted about the board. Focusing on the Kings, I noticed that Black was being checked by a White Rook on h3. Closer inspection revealed two White and two Black Bishops on squares of the same color, though it was not obvious whether those were white or black squares.

Holmes spoke, "Determining the move direction is not at all difficult, Watson. I suggest you check that out first. But complete definition of the last move is fairly

69. TERMINAL MOVE
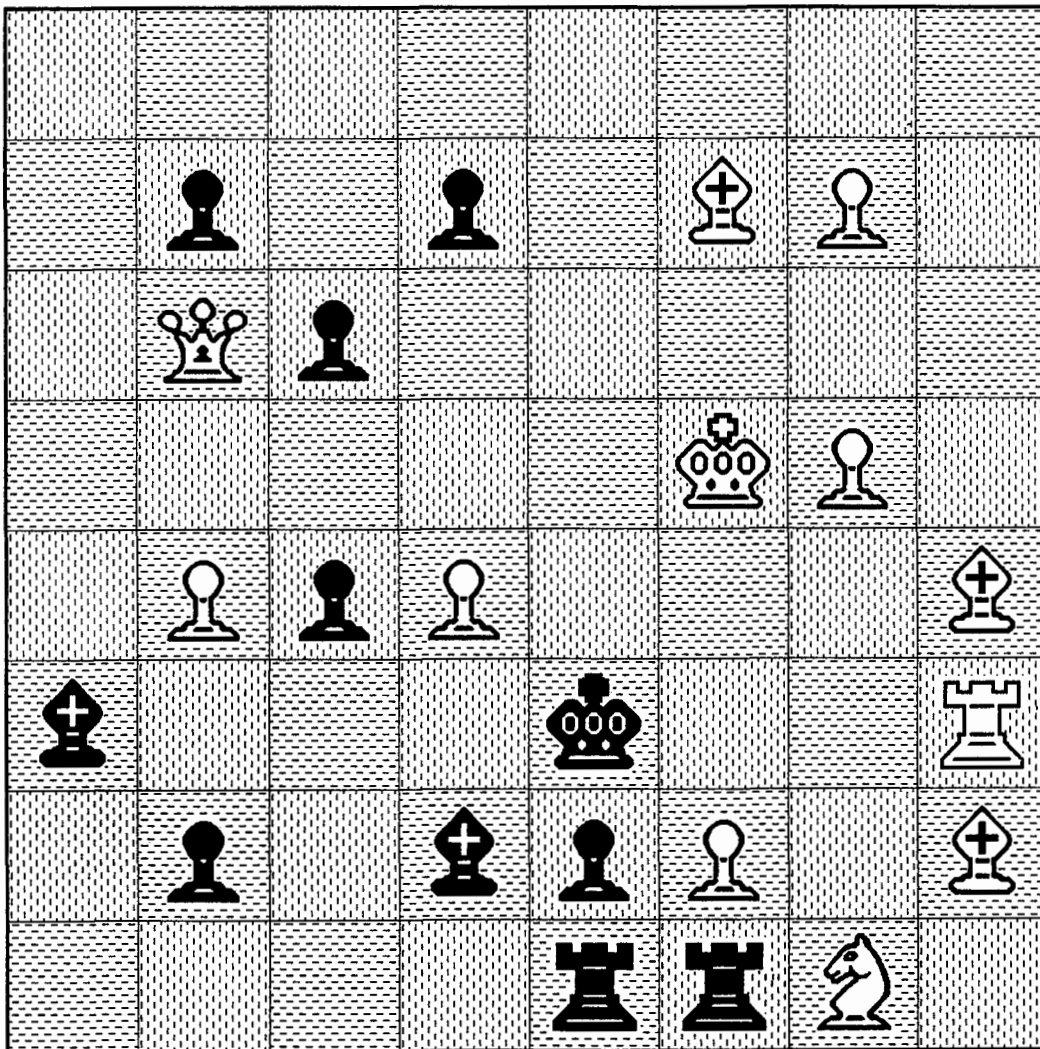
challenging, and you may need a hint or two. Care to try it?"

I first satisfied myself that I could identify the move direction, then considered potential last White moves to produce the check. Despite a small uncertainty, I felt I could work my way through it.

THE LADIES ARE SHAKEN

Mrs. Post and Mrs. Vanderbilt were sitting at this table. Both appeared somewhat shaken, and I inquired solicitously into their well-being. Had a problem arisen in their game?

Mrs. Post replied, rather reluctantly, "We were merely watching a game being played by two gentlemen. It would be in poor taste to identify them. Black made a terrible gaffe."

Mrs. Vanderbilt added, "He *did* apologize profusely, then took his departure as

70. BLACK'S GAFFE
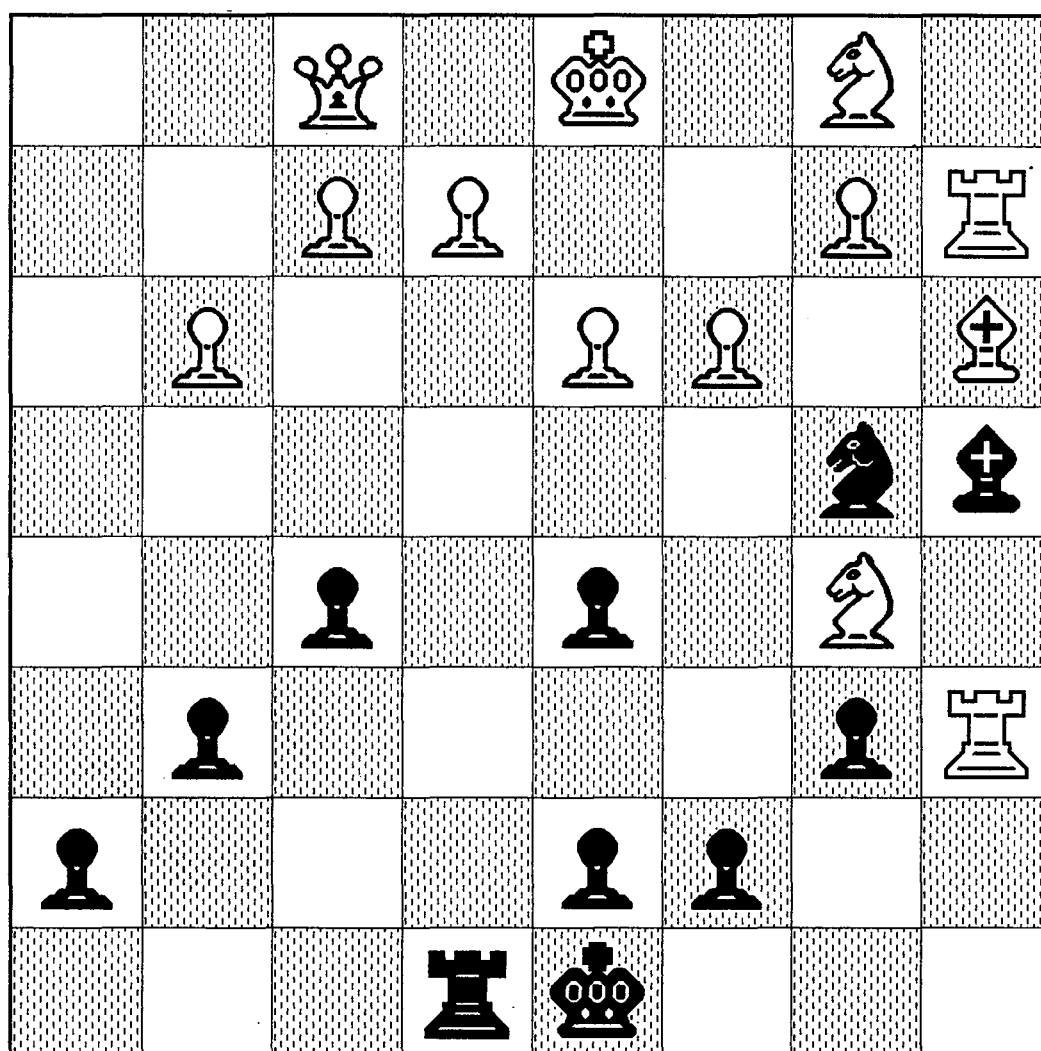
graciously as possible in the unusual circumstances."

Mrs. Post continued, "He took such pride in his impeccable Knight play. It was distressing to observe his humiliation. I tried to keep my composure, but my mask must have slipped a little after Black's last move - just enough for him to realize what he had done."

"Then, the gaffe was related to Black's play?" I wondered.

"The gaffe *was* his play," said Mrs. Vanderbilt.

Holmes begged the indulgence of the ladies for us to examine the position. A Black Bishop on h5 gave check. A Black Knight on g5 was poised to capture either of the two White Rooks. Even though White had g5 covered by both a Pawn and a Bishop, capture of the Black Knight was impossible; White had to deal with the Bishop check.

Holmes said soothingly, "I do understand why you are so concerned, ladies. It must be rare, indeed, for you to witness behavior like Black's."

"I confess that I cannot see the problem," I said. "Black's last move appears to put him in an admirable position. Holmes, what *was* his gaffe?"

SOLUTIONS

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1. CHAMELEON BOARD

"Is it clear who has the move?" Holmes asked.

I mused, "White's announcement of a one-move checkmate does not automatically indicate which player now has the move. If it is White's move, he can checkmate with any one of five moves. The Rook on d7 can move to e7; the Rook on f7 can move either to e7 or to f8; the Queen can move to e3 or e6.

"But if it is Black's move, the Black Knight on h8 can capture at g6. If White then moves a Rook to e7 or f8, the Black Knight can capture that Rook, preventing the one-move mate. If the White Queen moves to e3 or e6, the Black Knight can be interposed at e7, again delaying mate beyond one move. Thus, a one-move forced checkmate is possible *only* if it is now White's move."

"Since it *is* White's move," Holmes continued, "what was Black's last move?"

I considered the possibilities. "If the Black Knight at h8 had made the last move, it could have been only from f7 or g6, and one of those squares would now be unoccupied. If the Black Bishop at h7 had moved last, either g6 or g8 would now be unoccupied. The Black King could have moved only from f8, where it would have been in an impossible triple check (quadruple check if the White Pawn on g7 is moving up the board), or from e7, in quadruple check.

"Thus, Black's last move was with a piece no longer on the board. That could have been only a Black Pawn that disappeared because it promoted. Regardless of the direction of motion, no Pawn could have reached h8 to promote to the Black Knight. Promotion at h7 would have required that Black be moving to the right, the Pawn promoting to Black Bishop moving from g6, g7, or g8 and leaving one of those squares now vacant. No promotion of a Black Pawn to a Black piece being possible, the promoting Black Pawn necessarily became a White piece.

"You are right, Holmes. The existence of a forced one-move checkmate does prove that there has been a reverse promotion, and on the last move, at that."

Holmes pointed out, "For completeness, you should demonstrate that a reverse promotion was possible, as well as being required. Moving up the board, a Black Pawn from e7 could have captured at d8 and promoted to the White Knight. With Black moving to the right, a Black Pawn could have promoted to the White Queen; moving to the left, a Black Pawn could have promoted to the White Bishop.

"We have proved that Black *did* make the last move, that a reverse promotion on that move *was* possible, and that all possible last moves by Black are reverse promotions.

"Move direction remains indeterminable, with three of the four directions logically possible."

Soames-Forsyte mused, "I would not have dreamed one could weave a saga like that from such thin material. I must tell Galsworthy."

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2. A CHAMELEON CASTLING CONUNDRUM

I noted, "We need consider only the case in which Black is moving down the board, with the Black Bishop on c8 captured on its home square, and the Black Bishop now on the board a promoted Pawn.

"A reverse promotion of a White Pawn, in order to increase the number of Black pieces available for capture, could involve only the Pawns initially on a2 and b2. All other White Pawns are assumed to have moved to the g-file with a minimum number of captures. Allowing one of those Pawns to promote would require the Pawn from b2 to replace it on the g-file, increasing the required number of Pawn captures and defeating the purpose of the reverse promotion.

"There *are* bars to promotion by those White Pawns. The White Pawn from b2 would have to capture en route to promotion, to bypass the Black Pawn on the b-file. In doing so, it would consume the extra Black piece generated by the promotion, with no net increase in captures available for the Pawns moving toward the g-file.

"Only the Pawn from a2 could promote without making a capture, and only at a8, requiring the Black Rook to move from that square, thus ruling out subsequent castling."

"You have it!" Holmes said enthusiastically. "The White Pawn formation does not rule out Black movement down the chameleon board. But if he *is* moving down the board, Black cannot castle because his Rook has moved."

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3. QUEENS GALORE

I explained my reasoning to Holmes. "Regardless of the move direction, White is in check from either the Pawn on d4 or the Pawn on f6. White cannot be moving down or right, or the Pawn on b7 would be checking Black, with no possible last move by that Pawn. If White were moving up, the Black Pawn on f6 would be checking White; but, moving down, that Pawn had no possible last move.

"So White is moving to the left, and the last move was a move to the right by the Black Pawn, to its present checking location on d4. But did that Pawn move two squares from b4 or one square from c4?"

Holmes replied, somewhat mysteriously, "Sometimes, to understand what is close at hand, you have to explore the most distant corners."

The two corners distant from the checking Pawn held a White Queen at a8 and a Black Bishop at h8. How could either be relevant to the Pawn move?

Holmes resumed his deliberate questioning. "Is the Black Bishop promoted?"

"Clearly not," I responded. "With only six Pawns missing, there are six promoted Queens."

"So you were right, Watson, when you said there could be no question of underpromotion. Two final questions. Where did that Black Bishop originate, and how did it reach h8?"

"An original Black Bishop on that square color, and moving right, came from a3," I realized. "But I can't possibly trace its entire course from a3 to h8."

"Watson," said Holmes, "I have heard of a matron in Boston, who was asked what route she had taken to San Francisco. 'Through Dedham,' she replied. Dedham is a suburb of Boston, while San Francisco is three thousand miles distant.

"If she had known chess, Watson, that matron could have solved this problem."

I looked for the first leg of the Bishop's journey to h8. "Of course!" I exploded. "The Bishop could leave a3 only after a first move by the Pawn from b4, and left long ago. The Pawn's first move was to c4, where it remained until the last move, when it advanced from c4 to d4 for the check."

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4. HOW CAN THAT BE?

Holmes pointed out, "It is true that the Pawn from a2 could not have promoted to *White Knight* at a8. But it *could* promote to a Black Queen or Rook at a8, either piece being able to reach the board from a8. That reverse promotion created a seventeenth Black piece, allowing *six* captures by White Pawns."

"I see," I said, crestfallen. "The White Pawn from e2 could then promote at either d8 or f8, capturing from e7 with no move of the Black King required. The Knight created by *that* promotion could reach the open board from either d8 or f8."

Holmes agreed, and wrapped up, "Although the reverse promotion was essential in making a capture available to the promoting Pawn from e2, the piece captured by the promoting White Pawn was not necessarily the promoted Black piece, which may well have been among the five captures on the c- and h-files."

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5. WHITE'S LAST MOVE

With his "shades of grey," Holmes was reminding me that the problem was set on a chameleon board.

"Of course!" I agreed. "This board allows reverse promotions. White's last move was a Pawn capture to the last rank, with promotion to a Black piece. Except for the King, the only Black piece on a white square in that rank is the Rook at h1, which therefore

resulted from the reverse promotion.''

''And what was captured on h1?'' Holmes prodded.

I thought my way through the necessary sequence of moves. ''After an initial move of two squares, the White Pawn captured a Black Pawn on an adjacent file, the Black Bishop, the unmoved Pawn on g2, and... that leaves only the Rook originally on h1.''

''Quite so! The White Pawn captured the Black Rook and immediately promoted to a replacement Black Rook. Nothing changed except for the disappearance of the White Pawn.''

''I am quite apologetic, Holmes. I see now why you couldn't tell if the last move was a capture. It was if White made the last move, but if the Black King moved to b1 on the last move, it did not necessarily capture. In fact, with the White Queen or Bishop as the only possible occupants of b1, a capture there by the Black King would have been a move from an imaginary check - a check that could have been produced only by the forbidden move of the King into check at c2. So the last move was a capture if by White, but not a capture if by Black.''

''You still haven't exhausted the possibilities. It is entirely possible that Black made the last move and *did* capture, at a8, for a second reverse promotion and disappearing Pawn.''

''Then,'' said Lady Ashley, ''your one positive conclusion was that the board contains at least one promoted piece.''

''You have it,'' smiled Holmes. ''And now, I would love to see a heterochromatic game if you think you can find one.''

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6. WHOSE HUE?

''In normal, unconstrained chess,'' said Holmes, ''a Pawn can have three distinct moves. You might consider each, Watson, and test it against the heterochromatic constraint.''

''The most common Pawn move,'' I noted, ''is a one-square advance up its file. That move *is* heterochromatic. The initial move of a Pawn *can* be a two-square advance; that move is between two squares of the same color - homochromatic rather than heterochromatic. The third Pawn move would be a capture, by a forward diagonal move of one square. That move also would not be heterochromatic. So, in heterochromatic play, a Pawn move can only be a one-square advance on its file.''

''Then,'' asked Holmes, ''could the Bishop on c8 be a promoted Black Pawn?''

''The Black Pawn would have originated on c2,'' I realized. ''Unable to capture to leave its file, that Pawn could not bypass the White Pawn above it on the same file, thus could not promote. The Black Bishop is necessarily original, with Black moving down. Then, White is moving up, and the King checked by the White Pawn is the King on d5.

"The White King is on b3, represented by the White-Up ambiguous color. And the White Bishop on f8 is a promoted White Pawn."

"Exactly so, Watson. Heterochromatic constraints on Bishop and Pawn moves suffice to resolve the move direction and color ambiguity."

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7. FIVE TO RESOLVE

I considered first the ambiguous Bishop on c8, and said, "The Bishop on c8 is original. A Pawn on the c-file proves that no promotion occurred on that file; a Black Pawn could not have bypassed the White Pawn to promote, and a file can never contain two Pawns of the same color. If the ambiguous Bishop is White, then White is moving down the board. In that case, the White Bishop on c8 checks Black, and no legal move could have discovered that check by an unmoving Bishop. If the Pawn on b6 is White, it did not move from b7 to discover the check; a White Pawn on b7 would have been giving check *before* it moved - the imaginary check situation. Thus, White can only be moving *up* the board, and the ambiguous Bishop is Black. With Black moving down the board, the Pawn on b6 has to be Black, that on b3 White.

"Since the White Bishop on f8 is promoted, there can be no Pawn on the f-file. The ambiguous Pawn straddling a corner must be on either g4 or g5. If it is on g5, White is checked by the Black Bishop on c8, and no move could have discovered that check, with Black moving down. The Pawn has to be on g4. It cannot be Black, or the Pawn would be giving check, having made the last move; but before that move, White would already have been in imaginary check by the Black Bishop. The Pawn is White, and on g4, blocking the Black Bishop check.

"The last move created the only other possible check, of Black by the original White Bishop on f1. With White moving up, the White Pawn discovered the check by moving from c4 to c5, where it is now."

"Nicely thought through, Watson," Holmes complimented me. "Of course, despite the multiple ambiguities, no new principles are involved. It was necessary to consider only the restrictions on Bishop and Pawn moves in heterochromatic play."

Drumm beat Phyfe to the final note, "It is fascinating that you can bring harmony out of discord by stressing banned plays."

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8. EN PASSANT

The phrase to which Holmes had alluded quickly came to mind, "Checks and balances." What could it suggest in the present context?

"The phrase suggests more than one check," I mused. "Black is unquestionably in check by the Rook. If White is moving to the right, as I have assumed, balance requires Black to be moving to the left. I see! In that case, the Black Pawn on d4 checks the White

King, an impossible mutual check situation. So White is *not* moving to the right.

"But, then," I protested, *"no* last Pawn move could have been an en passant capture. Did you mislead me, Sutter?"

Sutter's laugh was a loud bray. "Watson, I merely said that an en passant capture *had* occurred - *not* that it happened on the last move."

Swallowing hard, I regrouped. "White is not in check, so Black is not moving up or to the left, and White is moving in one of those directions. With White moving to the left, no move could have discovered the Rook check. White is moving up the board, and required a capture to move from the e-file. The last move was a Pawn capture from e5 to f6.

"If the Pawn that made an en passant capture is not now on the opponent's third rank, it is because that Pawn has subsequently moved. To remain on the board as a Pawn, rather than a promoted piece, it has to be on the opponent's *second* rank. With Black moving down, the Black Pawn on f2, and only that Pawn, could have made an earlier en passant capture."

The problem *was* simple, if only I had not rushed to take the glitter for gold. A preconception, which was not logically necessary, had misled me.

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9. HOMOCHROMATIC?

Holmes explained, "The opposing Kings begin the game on squares of opposite colors. They are now on squares of the same color, showing that at least one King made at least one move that was not homochromatic.

"The Bishop in the center of the board demonstrates that play also was not heterochromatic. With conventional play, any of the four move directions is possible."

I apologized for my oversight, but added in self defense, "Even one trained in science can fail to notice the significance of so basic an element."

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10. AMBIGUOUS PAWNS

"The only Pawn move possible in heterochromatic play," I ventured, "is a move of one square forward on the file. There can be no two-square initial Pawn move. And no capture by a Pawn is possible."

"Apply that last restriction," Holmes suggested, "and you can fairly easily define the move direction."

"Column d," I observed, "contains one Pawn of each color. If movement is up/down, White can only be moving down. If White were moving up, one of the Pawns would have bypassed the other, requiring it to capture to leave the file, and again to return to the file.

"Rows 2 and 6 each contain Pawns of both colors, Black on the left, so side-to-side

movement would require White's moving to the left. But Row 4 shows exactly the opposite order of the two Pawn colors, requiring White to move to the right. We can rule out side-to-side movement, and are left with White moving down."

"Good!" Holmes approved. "What does the same principle say about the ambiguously placed White and Black Pawns?"

"With White moving down," I observed, "a White Pawn cannot be below a Black Pawn on the same file. With b5 ruled out, the White Pawn has to be on a5. The ambiguous Black Pawn cannot be on e4, where it would be above the White Pawn on e2; it has to be on f4, where it is properly below the White Pawn on f6.

"But that line of reasoning does not work for the two Pawns of ambiguous colors."

"Remember, Watson, I said we would have to mind the P's and Q's - the Pawns and Queens. It is time to consider the Queens, without forgetting about the Pawns. One of the two Black Queens is promoted. Could the promotion have occurred on the a-file?"

"With a White Pawn on a5," I stated confidently, "the Black Pawn on that file could not have bypassed the White Pawn, thus could not promote."

The light dawned. "Promotion could not have occurred on *any* file that still contains a Pawn. Files a through g all contain Pawns. Promotion had to be on the h-file, which thus cannot now contain a Pawn. The ambiguously colored Pawn straddling g3 and h3 has to be on g3. That leaves only the colors of the Pawns on g3 and g5 to be determined."

I could carry the analysis no further, and mutely appealed to Holmes for help. He concluded, "Two Pawns on the same file have to be of different colors, and the move direction dictates the color order. With White moving down, the Pawn on g5 has to be White, and the Pawn below it on g3, Black."

"That seems to leave only one possible question," I said. "Was the promotion to Black Queen a reverse promotion?"

"I have no idea, Watson," said Holmes. "Only the players can answer that question. And, if logical analysis cannot answer it, the question holds no interest for me."

Shaw said, "Holmes, I no longer wonder why you have remained a bachelor."

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11. RESTING PIECES

"The problem seems to revolve around move parity," I suggested. "With castling possible for each side one move ago, neither King nor King's Rook had moved before then. White's last move was with a Knight. And Black did not move his King or King's Rook on his last move, or that piece could not now be on its home square.

"Each Queen is between an unmoved King and an unmoved Bishop, thus has not moved. The Queen's Rooks have been limited to shuttling between two squares each. Otherwise, only Pawns and Knights have moved, while one Bishop on each side was captured without being able to move. The Black Pawn on g6 came from f7, and has

captured what could only be the missing White Knight. The ambiguously located White Pawn originated on h2 and has captured the missing Black Knight, on either g3 or g4.

"Let us assume that the ambiguous Pawn is on g3, meaning the Black Knight was captured there. Then, the Black Knights, both on black squares, have made an odd number of moves. Black Pawns show one move, an odd number. And the Rook on b8 has moved an odd number of times, so total Black move parity is odd. White Knights have made an even number of moves, as has the White Queen's Rook. White Pawns have made three moves, and total White move parity is odd. But White made the last move, checking with the Knight, so Black and White cannot have the same move parity.

"The missing Black Knight, then, was not captured on g3, but rather on g4, where the ambiguous White Pawn belongs. Black Knight parity is even, Pawn and Rook parity each odd, and total parity even. The White Pawn on g4 has made two moves, first to h3, then the capture to g4. Other Pawns have made two moves, and total White move parity is even. But again Black and White move parity are equal when it is Black's move!

"Holmes, help me. Whether the missing Black Knight was captured on g3 or g4, we have the impossible situation of equal move parities following a White move!"

"You have made an understandable but unwarranted assumption, Watson," Holmes chided. "You have assumed that the present location of the ambiguous White Pawn is where the missing Black Knight was captured. With the Pawn on g3, the capture would have to be on g3; but, with the Pawn now on g4, it could have captured on either g3 or g4. You have not considered the combination of a capture on g3 and present Pawn position on g4."

So I considered it. "That makes Black Knight parity odd, and total Black parity odd. The White Pawn captured to g3, then moved to g4. Those two moves plus two other obvious Pawn moves, make White parity even, and there is no conflict."

Holmes said, "I may have misled you, Watson, by mentioning a different slant. Actually, for the ambiguous Pawn on either possible square, the *slant* is the same - the diagonal capturing move from h2 to g3."

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12. COUNTLESS KNIGHTS

"With Black's Pawn formation," I realized, "a promoting White Pawn had to pass through a7, d7, e7, or f7. At d7 or f7 the Pawn would have given check - imaginary check since we know the Black King has not moved. Pawn access to a promotion square could only be through a7 or e7. In either case, the final move of the promoting Pawn had to be a capture; a noncapturing promotion would have required that either the Black King or its Queen's Rook move, denying future castling capability.

"The White Pawn from a2 would have required three captures to promote at b8, five to promote at d8 or f8. The Pawn from c2 could have promoted at b8, d8, or f8, with three captures in each case. The Pawn from g2 could have promoted only at d8 or f8, also

with three captures. Since two White promotions would have required at least six captures, while only four Black pieces are missing, there was only one such promotion, that to the third White Knight. That promotion to White Knight occurred on square b8, d8, or f8."

"And those possible promotion squares have one common characteristic, Watson." Holmes pointed out. "All are *black* squares. One original Knight also began the game on a black square, the other on a white square. Then before any White Knight move is counted, two (an even number) were on black and one (an odd number) on white. Any move by a White Knight reverses that *color parity*; after an odd number of Knight moves, an even number (possibly zero) of White Knights will be on white squares. After any even number of Knight moves, the number on white will once more be odd. The position shows two White Knights on white squares; that even number proves an *odd* number of White Knight moves, 23 rather than 24.

"Black's obvious promotions, to a second Queen and to the Bishop on a1, are really irrelevant, even for identifying theWhite Pawn that actually promoted."

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13. FALLEN QUEEN

"The Black King is in an apparent triple check," I mused, "from the White Queen on a2, the Rook on b8, and the Rook on g4. Placing the Black Queen to block one check can reduce the situation to a double check, which is possible if one check is discovered by a move of the other checking piece.

"A Queen move, of course, cannot discover a check. The only conceivable White Pawn promotion, to the Rook on b8, would yield one check but would not discover a second check. The Rook at g4 could have discovered the Queen check only by moving from c4; that move, between two vertical-grey squares, would not have been heterochromatic. It was the White Rook now on a horizontal-grey square at b8 that discovered the Queen check, by moving from vertical-grey square b3.

"The Black Queen must be positioned to block the third apparent check, from the White Rook on g4. The Queen could block that check if on g5, g6, or g7. But, at g5 or g7 the Black Queen would check the White King. Since both sides cannot be in check simultaneously, the Black Queen can be located only at g6."

"Absolutely right," Holmes complimented me. "The problem involved only the basic definition of heterochromatic play. And now, can you identify the move direction, and explain the players' dilemma?"

I could not, and Holmes explained, "Pawns in columns a and h rule out left/right movement. If White is moving down the board, the Black King could escape check with the heterochromatic move to g7, remembering that the Black Queen on g6 blocks check by the Rook on g4. But, if White is moving up, at g7 the King would be in check from the White Pawn on h6. No other heterochromatic move can negate the check. But the nonheterochromatic move of the Black King to h7 *would* negate it.

"The position with White moving up is not a checkmate under the rules of chess, but no heterochromatic continuation is available. The players must either drop their insistence on heterochromatic play, or abandon the game as inconclusive.

"The players would have no such continuation dilemma if White were moving down. The fact that they do have a dilemma indicates that White is moving up."

Brown added, "That kind of dilemma occurs fairly often in both heterochromatic and homochromatic play. Different players prefer different resolutions. Foreman and I have not encountered the problem in previous play together, so we have not established an understanding on continuation."

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14. PLACES FOR ALL

Holmes explained, "Only three Black pieces are missing, with the two Bishops necessarily captured on their home squares. With Pawns on all files, there have been no promotions, and the other missing Black piece is the Pawn originally on e7.

"The White Rook on e6 is checking the Black King. With no possible move that could have discovered the check, the Rook made the last move. Before that move, e6 had to be occupied, or White's move would have been from imaginary check by the White Queen. Only the missing Black Pawn could have been occupying e6, to be captured there.

"Watson, knowing the location of each Black piece, can you tell whether Black has made an odd or even number of moves?"

"Well," I temporized, "the Black Knights, on opposite square colors, have made an even number of moves. And we can count the Pawn moves, since a Pawn in heterochromatic play moves forward one square at a time; Black Pawns have moved 11 times, an odd number. But, with their passage relatively unrestricted, I can't count the moves of the Black King, Queen, and Rooks."

"Although we could, it isn't necessary to account for move parity separately for each kind of piece," Holmes proclaimed enthusiastically. "We simply note that Black began the game with 16 pieces, eight on each square color. Black's first move changed the square color of the moving piece, leaving seven Black pieces on squares of one color, and nine on the other -- both odd numbers. A second Black move would either restore the original eight and eight, or skew it further to six and ten, in either case an even number of pieces on each square color. *Every* odd-numbered move would leave an odd number of Black pieces, and every even-numbered move would leave an even number on each square color."

I counted Black pieces on white squares. Three were visible; the Bishop captured on c8 and the Pawn captured on e6 brought the total to five. "Black has made an odd number of moves," I concluded.

"And what about White's move parity?" Holmes asked.

"Of the 14 visible White pieces," I replied, "seven are on white squares. The Bishop

captured on f7 makes the total eight. The missing White Rook has been hemmed in by Pawns and the Bishop on c1, so was limited to the three squares in the lower left corner. It had to be captured on a1, a2, or b1, with one of those squares black and two white.

"White, giving check, made the last move, and has moved once more than Black, thus must have *even* move parity. If the missing Rook had been captured on either white square, a2 or b1, there would be nine White pieces on white squares, for *odd* move parity. So the Rook capture had to occur on a black square, a1."

"Exactly so," said Holmes. "In heterochromatic play, we can determine color parity when all piece locations are known. And it is color parity I had in mind as relevant to the Queen-side castling question. *Queen-side castling has the same effect on color parity as any unquestionably heterochromatic move.* We can account for color parity without knowing whether castling occurred.

"And color parity can be the basis for the ultimate definitions of homochromatic and heterochromatic play. Every move that *conserves* color parity is homochromatic. Every move that *changes* color parity is heterochromatic. Queen-side castling *does* change color parity, which is the ultimate reason for considering it a heterochromatic move."

Livingstone and Stanley had returned in time to hear that final pronouncement. They later repeated its substance to a club meeting, where it was formally agreed that color parity should be fundamental to definitions of *homochromatic* and *heterochromatic.*

Holmes was hard to live with for a week afterwards. He thanked Livingstone and Stanley for their role in reaching the agreement. Livingstone said modestly, "I did a little missionary work, but it is Stanley who is the persuasive negotiator."

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15. EACH HAS PROMOTED

"The Pawn position," I noted, "is consistent with up/down movement in either direction. The Pawn order on row 6 would also allow Black movement to the right, though not to the left. The promoted Bishop gives no clue to move direction. In a corner, it permits either row 1 or column h to be the rank where it was promoted; and a Pawn of either color could have promoted there."

"What of the other promotions, Watson?" Holmes asked.

"Besides the promoted White Bishop, there are three Black Rooks and three White Knights on the board, one of each necessarily promoted," I said. "With up/down movement, three files contain no Pawn, allowing three and only three promotions. If movement were left/right, there would be four files without Pawns, and a fourth promotion would be possible. The promotions still allow any of three move directions, including Black-down.

"I can't prove that Black *can* castle. It is rarely possible to prove the positive case. But there is no definitive proof that Black *can't* castle."

"You have not yet examined whether the Black King or its Queen's Rook has moved

during the game," said Holmes. His tone suggested I was overlooking an obvious point. "Clearly, you cannot establish with certainty where the promotions occurred. But you may have more success in determining where they could *not* have occurred."

"The only relevant case," I replied, "is that in which Black is moving down the board - the only case that could permit castling. Then, the White Bishop on h1 is a promoted Black Pawn. The other two promotions were necessarily on the a- and d-files, which contain no Pawns."

"On which file did a *White* Pawn promote?" asked Holmes.

"If the promotion was on the a-file, at a8, the Black Queen's Rook would have had to move. And," I realized in dismay, "if on the d-file, at d7 it would have checked, forcing a Black King move. *Any* White Pawn promotion rules out Black castling."

"Just so," said Holmes. "There is no possibility that Black can castle. You were correct in saying that the promoted White Bishop sheds no light on the move direction. But it is crucial in proving the impossibility of castling. If it is a promoted White Pawn, Black cannot be moving down the board. If it is a promoted Black Pawn with Black moving down, then at least one White Pawn promoted at a8 or d8, forcing a Black move that denied future castling."

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16. KNIGHT SITE

I began confidently, "The direction of movement is obvious. In heterochromatic play, White Pawns below Black Pawns show that White is moving up the board.

"The reference to 'even being odder' seems intended to make me focus on color and move parity. After any White move, White and Black have different move parities; one side has an odd and the other side an even number of pieces on each square color; for the *combination* of White and Black pieces, the number on each square color is odd. After any Black move, that number is even. Thus, if all 32 pieces are present or accounted for, the move parity on squares of either color will tell us whether White or Black moved last.

"In the present problem, the three missing Bishops were necessarily captured on their home squares, and the Pawn given as odds never moved from c2; those pieces should be counted as being on their original square colors. The third Black Knight shows that the missing Black Pawn promoted. We can account for the square colors occupied by all pieces of both sides, except for the ambiguously located Black Knight.

"Including the Pawn given as odds, White has seven pieces on white squares (two of them on f1). Ignoring the ambiguous Knight, Black has seven pieces on white squares. If the ambiguous Knight is on a black square (d2 or e3), there are 14 pieces on white squares - an even number, indicating that Black moved last. But, for either of those Knight locations, Black is checked by the White Queen on f1, and the last Black move would have been from imaginary check.

"Then, the Knight must be on a white square (d4 or e2). Move parity is odd (as

Holmes's comment suggested), and White moved last. It is Black's move, so the Black Knight cannot be on d3 where it would give imaginary check.

"The ambiguous Knight can only be on e2. It is Black's move, and neither side is in check.

"It would appear," I said thoughtfully, "that move parity contributes powerfully to retrograde analysis of heterochromatic games."

"Count on it, Watson," Holmes smiled.

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17. WHAT IS THE QUESTION?

To my mind, the promotion left the question wide open. The promotion *could* have been a reverse promotion, allowing any White or Black piece to be on f7. I considered the possibilities.

"A White piece on f7 cannot be a Queen or Bishop, or Black would be in check and unable to castle. A White Rook on f7 would check f8 and prevent King-side castling. A White Knight on f7 would check d8 and prevent Queen-side castling. So the piece on f7 has to be Black.

"A Black Queen or Bishop on f7, or an unmoved Black Pawn there, would check the White King, not allowing Black to castle. So the ambiguous piece must be a Black Knight or Rook, a third Black Rook possible because of the promoted Black Pawn. I see no way to decide which of those two Black pieces is on f7."

"Watson," Holmes smiled broadly, "I'm afraid Eliot led you up a garden path by giving information you did not need. If Eliot had not told you of promoting a Pawn, you would have counted Pawn captures to see whether a promotion was possible, and to what. I suggest that you do so now."

Chastened, I examined the Black Pawn positions. "The Black Pawns on a4 and b5 came from c7 and d7, with a total of four captures. If the piece on f7 is Black, as it has to be, five White pieces are missing. But the Bishop on f1 was captured at home, unable to move, and unavailable for capture by a Pawn.

"The Black Pawn from g7 would have required a capture to reach the h-file in order to promote. But we have already accounted for all possible captures of White pieces. That Pawn could *not* have promoted. Eliot was pulling my leg!"

"Let's be charitable, Watson," Holmes chided, "and assume that Eliot spoke only the truth. The Black Pawn from g7 *could* have promoted at h1, but only to a *White* piece. The reverse promotion made a seventeenth White piece, and allowed five Pawn captures plus the capture at f1 by another Black piece.

"You have pointed out, correctly, that no White piece can be on f7. So the reverse promotion of a Black Pawn does not widen the field of possibilities. On the other hand, the impossibility of promotion to a Black piece narrows the possibilities."

"I see," I conceded. "With no promoted Black piece possible, the piece on f7 cannot be a third Black Rook. It can only be an original Black Knight."

"Yes," said Holmes, "and you would have seen that much sooner if you had not milled around with Eliot's gratuitous floss about the promotion."

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18. WHAT FELL FROM WHERE?

I temporized by placing the coin on the line between squares e6 and e7. "Clearly, the fallen piece had to block the apparent Rook check of e8, or castling would have been illegal.

"A White Queen or Rook, on either e6 or e7, would have prevented Tennyson's castling by checking his King. A White Bishop or Knight on either square would have checked either f8 or g8, and prevented castling. A White Pawn on e7 would have checked f8. The only possible White piece would be a Pawn on e6.

"The White Pawn on e3 has captured once, from f2; one on e6 would have required four more captures from a2. And no more than three Black pieces are missing. No White piece, including a Pawn, can be on either e6 or e7. The fallen piece is Black."

I counted and recounted the obvious Pawn captures. "The Black Pawns on b5 and b6 have made three captures, from d7 and c7, respectively. The Pawn on e6 came from f7 with a fourth capture. White has six pieces missing, but that includes the Bishop that never left f1. Thus, only one more capture was available to permit promotion of a Black Pawn to a Black piece. The Pawn from h7 would have required two captures to promote at f1. The fallen piece is not a promoted Black Pawn.

"But I haven't ruled out the reverse promotion of a White Pawn. The Pawn on e3 has made one capture. The Pawn from a2 could have promoted at c8 with two more captures. I realize that only two Black pieces are missing, but the reverse promotion, by creating another Black piece, would have permitted three captures by White.

"With a reverse promotion, the fallen piece could be a Black Queen, Rook, or Knight, though not a Bishop. A Bishop promoted at c8 could have occupied e6 but not e7; and, at e6, it would check the White King, that imaginary check preventing Tennyson's castling. A Black Queen would have to be on e7, since on e6 it also would check the White King. But a Black Rook or Knight could occupy either e6 or e7. I am left with five possible combinations of Black piece and location."

"You came so close, Watson," said Holmes. "But you have overlooked one fact: the three Pawn captures required to permit White promotion at c8 include capture of the Black Pawn from the h-file. You have pointed out that the Pawn could not promote to a Black piece; and it could not be captured as a Pawn, on the b-,c-,or e-file. No Pawn of either color has promoted.

"With all other original Black pieces accounted for, only the original Black Queen can be the fallen piece. As you have shown, the Queen can only be on e7."

Shaking his head dubiously, Tennyson commented sotto voce, "Knowledge comes, but wisdom lingers."

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19. PROMOTED AND DEPARTED

"Of course!" I said. "In homochromatic play, the Black King is always on a white square, which requires movement to be up and down. If White is moving up, the Black Pawns on f2 and h2 have made six captures on black squares. Two of those could have been en passant captures of White Pawns on white squares of Row 3, leaving a requirement for capture of four White pieces that moved on black squares, those captures occurring on Rows 4 and 2.

"The White Rook on b4 has to be a promoted White Pawn, since an original White Rook, or one created by promotion of a Black Pawn, could not reach the fourth rank homochromatically. That leaves White missing three Pawns, an original Rook, and a Knight on black squares. The immobile Knight was captured on g1, not by a Pawn, leaving four White pieces for the four captures needed by Black Pawns.

"But the original White Rook, or one created by promotion of a Black Pawn, could reach only odd-numbered rows. thus could not be captured on Row 4 or 2 as required. And, with only five Black pieces on black squares available for capture, there could not have been a promotion of a *second* White Pawn on black.

"Reverse promotion of a Black Pawn to a White Queen or Bishop on black squares would seem to fill the need. But that promotion would require at least another three captures of White pieces on black squares, with only one Rook available. Thus, the position is impossible with White moving up the board.

"When White is moving *down* the board, all pieces now on the board can easily be original pieces. And the two Black Pawns have made no captures - indeed, have not moved. Where, then is the necessity for a promotion, Holmes?"

Holmes reminded me, "A Black Knight was on g1 at the start of the game, and has been captured there. By what? An original White Rook could not reach the first row; an original Queen could not reach the black square. Unmoved Pawns on f2 and h2 have prevented the White King and Bishop on black from reaching g1. The capture had to be by a promoted White Queen or Rook on black. A White Queen on black could have resulted from promotion of a Pawn of either color. A promoted White Rook, to reach g1, had to be a White Pawn promoted on the first row (White's eighth rank)."

"And," I realized, "neither of those possible promoted pieces is on the board now. The present White Queen is confined to White squares, and the present White Rook to even-numbered ranks. Ergo, the promoted piece that captured the Black Knight at g1 has subsequently been captured, as you deduced long ago."

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20. BOTH CASTLE?

"I assume," I told Holmes, "that your reference to the invisible hints at a Pawn that is no longer visible as such, having promoted. It is barely possible that a Black Pawn promoted to the Knight at a1 on the last Black move. The Pawn would have required five captures from its origin on f7. With one White Bishop captured at home on f1, the promoting Black Pawn would have had to capture the other five missing White pieces, including the Pawn from h2."

"Or that Pawn may have promoted to replace another piece captured by the Black Pawn," Holmes reminded me, before I could get the words out.

"Of the six missing Black pieces, two were Bishops that were captured on their home squares, hemmed in by Pawns. White Pawns could have made no more than four captures," I continued. "The White Pawn promoting at c8 would have required two captures from a2, leaving only two captures available for the White Pawn from h2. But that Pawn would have required three captures to reach the e-file, the nearest file on which it could have been captured as a Pawn by the Black Pawn from f7."

"But it would have required only the two available captures to reach the open f-file, permitting it to promote," Holmes pointed out.

"Yes," I assented. "But the White Pawn would have to pass through f7 in order to promote. There, it would have given check, requiring the Black King to move.

"You were right to object to my conclusion that the Black King or Rook made the last move. The last Black move *may* have been a King or Rook move, but could have been a Pawn promotion that depended on an *earlier* Black King move. In either event, Black cannot castle because his King or Rook has moved."

"So," said Holmes, choosing his words carefully, "if there *was* an invisible man, it was Ellison's and Black, not Wells's and White."

He allowed me the last word, "Of course, we would need a time machine to know whether Ellison had an invisible man."

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21. CAPTURE AND CASTLE?

"The seating suggests that White is moving up the board," I began. "Only the c-file contains two Pawns, and the Pawn order - White below Black - is consistent with that move direction but not with the reverse. It appears that the only question is whether the Black King or the Rook on a8 has been forced to move.

"With Pawns still on the d-, e-, and f-files, no promotion occurred on those files. So a promoting White Pawn did not force the Black King to vacate its home square. And there was no need for a White Pawn to promote at a8, forcing the Black Rook to move. A White Pawn could have promoted at h8, and promotion of a Black Pawn poses no problem.

"The Black King or its Rook *may* have moved; we can't prove the impossibility of such moves. But the usual approaches don't permit proof that either *did* move. Castling *may* still be legal."

Holmes said, in an exasperated tone, "For heaven's sake, Watson, *count* the promotions!"

I did so, "I find three, as indicated by two White Queens, three Black Rooks, and three Black Knights. But six files still contain Pawns, limiting possible promotions to no more than two. That doesn't make sense."

"You have fallen into a trap that the players may have set deliberately with their seating," said Holmes. A twinkle in Taliaferro's eyes made me suspect the truth of that supposition. "Even a *rank* amateur would question whether movement *could* have been up-down."

Chastened, I examined possibile left-right movement, with rows as files. "Five rows contain Pawns. The Pawn order on rows 2 and 3 would indicate White movement to the right, and nothing contradicts that indication.

"Three promotions could have occurred, on rows 1, 4, and 8 as files, but only if the last capture, at b4, was not of a Pawn. Since there *have* been three promotions, we know that last capture was not of a Pawn. It also could not have been the capture of a promoted piece, with a fourth promotion impossible."

"And what original Black piece remained to be captured," Holmes asked, rather unnecessarily.

"Only the Black Queen," I responded.

"And has either the Black King or the Rook on a8 moved?" was the final, gentle question.

"Clearly, both have moved far from column h, which was their home rank with White moving right. Black castling is absolutely impossible."

"You are right about the impossibility of castling, Watson, but there is still an inaccuracy in your penultimate sentence," said Cholmondely.

Holmes nodded. "I wondered about that, and was about to point out the possibility that the Rook on a8 is promoted, and has never moved as a Rook."

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22. POSITIONING MOVE

"With heterochromatic play," I observed, "the Black Bishop on h1 is necessarily promoted. But the Pawn on h5 does not allow a promotion to have occurred on the h-file!"

"Try looking at column h as a *rank*," Holmes suggested. "A Pawn moving to the right on file 1 could have promoted at h1."

"Then what is the Pawn at h5 doing on a first or eighth rank?" I inquired fretfully. "A Pawn *can* move to the eighth rank, promoting, but then must be exchanged for a piece."

The solution dawned on me, "Land, playing Black, advanced a Pawn to h5, then was called away before he could make the exchange. A move has been begun but not yet completed!"

"You have characterized the situation correctly," Holmes agreed, "though pure logic does not rule out the possibility that *Eastman* began that last Black move, interrupting it as a courtesy when Land was called away.

"Either way, you know the last move and the move direction. You should now be able to resolve the placement ambiguities."

"That last move would have been illegal if either side had been in check," I remarked. "So the Black Pawn must be on the c-column and the White Knight on the e-column to block both Rook checks. And the Knight cannot be on e7, where *it* would give check; it has to be on e6.

"But the Black Pawn can be on either c4 or c5, with no forbidden check in either case."

"Remembering that rows are files, can the Black Pawn be on c4?" asked Holmes.

"Yes, indeed!" I insisted. "A Black Pawn to the left of the White Pawn on f4 is consistent with the known move direction, Black moving to the right."

"Can the ambiguous Pawn be on c5?" Holmes continued.

I saw why it could not. "That would place two Pawns of the same color on file 4. The ambiguous Pawn has to be on c4."

Eastman flashed a congratulatory smile, "Excellent resolution, Watson. And Socrates would be proud of you, Holmes."

I didn't see why. All Holmes had done was pose questions.

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23. WHEN DID KNIGHT FALL?

I had been introduced recently to a number of problems in heterochromatic play, with color parity often central to the solution. I recalled more distant past experience, before being exposed to heterochromatic play, in which Knight move parity played a similar role. With no other promising approach, I thought I would examine that direction and see where it led me. It led me nowhere, and I confessed as much to Holmes.

"Perhaps a question can get you started," said Holmes. "What White and Black moves immediately preceded White castling?"

It took me a moment to realize that, before White castled, only his King's Knight could have moved. "The White Knight had to move. Since it is no longer on the board, it

moved to a square where it could be captured by the piece that moved last for Black.''

I continued, ''With four moves since Black castled, he could have moved his King, Queen, King's Rook, or even his Knight last. The Black Bishop, of course, has not moved. The Black Rook on a8 had no possible last move, with the Knight occupying b8. The White Knight was captured by the Black King, Queen, King's Rook, or the Knight on b8. I see no way to tell which.''

''Knowledge that the Black Queen has moved within the past four moves can eliminate one of those possibilities,'' Holmes suggested.

''Let me see,'' I mused. ''Since Black castled, the King has moved an odd number of times, actually only once, or the Queen could not have moved and returned to its home square within the four-move total. The Queen had to make at least two moves, with a maximum of one move by other pieces. So the King's Rook did not move, needing at least two moves to leave and return to the square it occupied immediately after castling. The White Knight could not have been captured by that Rook.''

''That leaves three possible squares where the White Knight could have been captured,'' said Holmes. ''What is the common characteristic of those three squares?''

''They are all on Black's first rank,'' I noted, ''and all are black squares. That square color may be a clue to move parity. The White Queen's Knight given as odds was on a white square, so the White Knights have even parity. The White Bishops did not move, and the Queen could not move before the last, castling move. The castling move, itself, makes total White move parity odd.

''It is Black's move, so Black move parity must be even. Without a Rook move after castling, the Queen was limited to two squares, so moved an even number of times, necessarily two. For a total of exactly four moves since castling, the Knight on b8 has moved once. When castling occurred, then, b8 was vacant, and the Rook on a8 has made no post-castling moves. The castling move, itself, makes Black parity even before considering Knight moves. To keep total Black move parity even, the two Black Knights must be on squares of opposite colors. Consequently, the missing Black Knight, which was the first Knight captured, died on a white square.

''The White Knight that was captured on Black's last move originated on g1, a black square. It made all White moves before the castling move, and changed square color with each move. It occupied a white square after any odd number of moves, a black square after any even number. Since it captured the Black Knight on a white square, it did so on an odd-numbered move - the 17th rather than the 18th move of the game.''

''You just need to be started in the right direction, then you do very well,'' Holmes complimented me.

''Why do I feel like a student of Socrates,'' I asked, ''who knew everything, but did not realize he knew it until the master asked the right question?'' (I had refreshed my hazy recollection of the Socratic method after an earlier reference by a player.)

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24. WHAT COLOR?

I began, "The two Bishops, which are immobile in heterochromatic play, demonstrate that movement is up and down the board. Both White Bishops may be original, with White moving down; both may be promoted White Pawns, with White moving up; or one or both may be promoted Black Pawns, with White moving down. We must consider both possible move directions.

"White appears to be in check by the Black Queen, and Black by the White Bishop. The fallen Pawn must be placed to block at least one of those apparent checks. If White is moving down, the Pawn on h5 is clearly checking Black, and made the last move; for that move direction, the fallen Pawn must be placed on e6, where it blocks *both* the Queen and Bishop checks. In that location, the fallen Pawn has to be White, to avoid two Pawns of the same color on a file. And a White Pawn above a Black Pawn on its file is consistent with the assumed move direction - White down.

"If White is moving up, the Pawn on h5 is not giving check; it is permissible for either the White Bishop or the Black Queen to be giving check, but not both simultaneously. The fallen Pawn must be positioned to block one of those apparent checks.

"With White moving up, both Bishops are *promoted* White Pawns, and there can be no other Pawn on either file c or file f. A second Pawn on the b-file would have to be Black, and be located *above* the upward moving White Pawn, not on b3, the only blocking square on the b-file. A second Pawn on the d- or e-file has to be White, and has to be below the Black Pawn. Only d5 both blocks one of the checks and is below the Black Pawn; it is the Black Queen check that is blocked. Black is in check from the Bishop, and has the move. We note that White's last move had to be a Pawn advance from c7 to c8, promoting to Bishop to create the check.

"In summary, we do not know whether White is moving up or down the board. The fallen Pawn may belong on d5 or e6. But, in either case, the fallen Pawn is White, and Black is in check, thus has the move."

"And we know that the last, checking move was made by a White Pawn," Holmes added. "Since Landor now has the move, it is Hood who is playing White."

Landor's nod confirmed that the move was his, and he said, "Is it not time then to be wise?"

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25. GET THE NUMBER

"We recently analyzed a problem involving Knight move parity," Holmes began. "And we have seen that move parity can be analyzed in any heterochromatic game, so long as we can account for all pieces of at least one color. We can account for only 14 of the White pieces, but for all 16 Black pieces. What do their positions tell us, Watson?"

"In the present position," I responded, "we count eight Black pieces on white squares (and eight on black squares, including the immobile Bishop on f8). Thus, Black has made

an even number of moves. Given the possibilities of 33, 34, or 35 moves, only 34 moves would leave an even number of Black pieces on each square color. And Black moved last, so each side has made 34 moves.''

Holmes pointed out, ''White must also have an even number of pieces on white squares. We count five visible White pieces, plus the Bishop from f1, on White squares. Of the other captured pieces, a Bishop was captured on c1 and a Rook on e1, both black squares. The missing White Knight and Pawn, then, were both captured on white or both captured on black squares.''

''What is the special significance of the Black Pawn move from g7 to g6?'' I inquired peevishly.

''That was only an example of a minimum change that would make the problem insoluble,'' laughed Holmes. ''The number of Black pieces on white squares would have been seven, an odd number, allowing either 33 or 35 moves, though not 34. Changing the square color occupied by *any* one Black piece would have allowed two possible solutions rather than a unique solution.''

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26. NUMBER UPPED?

''Concentrating on the check,'' I said, ''suggests heterochromatic play. The Black Queen could have moved heterochromatically from h7; the Black Knight could have moved from f6 to discover check, or the black King from f6 or g7. The immobile Bishops indicate left/right play. The two Pawns show that White would have to move to the left in heterochromatic play. Then, both Bishops are promoted. But, in heterochromatic play, two promotions on the same file are an impossibility. Play has been *homochromatic*, and the immobile Knights prove that movement was up/down rather than left/right.

''With homochromatic play, no piece now on the board could have made the last, checking move. Knights do not move homochromatically. Either King move to discover check would have been heterochromatic. The Black Bishop could not have discovered check from g7, where the Bishop would have given imaginary check. And any homochromatic move of the Black Queen would have been from imaginary check somewhere on the long diagonal.

''Thus, Black's last move was the promotion of a Pawn. A Pawn promoting to the Knight on g8 could have moved homochromatically only by capturing from h7, and that move would not have discovered the Queen check. The promotion was to the Black Queen, the Pawn capturing from g7.

''Despite your suggestive question, Holmes, the promotion could not have been the reverse promotion of a White Pawn. Even when reverse promotion is permitted, the rule against moving into check is overriding.

''With Black moving up, all three Knights on the board also are promoted, each from a Pawn of its own color. So, the answer to your more difficult question is that four Pawns

have promoted, with none of the promotions to a piece of opposite color."

"All you have shown thus far, Watson," Holmes chided, "is that *at least* four Pawns have promoted to pieces of their own colors. Can you prove that no additional promotions have occurred?"

"I confess that I can't," I replied. "Too many Pawns are missing that might have promoted."

Holmes seemed pleased with himself as he pointed out, "After an initial two-square advance, each promoting Pawn must make four captures. Each side has two visible promotions, requiring eight Pawn captures; five pieces of each color are still on the board. The remaining three pieces of each color do not permit another four captures by either side for a third promotion."

Astonished but gleeful, I replied, "Holmes, you have forgotten that a reverse promotion increases the number of captures possible! A Black Pawn could have promoted to a White piece, the additional White piece permitting the one additional capture needed."

"I apologize, Watson," said Holmes, "for my abbreviated discussion that let you fall into that trap. While the reverse promotion of a Black Pawn would have provided a seventeenth White piece as you say, it would have reduced the number of Black pieces to fifteen. But note that the White Pawn on c2 has required three captures to reach that square. The eight captures further needed for two promotions, plus the five Black pieces still on the board, account for all of the original sixteen Black pieces. No Black piece can have changed color. And, with no further Black pieces available for capture, no White Pawn has made a third promotion.

"You were entirely correct when you said there have been exactly four promotions, none of them color-changing. My objection was merely a demand for proof."

Holmes added, "In retrograde chess analysis, we normally accept as a fact that the problem position *did* result from legal play. If we were manufacturing *artificial* problems, as for a book, we should have to demonstrate that the problem position *could* be reached with legal play. That can be especially difficult when play is homochromatic."

Author's Note: Holmes makes a valid point. Every homochromatic problem in this book *has* passed the test of constructing a feasible sequence of homochromatic moves, from initial setup to the problem position. Where less restrictive rules apply, less complete demonstration is accepted, often limited to assuring that Pawn formations could be reached without impossible numbers of captures, and that routes to the open board could have existed for Kings, Rooks, and Bishops.

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27. THERE IS A SOLUTION!

I saw no possible solution if the Rook check was not blocked, and refused to consider the problem further until Holmes wheedled, "Come now, Watson. You suggested that

another piece belonged on the board, and Longstreet said what?''

"He said, 'No, quite the contrary.''' I said ungraciously.

"And what is the contrary of an additional piece?''

"I suppose," I responded, "a piece *less*.''

"Ask yourself two questions," Holmes went on. "What piece could be removed to make the situation legal? And how does the need to remove a piece relate to Meade's being interrupted?''

I grudgingly consented to examine the board again. Every time I had seen a problem that appeared so impossible, the solution had involved an en passant Pawn capture. Could that be the situation here? If so, that capture had to be compatible with the very real Rook check.

"Suppose the last move was by White," I essayed. "A White Pawn, moving right, could have captured from e6 to f5, giving check and discovering the Rook check. If that capture was en passant, the Black Pawn on e5 had made the preceding move, from g5. It is necessary to remove the Black Pawn from e5 to complete the en passant capture!

"Of course! Meade, playing White, had begun the en passant capture by moving his Pawn from e6 to f5. He was interrupted before he completed the move by removing the Black Pawn from e5.''

"That's it!" said Holmes. "You saw my relief when Longstreet ruled out the more mundane case of another piece to block the Rook check. The interrupted en passant capture is far more pleasing.''

Longstreet wrapped up the discussion, "The double check would have forced me to move the King, probably to capture the Pawn on g5. The White Rook could then capture my Queen, leaving my remaining Pawn easy prey. My lone King would have been a sure loser to a Rook and King, so of course I conceded.''

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28. CHECKING ACCOUNT

I had mentioned only that the situation was a check, and decided I had better explore the ramifications of that check, by the White Bishop on c8. "A Pawn did not just promote to Bishop to create the check," I said. "It could have moved only from c7, which is occupied. So the check has to be discovered.

"A move by the White King did not discover the check. It could have moved only from d7 or e6; on either square it would have been in an impossible double check by two Knights. The Rook on d5 could have discovered check except that a move from either d7 or f5 would not have been heterochromatic.

"That leaves two possibilities. The White Pawn on g5 may have moved from g4 to discover the Bishop check. That would require White to be moving up, with the Bishop on c8 promoted.

"But it is equally possible that the White Knight now on g7 moved from either e6 or f5 to discover the check, and the Knight move is compatible with either move direction."

"If the Knight made the last move," Holmes asked, "what was the situation before the Knight move?"

"That would depend on whether the Knight made a capturing or noncapturing move," I replied. "Ah! If the move was *not* a capture, then before the Knight move, the White King was in check by the Black Rook on h7, with no possible Black move yielding that check. To prevent the imaginary check, g7 had to be occupied, and the Knight move was a capture. A Bishop could not have been on g7 in heterochromatic play, and Black still has a Queen, two Rooks, and two Knights; a Black Pawn was on g7 or there had been a Black promotion."

"And *was* there a Black promotion?" Holmes persisted.

"No," I realized. "Every file except the c-file contains a Pawn, ruling out promotion on the file. And the promotion on the c-file was the White promotion to Bishop. With no Black promotion, the piece captured on g7 had to be a Black Pawn. With a White Pawn still on that file, and below g7, White has to be moving up, and the Bishop on c8 is promoted.

"But which *was* the actual last move, Holmes?"

"It is impossible to tell," he replied, "but does it matter? Whether the White Pawn or White Knight moved last, White is moving up, and the Bishop on c8 is promoted."

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29. FROM WHERE?

I realized that displaced Pawns called for an analysis of Pawn captures, and began briskly, "Doubled Black Pawns on the a-file require one capture, and those on the h-file a second. The ambiguous Black Pawn requires a third capture if on b5.

"Three Black Rooks prove a promotion; if the promoting Pawn was Black, it came from e7 and required one capture, the fourth by Black. If the ambiguous Bishop is Black, the last move involved another capture by Black, making a maximum of five Black captures required, with six missing White pieces. No problem there.

"However, if Black made the last move, a check, then Black cannot also be in check by the White Queen; to block that check, the ambiguous White Pawn has to be on g4. Doubled White Pawns on the a-file have made three captures, the doubled Pawns on the h-file a fourth capture, and the Pawn at g4 a fifth capture. With the ambiguous Bishop Black, only four Black pieces were available for capture. So the ambiguous Bishop cannot be Black. Unless," I faltered, "a White Pawn promoted to a Black piece."

"That would change nothing essential," said Holmes. "The Pawn from d2 would have required a capture to promote at c8 or e8; the supply of captures would increase by one, but so would the demand, leaving one capture too few. The ambiguous Bishop is definitely White."

"Then," I continued with relief, "Black is in check from the Bishop on e4; no White move could have discovered that check, so the Bishop moved last, capturing at e4. White Pawns on the a- and h-files still require four captures. The ambiguous White Pawn could not capture to reach g4, so belongs on f4. Black, then, is in check by the White Queen as well as by the Bishop. The double check proves that the last move was a White Bishop move from g2 to e4, capturing, checking, and discovering the Queen check."

Holmes said, "You might tidy things up by resolving the ambiguous Black Pawn."

"Oh, yes," I said, "though it is pretty obvious now. White cannot be in check, so the Black Pawn belongs on b5, where it blocks the apparent check. Black Pawns, including the Pawn promoting to Rook, needed only four captures, with five missing White pieces."

"Bullseye!" said Cross. Having been loudly wrong on occasion, I held my breath until he uttered that second syllable.

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30. THE ROAD TAKEN

I mused, "The Pawns show that White is moving up the board. Then the White Pawn on h6 is giving check, and made the last move.

"Counting the Black Bishop necessarily captured on c8, the total number of pieces known to be on white squares is 13. That 'color parity' has to be odd following the White move, so the fallen White King belongs on a black square. Examining each black square, we find that the King can be only on a3 or d8 and avoid a situation of mutual check. I don't believe the White King could have reached d8 by any route while avoiding imaginary check; but I can't quite prove that."

Holmes took over, "It is easy to show a feasible route for the White King from e1 to a3, through d1 and d3, before the Pawn now on c3 advanced from c2. But it could reach the eighth rank only from e7, d7, or c7, and could reach the latter two squares only through e7, which therefore appears in all possible routes. And the White King could never have occupied e7; with the Black Bishop always on f8, a King move to e7 would have been a move into check.

"In fact, the King could never have reached e6, en route to e7, without encountering an imaginary check. But it is a little simpler to concentrate on e7 as a square the King would have needed to reach, but could not.

"Therefore, a3 is the only location the White King could have reached heterochromatically that satisfies move parity without mutual check."

I nodded in understanding, and said, "The need to examine a specific route segment for heterochromatic feasibility is a new element in our analyses, Holmes. We may encounter that necessity again some time."

"I would almost bet on it," Holmes chuckled.

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31. DUAL PROMOTIONS

"Black is in check from the White Rook on h6," I began. "That Rook could not have moved from h7, an imaginary check position, and could not move from any other square as a Rook. No White piece now on the board could have moved from h7 to discover the Rook check. So the last move had to be made by a White piece no longer on the board -- a promoting Pawn. There are two possibilities.

"If White is moving up, the Rook check could have been discovered only by a White Pawn from h7 capturing on g8, and promoting to the Black Bishop. What Black piece could have been captured on g8? Not a Queen or Rook; either would have checked the White King, and could have produced the check only by moving from an imaginary check position. With two Black Knights on the board, a third Knight captured at g7 would have proved a promotion prior to the reverse promotion on White's last move.

"Aha, Holmes! You have overlooked a possibility. The Black piece on g7 could have been an original *Bishop*, replaced by the promoted Bishop there now. Nothing would have appeared to change except the disappearance of the promoting White Pawn. No second promotion is necessary."

"I will grant you, Watson, that the piece captured on g8 could have been a Bishop. But not an *original* Bishop. With White moving up, the original Bishop was confined to c8 by Pawns, and died at home. If the piece captured at g8 was a Bishop, it was a promoted Bishop that was captured, and there have been two promoted Black Bishops. Capture of either a Knight or Bishop on g8 proves two promotions."

"Agreed," I said sheepishly. "Let's consider the other alternative. White could have been moving right, a White Pawn from g5 capturing to h6 and promoting to White Rook. The Pawn could not have captured from g7, where it would have given imaginary check. What could have been captured on h6? A Black Queen there would have checked White, and could have moved to h6 only from imaginary check positions g7 or h7. (At first, I almost concluded that the Black Queen could have moved from the diagonal through g5, until I remembered that the White Pawn was on g5 at that time.) The piece captured on h6 had to be a third Rook, Bishop, or Knight, any of those proving a promotion to a Black piece preceding the White promotion to Rook."

"So, Watson," said Holmes. "Now you understand why I didn't know what promoted piece is still on the board, or even what color it is. If White is moving up, the Black Bishop on g8 is promoted. But if White is moving right, the White Rook on h6 is promoted."

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32. UNMASK THE BISHOP

The Pawn formations betrayed an almost homochromatic pattern, marred by the White Pawn on e3. I said, "The White Pawn on d3 clearly captured from c2, so the Pawn on b7 came from g2, capturing on every move. The six captures by White Pawns were all on white squares. The Black Pawn on h6 captured from g7, and that on g3 made four captures from c7, all five of those Pawn captures being on black squares."

"It could be instructive," Holmes suggested, "to consider what pieces they captured."

"That is interesting," I said. "Only six missing Black pieces *could* move on white squares - the Queen, a Rook, a Bishop, two Knights, and a Pawn. All were captured by White Pawns. Clearly, the fallen Bishop is not a Black Bishop on a white square.

"The White Bishop from c1 was unable to move. If not the fallen Bishop, it was captured at home; it was not among the five White pieces captured by Pawns on black squares. That leaves one White Pawn, the Queen, one Rook, and two Knights capable of moving on black squares, all of them captured by Black Pawns."

"What do the opening moves tell you, Watson?" Holmes asked.

"The only White Pawn that could have made an initial two-square advance is the Pawn from h2," I observed. "That means it was not captured on g3, as appeared possible. It may have remained on the h-file, being the piece captured at h6. Or it may have captured to the g-file, promoted, and been captured subsequently as a promoted piece. The only piece it could have captured for promotion was the Black Bishop on black squares.

"It appears that the Black Pawn from b7 made Black's initial two-square advance. Or the Pawn from a7 could have done so, the Pawn now on a6 capturing from b7. In either event," I realized, "the Pawn from b7 has moved, and was not captured as a Pawn at b7. A Pawn from either a7 or b7 has promoted to provide the sixth capture on white squares."

"And where did the promotion occur?" asked Holmes.

"We have accounted for the capture of five White pieces, with the Bishop on c1 a possible sixth," I said. "At most one more White piece was available for capture, the Bishop from f1, on white squares. The Black Pawn, with only one capture available, had to originate at b7 and promote at c1. That means the Bishop originally on c1 *was* captured earlier.

"We have now accounted for the capture of both White Bishops, and the Black Bishop on white squares. The fallen Bishop can only be the Black Bishop on Black squares. That means the White Pawn from h2 was unable to capture in order to promote, and is the piece captured on h6.

"But, following the capture on h6, the door was open for the Black Bishop to leave f8, and it could be on any black square not obviously occupied, except c1."

"How many Pawns have been captured?" Holmes asked me.

"The Black Pawn from b7 promoted, so was not captured as a Pawn. Why," I realized, "only the White Pawn from h2 was captured as a Pawn. The last move was capture of a Pawn, and was necessarily at h6. Before that capture on the last move, the Black Bishop on f8 could not move. It has fallen from its home square, f8."

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33. ROOK DOWN

I had noticed the apparent double check of White, by the Black Queen on a5 and the Rook on e6, but had overlooked its impossibility. It was clear, now, that the Rook could not have made a move that both produced its own check and discovered the Queen check. In fact, no last move could have created the Rook check alone (with Black moving up, a Pawn capture from e5 could have discovered the Rook check, but too few Pawn captures were available to sustain that move direction). I now understood how Holmes could be confident a piece had fallen. A fallen piece belonged on the e-file, to block the impossible Rook check.

"The fallen Rook," I said, "of course belongs on the e-file, blocking check by the Black Rook on e6. With White Rooks on a1 and h4, another on the e-file requires promotion of the missing Pawn to a third White Rook. The last move was by the Black Knight, from b4 to a6, discovering the Black Queen check."

Holmes held up a commanding hand to keep Gatsby silent, and said, "That is a logical possibility, Watson, but not a logical necessity. You have not proved that the position is *not* a double check. You have not ruled out the possibility that the fallen Rook is *Black*. And you have not addressed the significance of the Bishop positions."

I thought aloud, "The fallen Rook, now viewed as Black. conceivably *could* have moved to create a double check, but only by moving from d2, where it blocked the Black Queen check, to e2. Before that move, e2 had to be occupied, to block the apparent check by the Black Rook on e6. The last Rook move, then, captured a White piece on e2.

"At first blush, it would appear that the White piece captured at e2 might be the Pawn that originated there. And that, of course, is the significance of the Bishop positions. The Bishop originally on f1 could not have left home and reached h3 until after the Pawn moved from e2. So the capture at e2 was not of the Pawn. With the full complement of other White pieces still on the board, any capture at e2 would prove a promotion."

"That's the other half, Watson," said Holmes. "Either the fallen Rook is a third White Rook, proving a promotion, or it is a Black Rook that captured at e2, also requiring promotion of the White Pawn."

"Would you like to know which?" asked Gildersleeve.

"Good Lord, no!" said Holmes. "What *could* have happened is so much more real than what actually happened."

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34. PROMOTED TO TRAVEL

Holmes explained, "None of the usual indicators of move direction can be found in this position. There are no Pawns on any outer row or column to rule out any move direction. No row or column contains two Pawns of the same color to rule it out as a file. No row or column contains two Pawns of opposite colors, to rule out one direction of movement. There are no Pawn 'checks' to give motion clues, and no immobile Bishops are present.

"Black is in double check from the White Queen on a4 and the Rook on e5. Clearly, the checking Rook made the last move, from b5, discovering the Queen check. The moving Rook had to capture at e5, or Black would have been in imaginary check from the White Rook on e4. The captured piece could not have been a Bishop. It could not have been a Pawn, which would make a second Black Pawn on a file, whether movement is up/down or right/left. The piece captured had to be a Black Queen, Rook, or Knight. Since the Black pieces on the board after the capture included a Queen, two Rooks, and two Knights, the piece captured was a second Queen or third Rook or Knight, proving that a promotion occurred.

"Either the promoted piece or its original counterpart could have been captured at e5, leaving us uncertain as to whether the promoted piece is still on the board.

"Although the promoted piece is Black, the promoting Pawn could have been of either color. We know that the Pawn promoted at one of the four corners of the board, since only the outside rows and columns presently contain no Pawns. Since no corner is now occupied, the promoted piece did move."

"I am convinced," Haggard admitted. "I will concede that you have the mind of a Solomon. But I will not grant you his heart until I have seen your poetry."

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35. PAWN OR BISHOP?

I interpreted Holmes's wording to suggest, in part, that I should count pieces on one color to establish color parity. "White made the last move, checking with a Knight to d6. Following a White move, color parity has to be odd. There are five White pieces on white squares," I began.

"There are *six*," Holmes corrected me. "You are overlooking the Bishop captured on f1."

"Correction accepted," I said. "So an odd number of Black pieces must be on white squares. There are eight visible Black pieces on white squares. Whether fallen or captured at home, the Bishop originally on f8 is on a black square. So the Black Pawn is on a white square or was captured on a white square."

"You have heeded well my reference to counting," Holmes said, approvingly. "What about paving the way?"

I realized, "Of course! The Bishop could not still be on f8. Its capture was necessary to

clear the way for the Rook from h8 to reach the board. So the fallen piece is a Black Pawn, originally on e7, and now on a white square.

"And, a Pawn cannot leave its file heterochromatically, so it belongs on either e4 or e6. But which?"

Holmes smiled. "The Black Rook needed *two* barriers to be removed before it could reach the board. Every possible route was through g8, f8, f7, e7, and e6. The Black Pawn from e7 had to reach e5 before the Rook could reach its present position. It cannot be on e6, so is necessarily on e4."

"With the Black Pawn on e4," I pointed out, "the White Rook on e3 is not giving check. Black is in check only by the Knight on d6; that Knight made the last move, from c4, f5, or f7, checking but not also discovering a second check."

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36. TRUTH AT THE TABLE

Holmes drew the following table. "The first column shows, in abbreviated form, the possible facts concerning a fallen piece. The second identifies each brother. The third columshows how each would describe the facts. The final column shows how the *other* would describe his brother's comment.

Fact	Brother	Says	Reported As
Fallen	Liar	Not Fallen	Not Fallen
	Truthful	Fallen	Not Fallen
Not Fallen	Liar	Fallen	Fallen
	Truthful	Not Fallen	Fallen

"Regardless of which is speaking about the other, what he reports his brother as saying is the *opposite* of the facts about which his brother spoke. We can know the facts without knowing whether or not the speaker is truthful! The 'Not Fallen' report indicates conclusively that a piece checking one King *has* fallen from the board."

"I am not sure I understand fully, Holmes," I said, "but I will accept your conclusion. A piece checking one King has fallen from the board. But what do we know about whether the game was homochromatic, heterochromatic, or neither? The liar and

truth-teller used the same words."

"Just as they would have about the fallen piece," said Holmes. "And it is equally possible to ascertain the truth from their statements. Consider *this* table," and he sketched again.

A	B	A or B	
True	True	True	
True	False	True	←—Truthful
False	True	True	
False	False	False	←—Liar

"The table is valid for any pair of statements, A and B. The statement 'A or B' is true unless both A and B are false. In our present case, A means 'I played Black,' and B means 'The game was neither homochromatic nor heterochromatic.' 'A or B' has to be false when spoken by the liar. So the liar did *not* play Black, and the game *was* either homochromatic or heterochromatic.

"The truth-teller used the same words, but with a different meaning. 'B', a factual statement about the game, remains false; but 'A' now means 'The truthful brother played Black'. The truth of 'A' makes 'A or B' true when spoken by the actual player of Black.

"We know, then, that the truthful brother played Black, and that the game was either homochromatic or heterochromatic. Our normal retrograde analysis can determine which.

"For example, Watson, does the assumption of homochromatic play reveal any contradiction?"

I was relieved to be back in the more familiar world of retrograde analysis, and examined the position closely. "Play can only be left/right homochromatically, or the Kings would be on squares of the wrong colors. Two of the White Rooks are on white (horizontal grey) squares, so one is promoted if White is moving to the right. And it has to be a *reverse* promotion, since a Rook promoted on White's eighth rank could not reach the first or third rank.

"At the same time, all three Black Rooks have to be promoted Black Pawns, since none could have reached its present rank from Black's first rank. That makes four promoted Black Pawns, requiring 16 Pawn captures, and only eight White pieces are missing.

"With White moving to the left, all three White Rooks are promoted White Pawns, as is one of the two Black Rooks on black squares. Again, too few Pawn captures are available for four promotions by one side. Play cannot be homochromatic, so must be heterochromatic."

"That wasn't too difficult, was it?" asked Holmes. "And you can easily determine the direction of movement in the heterochromatic game."

"Yes," I replied. "Two White Pawns on the c-column rule out up/down movement. Rows are files, and the Pawn order on file 4 proves that White is moving to the right. One Rook of each color is promoted, those promotions taking place on files 1 and 8, the only files that do not contain Pawns now. With no other promotion possible, the fallen piece cannot be a Queen or Rook of either color. We need consider only possible Pawn or Knight checks."

"You haven't specifically mentioned Bishop checks, which could only be discovered checks," said Holmes. "But only original Bishops would be possible, limited to their home squares, and unable to check any square on the eighth rank."

"There can be no White Pawn on g5 or g7," I said, "Where it would be to the right of a Black Pawn on the same file. And there can be no Black Pawn on b3 or b5 where it would be left of another Pawn on its file. The fallen checking piece is necessarily a Knight.

"A Black Knight could check the White King only from b2, b6, c3, or c5, and all of those squares are occupied. Squares f7, g4, and g8 also are occupied, leaving f5 as the only square where a White Knight can be located to check the Black King.

"The fallen piece is, then, a White Knight, fallen from f5. Giving check, that Knight made the last move. And it was moved by the liar, since his truthful brother played Black."

"Quite right, Watson," said Holmes. "Solving the problem wasn't nearly as difficult for you as discovering what the problem was."

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37. WHITE'S THIRD MOVE?

I turned my attention to the position, and voiced my train of thought. "It is obvious that each side has made three captures with Pawns. The White Pawn on f3 captured once from e2; that on f4 captured twice from d2. The Black Pawn on c6 captured once from d7; that on c3 captured twice after an initial move from a7 to a5. The Black Pawns on g5 and h5 obviously made initial two-square advances; so did the White Pawn on h4, and possibly the missing White Pawn from c2.

"Since Knights do not move homochromatically, and Pawns block other pieces, each side's first move in a homochromatic game must be a two-square advance by a Pawn. White may have moved either of two Pawns on his first move, and *may* have moved the other Pawn on his second move. With c2 vacated, the White Queen could have moved to

any of three squares on its open diagonal. With h2 vacated, the White Rook could have moved from h1 to h3. It is possible that the Queen or Rook made the second and third moves, or that the second Pawn made the third move.

"There are too many possible third moves, Holmes, to say that a specific third move *must* have occurred."

"Come now, Watson," Holmes chided. "Consider *what* was captured, and you can eliminate some of those possibilities."

I mused aloud, "The White Pawn on f3 necessarily captured the Black Bishop from c8. The Pawn on f4 captured two of three pieces, the Black Queen, Bishop from f8, and Rook from h8. The Black Pawn on c6 captured the White Queen or the Bishop from f1. The Black Pawn on c3 captured the Bishop from c1 and -- no other missing White piece moved on Black squares!

"Therefore, the capture at c3 was an en passant capture of the White Pawn from c2."

"Very good, Watson," Holmes approved. "So the White Pawn from c2 did not move until the Black Pawn had already reached b4 by capturing the White Bishop from c1. And a Black Queen or Bishop had to reach e3 and be captured by the Pawn from d2, before the White Bishop could move. The Pawn from c2 did not move within White's first three moves."

"That *does* simplify the problem!" I exclaimed. "Only the Pawn from h2 could have made White's first move, to h4. No White Pawn could capture within the first three moves, so only the Rook from h1 was mobile. It made the second move, from h1 to h3, and the third, from h3 back to h1. It continued to shuttle between those two squares until a Pawn capture became possible. But I see nothing unique; we have seen en passant captures many times."

"There is one flaw in your analysis, Watson. It fails to explain how the White King reached its present position."

"I didn't realize I was supposed to explain *that*," I said somewhat impatiently. "After the Pawn capture at e3 vacated d2, the King could move through d2, c3, d4, e5, f4, and g3 to reach h2. That had to happen before the Black Pawn from g7 advanced to g5, so the White King would not move into check at f4."

Holmes shook his head in mock sadness, "But the Black Piece captured at e3 was the Bishop from f8, which could move only after the Pawn move from g7 to g5. Thus, once d2 was vacated to allow a move by the White King, it was already too late for the King to pass through f4 en route to h2.

"You may think that the Black Queen was the piece captured at e3. That Queen's access to the board was blocked initially by the Pawn on d7, the Bishop on c8, and the Bishop on f8. The Queen could not reach the board through f8 until after the Pawn move from g7 to g5 allowed the Bishop to move from f8. The Pawn on d7 had to capture to c6 before either the Black Queen or the Bishop fron c8 could pass through d7. The piece captured at c6 was the White Queen, which could not reach the board until d2 was

vacated by the Pawn capture at e3. Whether the piece captured there was a Bishop or Queen, it had moved through g7, and a Black Pawn was already at g5."

"But that leaves an impossibility, Holmes," I protested. "The only other way for the White King to reach h2 was by castling to g1. And castling would have been illegal after the Rook from h1 made the second White move of the game."

"You are right, Watson," Holmes chuckled. Castling *would* have been illegal after the second White move. But castling *did* occur. It follows that White castled no later than the second move, and quite possibly on the *first* move of the game. Clearly, White gave odds of the King's Bishop and King's Knight. Castling was one of White's first two moves, the other being the Pawn move from h2 to h4. The only possible third move was of the King, from g1 to h2."

"You are amazing, Mr. Holmes," said Lady Ashley. "I wanted to tell you of the odds, thinking you couldn't possibly deduce what happened otherwise, but you wouldn't let me."

Holmes accepted the compliment with a bow of his head, "I often say that, when the impossible has been eliminated, what remains must be true. The rules of chess allow White to give odds. In the absence of odds, the situation was impossible. Ergo, odds were given, and the specific odds can be deduced."

"I would wager that castling was the very first move," I ventured. *"That* would certainly qualify as unique."

"You would win your wager, Dr. Watson," said Lady Ashley. "I am somewhat surprised that Mr. Holmes did not arrive at that conclusion."

"I would have been willing to make the same wager," said Holmes. "But that would have been betting on a psychological probability, whereas retrograde chess analysis deals only in logical certitude, not in probabilities."

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38. LAST NONCASTLING MOVE?

"Stratton moved after Briggs moved his Queen," I realized. "So his last move was not by a piece subsequently captured by the Queen. If the ambiguous Queen is White and Stratton is Black, he did not move his King or Rook, both still in position to castle. And three of the four Black Pawns have not moved. The only possible move was of the Pawn now on d6, from d7."

"Come now, Watson," Holmes chided. "If that Pawn had been on d7 before Stratton moved, the White Queen on g4 would not have prevented Black castling."

"Then there was no possible last move for Stratton, playing Black. Stratton is playing White, and the ambiguous Queen is Black."

"How did a Black Queen reach g4?" asked Holmes.

I realized, to my consternation, that a Queen from d8 could have reached only squares

b8, c8, and d7 without a King or Rook move. "The Black Queen has to be promoted," I asserted.

"And where did it promote?" Holmes persisted.

"Not on any of the five files containing Pawns," I began confidently. "That leaves the a-file; but neither Rook on that file has moved. The f-file contains no Pawn; but a White Pawn could not advance through f7 en route to promotion without giving imaginary check to an unmoving King, and a Black Pawn could not pass through f2. Promotion was possible only on the h-file."

"And how do we know that no Pawn promoted to a Black Queen on the h-file?" Holmes went on relentlessly.

Sheepishly, I acknowledged, "One of three White Rooks is promoted, accounting for the only possible promotion."

"Then," said Holmes, "The ambiguous Queen is White. I repeat the original question. What was Black's last move?"

"No Black piece still on the board moved last," I managed, "and there was no subsequent White move that could capture it. The Black move could have been only a Pawn move, from h2 to h1, with reverse promotion to the White Rook now on that square."

I had not noticed when Stratton joined us, but he now said, "Bravo, Watson. You roar to life when Holmes yanks your chain."

Briggs added, "You tend to cut a narrow swath through the details. But you do power your way through to the end. If your ability to cut it is ever questioned, refer your critics to the two of us."

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39. HOMOCHROMATIC PROMOTION

"First," said Holmes, "I will review for the benefit of Lord and Lady Ashley the logic that led Watson to his conclusion that White could not be moving up the board, with the White Rook promoted.

"The Black Pawns do have enough captures available to reach their present positions, moving down. One feasible sequence has the Pawn now on g3 originating on g7, advancing to g5 in one move, capturing at h4 the White Pawn from h2, and capturing at g3 the White Pawn from g2, en passant. The Pawn on d4 originated on c7, advanced to c5, and captured at d4 the White Pawn from d2. The Pawn on f2 originated on e7, advanced to e5, captured a White Bishop on f4, the White Pawn from e2 on e3, en passant, and the unmoved White Pawn on f2.

"But could any White Pawn have promoted? The missing Black pieces on black squares are one Pawn, from a7, one Rook, from h8, and the Knight on b8. The Black Pawn from b7, though on white, can be captured en passant at b6, seeming to provide the

fourth capture necessary for a White Pawn promotion. However, the Black Rook from h8 can be captured only on White's eighth rank, which rules out capture of the Black Knight by the promoting Pawn, or on White's sixth rank, where it substitutes for the en passant capture. The Pawn from a7 can be captured there or at a5, but no capture is possible at the other of those two squares. With no White promotion possible, White cannot be moving up the board."

"That was not an easy analysis," said Lady Ashley. "Doctor Watson's achievement *was* praiseworthy."

Holmes continued, "With White moving down the board, the White Rook is an original Rook, The Black Queen and Bishop can easily be original pieces. No piece presently on the board is necessarily promoted.

"But the Rook on b4 made the last move; its check clearly was not discovered. The Rook could move homochromatically only from b8 or b6. To prevent imaginary check, a Black piece had to be occupying b4, that piece being captured by the Rook.

"The piece captured on b4 could not have been the Pawn from b2. On b4, the Pawn would have given check, thus would have made the preceding move. That move could not have been a two-square advance from b2, or the Black Bishop would have given imaginary check before the Pawn move. The Pawn could have reached b4 only by capturing first at c3 (to block the Bishop check), with a second capture to b4. The Pawn capture at g3 would have made three Black Pawn captures on black squares. The only missing White pieces that could reach the capture squares were the Queen, one Bishop, and four Pawns. Thus, a White Pawn had to reach Black's fourth rank, requiring the Pawn to capture on that rank.

"In the absence of a Black promotion, the piece captured by the White Pawn on the fourth rank could not have been a Black Queen or Bishop. It was not the Pawn from b2, which would capture later on b4, and the other Black Pawns on black squares are still on the board. The only other missing original Black piece that moved on black squares was the Rook from a1, and it could not reach Black's fourth rank. Thus, capture of a Black Pawn at b4 was impossible without a prior Black promotion. And a prior Black promotion would have been of the Pawn from b2, making it unavailable for capture by the White Rook on the last move."

"A Black Rook could have been the piece captured at b4 by the White Rook, but only a *promoted* Black Rook. And, of course, a Black Queen or Bishop captured at b4 would have required a Black promotion to create the second Queen or Bishop."

"To summarize your able analysis, Holmes," I said, "the White Rook on b4 captured on the last move. A Black Rook captured at b4 had to be promoted; a Queen or Bishop captured there would have required a promotion to create a second Black Queen or Bishop on black. A Black Pawn captured on b4 would require an absolute absurdity - an earlier promotion of that same Pawn!"

"Well summarized, Watson," said Holmes. "And that leaves only the question of feasibility of the Black Pawn promotion."

"Easy enough," I replied. "There were an adequate number of captures available on black squares. One feasible move sequence is an initial noncapturing move from b2 to b4, capture at a5 of the Pawn from a7, capture en passant at b6 of the Pawn from b7, capture of the unmoved Pawn at c7, and capture of the unmoved Knight at b8. The order of capture of the Pawns from a7 and c7 could have been reversed, but the promotion had to occur on b8. Although a White Bishop also was available on black squares, it could not have been captured by the promoting Black Pawn, since only that Bishop could reach g3 to permit the Pawn capture there."

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40. UNCHANGING CHAMELEON?

"No last move by either the Bishop or the Rook could produce this double check," said Holmes, "but the double check is real. It follows that *some other* Black piece moved last, discovering *both* checks of the double check."

That suggestion was absolutely unprecedented. Fortunately, I remembered other maddening situations, and said, "When the situation seems completely impossible, look for an en passant capture."

"Bravo!" said Holmes. "An en passant capture was possible only if the Black Pawn was on e4 one move ago, blocking the Rook check. With White moving to the left, a White Pawn advanced from g5 to e5, to block the Bishop check created by Black's preceding move. The Black Pawn then captured en passant, from e4 to f5, removing both blocking pieces and discovering both checks."

"With White moving left," I said, "the White King is on a white square, and the Black King on a black square -- the reverse of the situation in a homochromatic game. The game has *not* been homochromatic."

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41. FIND THE CHECKMATE

I reexamined the logic that had led me astray. "White cannot be moving up, or the Pawn on d4 would be checking the Black KIng, with no possible last move. White is not moving to the right, or that same Pawn moved from c4 or captured from c5 while Black was already in check by the Queen.

"Black is not moving right, or both Kings are in check, White by the Pawn on c3. So White is moving *down* the board."

"Certainly," Holmes agreed. "Therefore, the position is not a checkmate by White."

"Then the checkmating move has not yet occurred!" I realized. "The Black Pawn on f2, moving up, can and must capture the White Queen."

"And that *is* the checkmating move," said Holmes. "It gives new meaning to 'forced checkmate'; Black is forced to make the Pawn capture that gives checkmate. The Black

Pawn capture discovers check by the Black Rook on g2. No White piece can capture that Rook; none can be interposed between Rook and King. And every possible White King move would leave it in check."

He continued, "Your analysis of possible White checkmating moves, though on the wrong track, *is* pertinent to determining the move that immediately preceded the checkmate."

I tried to get back on track. "Doubled White Pawns on the b-, d-, and f-files have made three captures, accounting for the missing Black Bishop and two Pawns. One Black Pawn had to promote to permit its capture, and could do so on the a-file without capturing.

"Doubled Black Pawns on two files have made two captures; the third missing White piece, the Bishop from f8, was captured at home, and not by a Pawn. One of the two missing White Pawns also was promoted to permit capture by a Black Pawn.

"With all missing Black pieces accounted for, no Black piece remained to be captured at e3 on White's last move; without capturing, the Queen could not have made the last White move, which would have been from imaginary check. The Queen check had to be discovered. With White moving down, the only move that could have discovered the check is a move of the White Bishop from e4 to f5. *That* is the move that preceded the checkmate."

Holmes concluded, "It is unusual, though not without precedent, for a retrograde analysis problem to involve a move that has not yet occurred."

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42. DEJA VU AGAIN?

"We know the White Bishop moved last," I began. "It *could* have moved from b4, in which case the preceding Black move could have been by the King, from b3. If the Bishop moved from c1, the Black King could have just moved from a1.

"Also, for either move direction, Black's last move could have been a Pawn move. We *can* rule out a move by the Black Knight, since both squares from which it might have moved are occupied.

"Still, looking back only one move, I see multiple possibilities. Looking back many moves, to a possible previous occurrence of the position, would involve more move combinations than I could keep in mind."

"If this position has occurred before," said Holmes, "Black's last move was necessarily a *King* move. Watson, can you see why a Pawn move must be ruled out?"

Considering move legality got me nowhere. Finally I realized, "Of course! *Any* Pawn move creates a position that has never occurred before. Forever afterwards, it is impossible to return to any position that occurred before the Pawn move."

"Quite so," Holmes assented. "You might say that a Pawn move is irreversible. But to

answer the question about the past of this game, you really should be considering Black's *next* move, rather than his last."

"How can that be pertinent?" I queried.

"If this situation occurred earlier," Holmes replied, "the next Black move was one of a number that eventually led back to this position. What next move *could* Black have made?"

I considered, "Every possible move of the Black King would be a move into check. And we have noted that any Pawn move would have prevented a return to the position. That leaves only a move by the Black Knight, capturing the Pawn on f7. But..."

Holmes chuckled, "You have realized that a capture, like a Pawn move, is irreversible, preventing any future recurrence of any prior position."

"Then," I said, "this position, with Black having the move, is occurring for the *first* time in the game. And the *last* time. No sequence of moves, from this position, will permit the position to recur.

"With his material superiority, White would not have wanted to claim a draw. And he clearly could *not* claim a draw under the 'threefold repetition' rule."

"You have answered my question correctly," Holmes conceded. "But notice how different the situation would be if *Black* had moved last. He could have made a reversible move of the King, from a1 or b3, following a White Bishop move from c1 or b4 that discovered a Rook check. It is easy to visualize the White Bishop's next move as a return to the square from which it moved last, with the Black King also returning to its previous square, the Bishop blocking any check. That sequence could be repeated indefinitely. Just before moving to create a third repetition, Black could and *should* claim a draw."

"I see now what your conditional statement meant," I acknowledged. "If White moved last, this position never existed earlier. So, if the position is a third repetition, it can only be because Black moved last. And that *was* a possibility until Rathbone gave us additional information."

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43. TRUTH BE TOLD

"Allow me to review my thinking on White movement to the left," I said. "With Black moving right, the two Black Pawns, both on white squares, have required three captures each. In each case, one capture could have been an en passant capture of a Pawn on a black square, but two pieces on white squares had to be captured - a total of four for the two advancing Pawns. The Black Queen on h6, on a white square, is necessarily promoted, and would have required at least three captures of pieces moving on white squares, for a total of seven such captures. But White is missing only five pieces that move on white, and one of those was given as odds."

"Could the promoted Queen have resulted from reverse promotion of a left-moving White Pawn?" Holmes asked.

I knew my analysis on that score was sound. "The Pawns on white squares b2 and b4 would have required at least four captures other than en passant captures. Aside from the Black King, seven Black pieces were confined to white squares, and three are on the board now. No additional captures were available for a White Pawn to promote to a Black Queen on white"

"That was a solid analysis," said Holmes, "and I repeat my congratulations. Now, how do you account for the Black Queen on h6 if White was moving to the *right*?"

"It isn't even necessary to consider a reverse promotion," I responded. "A Black Pawn could easily have promoted to Queen. The Black Pawns on g5 and g7 have not moved. The only required captures of pieces moving on white are the three captures by the promoting Pawn, with enough captures available."

"True enough," said Holmes. "But how does it happen that h6 was vacant to permit the Queen to move there?"

"The Bishop originally on h6 was captured there, of course, unable to move with Black Pawns on g5 and g7."

"And what captured the Bishop?" asked Holmes, very quietly.

"Not a White Rook, of course," I responded, "which could not reach its eighth rank homochromatically. Not a White Bishop, which could not penetrate the Pawn formation. Not an unmoving Knight. Only the White Queen could have captured the Bishop homochromatically. And, zounds! That Queen was given as odds."

I thought frantically, and saw an out. "White could have promoted to a White Queen or Rook, which captured the hemmed-in Bishop. It is even possible that each side made a reverse promotion to Queen."

Holmes pointed out, "With White moving right, the White Bishop on the board is promoted, the original Bishop captured on a3, presumably by the Black Queen now on h6. White had enough captures available to promote *one* Pawn on white squares homochromatically, but not two. The Black Queen on white resulted from a Black Pawn promotion. Black also could not promote two Pawns on white, so the White Bishop now on the board is not a promoted Black Pawn in a homochromatic game.

"With the White Bishop the only promoted White Pawn on white, a White Queen was not available to capture the Bishop on h6, and White movement to the right is ruled out. No move direction could yield this position with homochromatic play."

"Then," I said, "the brother who said play was homochromatic was ..."

"Yes," Holmes interrupted, "the lying Basset."

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44. AT WHOSE HANDS?

Holmes pointed out, "With only 20 pieces visible, we must account for the fates of the other 12. From Lord Ashley's statement, and our observation that he did continue play,

"Before the White Pawn captured at g3, it was on f2 and the White King was on e1. There was no Black access to c1 or g1 for the captures there. The Black Queen had to be alive after the capture on g3, to capture later on c1. Therefore, the piece captured on g3 was the Black Bishop rather than the Queen; the Black Queen subsequently made both captures on White's first rank, and was one of the two pieces captured by the White King. The capture of the Black Queen was on either e1 or e3, since the Black Rook and White Knight were captured on the only other squares reachable by the White King.

"Lady Ashley has told us that she captured some other Black piece before the Bishop. That was not the Black Queen, which made subsequent captures. It was not the Rook, which could be captured at f2 only after the Pawn from f2 captured the Bishop at g3. It had to be the Knight on g8. Access to g8 was only through h7, so the Pawn from h7 had already made its first capture, to g6. It had *not* made its second capture, of the White Rook at f5, since that Rook was still blocked at h1 when the Black Bishop was captured at g3. With the Black Pawn on g6, the White Bishop could not reach h7 to capture at g8; only the White Queen could reach h7, through h5. Therefore, the capture at g6 which opened up h7 and g8, was capture of the White Bishop, with the White Queen subsequently capturing the Knight on g8. The White Queen, then, was the only piece remaining to be captured at a6 on the final move of the game.

"We have identified what captured each piece; except for a two-square uncertainty involving the Black Queen, we have determined where each piece was captured. We have not yet determined the chronological order of the captures, though we do have some information of the 'A before B' type. In particular, we still need timing information relative to the captures of the White Bishop, Knight and Rook, and of the Black Rook and Queen.

"The first of the two captures by the Black Queen was of the Knight on g1. While the Knight was there, the White King was restricted to e1, f2, and e3. To reach c1 to capture the Bishop, the Queen would have to move to e1, and change direction for the move to c1. With the White King on f2, the Queen had to change direction at e3 en route to e1, giving check with no homochromatic escape route for the White King. With the King on e3, the Queen would have had to change direction at f2, again checking with no escape route for the King.

"But the White King could be on e1, with a blocking piece on f1, when the Black Queen swept down the long diagonal to g1, to capture the Knight. The blocking piece had to be the White Queen, since the Rook was still on h1, locked in by the Knight.

"After capturing the Knight on g1, the Black Queen had to return to the open board, off the long diagonal through g1, while White pieces were reorganized. When that Queen returned, the White King had to be on g1; some piece had to be on f2 to block check when the Black Queen reached e3, and only the Black Rook could be there. Another piece had to be on f1 to block check when the Black Queen reached e1, with the White Queen and the Rook from h1 as possibilities. Once the Black Rook reached f2, it never moved again; any possible move would have been a Rook retreat, which we know never happened. The White piece blocking at f1 had to remain there for several moves, while

the Black Queen moved to c1 for the capture, back to e1, and up to e3. The other White piece still mobile on white squares (Queen or Rook) made all White moves during that time.

"It would appear that, with the Black Queen on c1 after its capture there, the White King could have captured the Rook on f2, then captured the Black Queen when it returned to e1. But timing rules that out. The Black King would have required two subsequent moves to reach g1 as its final location. In the most favorable circumstances, the White Rook was already on f5, and the White Queen, which had been blocking on f1, required one move to reach a6 where it was captured. But only two Black moves were available to precede those three White moves - captures on f5 and e4 by the Pawn from h7. The only alternative was for the Black Queen to exit to the open board before returning to be captured.

"With the Black Queen the final capture by White, only two subsequent White moves were possible. The White King required those two moves, to reach g1 from either e1 or e3. Thus, the White Queen had to be on a6 when the Black Queen was captured.

"Once the Black Queen was again on the open board and not checking f2, the White King was free to capture the Rook on f2, and had to do so before it could capture the Black Queen. That Queen had to be captured on e1 or e3, the only squares reachable by the White King where no capture had yet occurred. The White King had to be positioned on f2 when the Queen moved to e3 or e1, since that was the only location from which it could capture on e1 or e3. From the open board, the Black Queen had to move to e3 first, changing direction if headed for e1. The White King had to capture the Queen once it reached e3, with no alternative that removed the Queen check.

"We are now prepared to lay out the complete sequence of captures. It began with capture of the White Bishop at g6, by the Black Pawn from h7. The White Queen then captured the Knight on g8. The White Pawn from f2 captured the Black Bishop at g3. The Black Queen captured the White Knight at g1, the same piece next capturing the White Bishop at c1. The White King captured the Black Rook at f2, then the Black Queen at e3. The Black Pawn then on g6 captured the White Rook at f5, then the White Pawn at e4, remaining on e4 and falling from the board. Finally, the Black Pawn from b7 captured the White Queen at a6. The White Knight on b1 was a second fallen piece.

"No other combination of fallen pieces and captures satisfies all of the constraints, and the order of the captures also is unique."

"That analysis is a remarkable tour de force, Mr. Holmes," said Lord Ashley.

"And the pieces of the analysis are so interlocked," I commented, "that breaking it into several bite-size segments, as you usually do to allow me to contribute, verged on impossibility."

"Not impossible," said Holmes. "But it would have made the proof even more lengthy, and would have risked loss of the continuity needed for understanding."

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45. AN OPEN FILE

Stung, I concentrated my attention. "Counting moves after each side castled may help," I said. "Castling put the Black Rook on f8. Two moves, the first to f3, were required for the Rook to reach b3. Thus, the capture at b3 was at least the third White move following Black's castling. The White Knight on h6 is checking the Black King, thus made the last White move. That was the fourth of four possible White moves after Black castled. The White Rook originally on a1 had no opportunity to move after the capture at b3 opened the board to it. Before then, the Rook could move only by shuttling between a1 and b1, so the Rook was captured on one of those two squares. But I don't know which."

Holmes said, gently for him, "That's a good start, Watson. If you look more closely at the Knights, you should be able to reconstruct the complete sequence of the last four moves."

"The Black Knight on g3," I began, "has moved there since White castled, or it would have prevented castling by checking f1. So Black's last four moves have been castling, two Rook moves, and the one Knight move. That means the Black Knight on g4 was there four moves ago.

"The White Knight's last move, then, was from f5 or f7. It could not have been on the f-file when the Black Rook moved down that file from f8. So it has made two moves, to the f-file and then to h6, since the Black Rook moved to f3. White's last four moves have been castling, the capture at b3, and the two Knight moves."

"Though not necessarily in that order," Holmes pointed out.

"No," I agreed. "Black castled, then White castled. The Black Rook moved to f3, then the White Knight to f5 or f7. The Black Rook moved to b3, and the White Pawn captured it there. The Black Knight moved to g3, and the White Knight to h6. No other order allows all of those moves within the last four moves.

"The answer to your second question is, Black's last move was the Black Knight move to g3. And the move was from e4 or h5. It could not have come from f5, or it would have blocked the first move of the Black Rook. And it could not have been on f1 or h1 without interfering with White castling."

"Nicely thought through," Holmes approved. "Now, knowing where everything else was captured, and what moves have followed castling, you have enough information to decide whether the White Rook was captured on a1 or b1."

"Of course," I said. "That problem now involves only move parity. Black has made one castling move, an even number of Rook moves, an even number of Pawn moves, and an even number of Knight moves. Neither Bishop moved, and we now know the Black Queen has not moved (it couldn't before Black castled, and has had no time to move since). Black move parity is odd. Since Black is in check and has the move, White parity has to be even.

"The White Pawns have even move parity, as do the White Knights. The White Queen and Bishops have not moved. Castling makes move parity odd. To make it even as it must

be, Rook parity has to be odd. The missing Rook was captured on b1 rather than a1.''

Burns was lighting a cigar. ''Listening to this analysis,'' he said, looking at Holmes, ''it appears that we gave you precisely the information needed to solve the problem.''

Apparently, smoke got in an eye, and he blinked to clear it.

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46. MORE THAN ONE WAY

''You might examine the sensitivity of promotion square colors to which side promoted,'' Holmes suggested. ''Start with the question of where the missing White Pawn could have promoted.''

''The missing White Pawn started from d2,'' I pronounced. ''It did not promote at d8 without capturing, since it would have given imaginary check at d7, preventing the subsequent Black castling. It had to capture to the e-file and ...'' I trailed off.

''Doesn't work, does it?'' Holmes said cheerfully. ''The Black Pawn on e7 prevents capture from e7 to d8 or f8. Both alternatives, through d7 or f7, involve imaginary check and must be ruled out.''

''Then,'' I puzzled, ''there had to be *two* reverse promotions of Black Pawns, those missing from d7 and f7. The Black Pawn from d7 could have promoted at d1 without capturing, but would have given check at d2, requiring a White King move that did not happen. To avoid that imaginary check, the Pawn from d7 could capture once to the e-file, then capture a second time from e2 to d1 or f1. Alternatively it could have captured twice to reach b2 and promote at b1, or three times to promote at c1 with a capture from b2. The Pawn from f7 could have promoted at d1 or f1 with two captures, or at b1 with four captures.

''With two reverse promotions, White had a total of 18 pieces, of which Black Pawns could capture 4 to leave the present 14. So each promoting Pawn was limited to two captures, ruling out promotion on c1. The three possible promotion squares - b1, d1, and f1 - are all white.

''Thus, one original and two promoted White Knights originated on white squares - an odd total. There are currently an even two on white squares. For current color parity to differ from the original value, the White Knights have to have odd move parity.''

''Just so,'' Holmes agreed. ''Note that we really don't care whether promotions were normal or reverse promotions, except as that fact affects the promotion square colors.''

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47. ONE WITHOUT THE OTHER?

''Assume that the piece on d3 is White,'' Holmes urged, ''and examine White and Black move parities.''

"If that piece is White," I stressed, "there are 11 White pieces on white squares, including the Bishop captured on f1 and the Pawn on c2 given as odds. Black pieces on white squares number 6. White has odd move parity, Black even; the inequality means that White moved last and has made one more move than Black. But what if the Knight on d3 is Black?"

"Work through that possibility," Holmes almost commanded.

"In that case," I said, "White would have 10 pieces on white squares, an even number. Black would have 7 pieces on white squares, an odd number. Again, White and Black have different parities, which is possible only when White has moved last."

"So," Holmes summed up, "all that mattered was the *total* number of pieces on white squares, regardless of the piece colors. When that total is odd, White and Black have made different numbers of moves, with White moving last, though we don't know which had the odd parity and which the even. When the total is even, White and Black have the same move parity, though we don't know whether that is odd or even; with equal parities, Black has moved last."

"For determining who moved, we have used that 'total parity' approach before," I realized, "not distinguishing between piece colors, although we did *know* their colors. For some reason, not knowing the piece colors made the problem appear different."

I had a further nagging question, "What would the situation be if this had been a chameleon board, permitting a reverse promotion?"

"Then, of course," said Holmes, "the piece on g7 might be either a Black or a White Knight, Rook, or Queen. We should not know how many pieces of each color are on the board. But we could still count an odd total number of pieces on white (or on black), and the parity of that total is all we need to tell who made the last move.

"If we particularly wanted to know whether White had made an odd or even number of moves, the existence of a reverse promotion *could* complicate matters, even though we knew the color and identity of every piece."

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48. TO HIDE A HIPPOPOTAMUS

"How did a White Rook reach d4?" Holmes asked.

No juggling of White Pawns between second and third ranks provided any path from h1 to d4. "If the White Bishop were not on c1," I complained, "that question would have a reasonable answer."

Holmes continued, "If c1 had to be unoccupied for the Rook to reach d4, then we can be certain it *was* unoccupied."

"But the Bishop could not have left c1 and returned, in heterochromatic play. Unless," I realized, "it left c1 by being *captured*, and the Bishop now on c1 is a promoted Black Pawn!"

"That *is* the only possibility," Holmes chuckled. "And now you should understand the hippopotamus allusion."

"You hide a hippopotamus among others of its kind," I agreed. "The missing Black Pawn was hidden, as a promoted White Bishop, where it could masquerade as the original White Bishop."

"And now you know enough to evaluate move parity for each combination of Knight colors and locations," said Holmes.

"There are 28 visible pieces," I counted. "Original Bishops on c1, c8, and f1 bring the total to 31; the Pawn on c2 given as odds and the Black Pawn that promoted on c1 make 33 pieces. How can I count move parity with more than 32 pieces?"

"First, recognize that the game *has* included 33 pieces - the original 16 of each color plus the promoted White Bishop. But the promoting Black Pawn and promoted White Bishop did not coexist; that Bishop is really a continuation in time of the Black Pawn. Use that continuity to reduce the total to 32 pieces," Holmes advised. "For total parity, count the promoted piece, but not also the Pawn that promoted."

"Then," I said, "13 pieces are clearly on white squares, 18 on black, and one Knight may be on either color. If the ambiguous Knight is on black square c5, move parity is odd and White moved last. But, with the Knight on c5, the Black Queen is giving check and White could *not* have moved last. The ambiguous Knight has to be on d5, a white square. Move parity is even, and Black moved last. The Knight on h4, giving check, moved last and must be Black. The Knight on d5 is White."

"To sum up," said Holmes, "move parity is inconsistent with the existing Knight check for a Knight of either color on c5 or a White Knight on h4. It is consistent only for a White Knight on d5 and Black Knight on h4. Black *did* make the last move."

"Did each side make an odd or even number of moves?" I asked. "I get different answers depending on whether I consider 15 Black pieces, as we just have, or 16 including the Pawn that promoted."

Holmes replied sternly, "To assess Black move parity, it is essential to consider all of Black's 16 pieces, the promoting Pawn being considered on c1, the last square to which it moved. Then, 8 Black pieces are on white (V-grey) squares, showing an even number of Black moves, consequently an even number of White moves also."

"To assess White move parity directly," I inquired, "don't I have to count all of the 17 White pieces that have existed? If I do, I count an even 6 pieces on white squares, including two Bishops on c1. But I count an odd 11 pieces on black squares. How can I compute move parity from one odd and one even color parity?"

"Remember, Watson," Holmes reminded me, "that we have faced the same question in addressing Knight move parity with more than two Knights of one color. What was the original White color parity on black squares, given the reverse promotion at c1?"

"Eight original White pieces were initially on black squares; the reverse promotion on a black square made the total 9, an odd number. So when the present White color parity

on black is odd, equal to the initial color parity, White has moved an even number of times. On white squares, White had eight pieces initially, an even number, and has six now, also an even number. So black and white squares give the same answer, an even number of White moves, agreeing with the even number of Black moves.''

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49. TO RESOLVE A PAWN

I considered each Black Pawn that could have promoted on a black square. "The Pawn from a7 could have promoted at a1 or c1, in either case requiring two captures. The Pawn from d7 could have promoted at a1, c1, e1, or g1, requiring three captures for each possible promotion site. If Black, the Pawn on g5 came from f7 and required a sixth capture by Black Pawns. But White's six missing pieces include the Bishop on f1, not captured by a Pawn.

"You were right so far, Holmes. The Black promotion details rule out a Black Pawn on g5; the ambiguous Pawn is White."

"Yes," said Holmes. "And now you had better revise your analysis of Black promotions to avoid being misled later."

"With the ambiguous Pawn White, promoting Black Pawns could make no more than *four* captures," I realized, "so not both of the Pawns from a7 and d7 promoted. The Black Pawn from f7 promoted, at e1 or g1, with one capture."

"Of course," Holmes pointed out the obvious, "the Black King is in check, and the Pawn on g5 made the last move. From which square?"

"The Pawn on g5 originated on f2 or h2," I said. "If on f2, it could have captured from f4; and for either origin, it could have captured to the g-file earlier, the last move being a noncapturing move from g4."

Holmes pointed out, "The White Pawn from f2 had to vacate its file to allow the Black Pawn on that file to promote without exceeding the captures available to Black. Thus, if the Pawn on g5 captured from f4 on the last move, it had made two prior captures, whether it originated on f2 or h2."

He continued, "With that clear, you might profitably analyze the White promotions."

I proceeded cautiously, determined not to overlook any possibility. "The promotion to White Bishop was on a white square, and not on a8, from which the Bishop could not have moved. The Pawn from b2 could have promoted at c8 or e8, with three captures in each case. The Pawn from h2 could have promoted at e8 or g8, requiring three captures.

"The White Pawn from f2 could have moved up its file and captured to e8 or g8 to promote. But," I added, "bypassing the Black Pawn moving down the file added two captures. The Pawn from f2 could promote at c8, e8, or g8, but required three captures in each case. The promotion to White Bishop required three Pawn captures in each possible scenario.

"The White Pawn on g5 required a fourth Pawn capture, from either f2 or h2. The promotion to White Knight could have occurred only at a8, with one Pawn capture; at f8 it would have required two Pawn captures for a total of six, with only five Pawn captures available.

"The Pawn on g5, then, has made only *one* capture; we have seen that a last-move capture from f4 would have been a *third* capture by that Pawn.

"Thus, the last move by the White Pawn on g5 could only have been an advance from g4. Before that move, the Black Bishop on h6 checked the White King, and could not have moved to create the check. The Bishop check was discovered. The Black Rook on d5 moved from g5 to discover the Bishop check. The only other Black piece that would have blocked the Bishop check, if on g5, is the Black King. And, on g5, the King would have been subject to an impossible double check by two Knights.

"Summarizing, the ambiguous Pawn is White, and made the last move, from g4 to g5. Black, in check now has the move, and last moved his Rook from g5 to d5."

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50. HOMOCHROMATIC AMBIGUITIES

As directed, I turned my attention to g1, where an immobile Black Knight had been captured. "With Black moving up, the Pawns on f2 and h2 have not moved. The Knight could not have been captured by a diagonal move, thus not by a White Pawn, Bishop, or King. An original White Rook could not reach row 1. The original White Queen could reach that row, but not the black square g1.

"Only a White Queen or Rook, promoted on a black square, could have captured the Knight on g1. That fact is enormously helpful. White Pawns are on black squares a5, e7, and f4; the fourth and last White Pawn on black promoted. So the ambiguous White Pawn is necessarily on a white square, c4 or d3.

"We have already seen that too few White captures on white squares were available for both a promotion and *any* capture by the ambiguous Pawn. Since that Pawn *has* captured at least once on white, there can be no promoted White piece on a white square. The ambiguous Rook on d7 is Black."

"Excellent, Watson," Holmes approved. "You have narrowed the problem down to placement of the ambiguous White Pawn on either c4 or d3. Can you explain the connection between the captures on g1 and e3?"

"Rather easily, now," I affirmed. "The only missing White pieces that were confined to black squares are a Knight that could not move, a Rook that could not reach e3, and the promoted Queen or Rook that captured the Knight on g1. The Black Pawn from d2 captured, on e3, the same piece that made the capture on g1."

"Finally," Holmes continued, "you should be able to place the ambiguous Pawn precisely."

"Indeed so," I said with deep satisfaction. "To be on d3, the Pawn would have

captured from c4 or e4. With the Rook on d7 Black, only an en passant capture to d3 was a possibility. That would have captured the Black Pawn originally on d2 and advancing to d4. But that Black Pawn, instead, captured on e3. No capture was available to allow the ambiguous Pawn to reach d3. It is on c4, reached by capturing the Black Pawn that had advanced from c2 to c4."

"My congratulations to both of you," said Lady Ashley warmly, "for tracing through all of the complexities. What would you have concluded if Mr. Holmes had not recognized the significance of the Knight missing from g1?"

I thought for a moment, and replied, "I would have assumed that the White Pawn from c7 advanced to c5, captured the missing Black Queen and Bishop on black to reach e3, and was the piece captured there by the Black Pawn. The ambiguous White Pawn *still* could not be on a black square. On a white square it still would have required the ambiguous Rook to be Black. And the White Pawn still could not have reached d3. The *same* resolution of the ambiguities would have been reached.

"Holmes, doesn't that mean the problem was so simple it could be solved despite imprecise logic?"

Holmes harrumphed, "You *could* have reached the same resolution," he conceded. "But obtaining the right answer through faulty analysis is *not* solving the problem."

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51. AFTER CONSECUTIVE CASTLING

I suggested to Holmes, "Let us divide the analytical chores. I will locate half of the missing pieces, and leave the other half to you." Holmes looked dubious, but agreed.

"The Bishops on f1 and f8 were unable to move, hemmed in by Pawns, and were captured on those home squares," I said. "Only a White Knight could reach g6, where some White piece was captured by the Pawn there now. So the missing White Knight was captured at g6. There, Holmes, is my half of the analysis. I trust you will be as succinct."

"Don't count on that," Holmes grimaced. "A comprehensible but persuasive analysis has a lot of ground to cover. The other three missing pieces are all Black - the Queen and two Rooks. The King's Rook participated in Black castling, thus was captured after that castling. The other Rook and the Queen *may* have been captured earlier.

"Since White castled first, the two sides have made equal numbers of moves subsequently, though Black was limited to one noncastling move fewer than White, thus to a maximum of six noncastling moves since castling.

"White has made four moves that necessarily followed White castling - one each by his King, King's Rook, and Queen, and the final checking Knight move to d6. Black shows four moves that had to follow Black castling. The King moved at least twice, from g8 to e8. The Knight on h8 moved to that square, previously occupied by the Rook, after Black castled. The Knight could move only from g6, so the Pawn capture at g6 followed the Knight move, thus followed Black castling.

"In addition to the final checking move, a White Knight move to g6 had to follow the Black Knight move from g6, thus followed Black castling. And capture of the Black King's Rook requires additional moves by White after Black castling. Those moves are by White Knights, the only pieces that could capture the Rook. Thus, noncastling White moves after the first castling total five plus the additional Knight moves involved in the Rook capture, limiting the latter moves to a maximum of two, including the actual Rook capture move.

"Black *may* have made up to two postcastling moves in addition to the four already identified. With one additional move, the King's Rook could have reached d8 or e8, and may have been captured on any of three squares, including f8 to which it moved in castling. A capture site can be ruled out if capture there required more than two White moves not previously counted.

"Capture of the Rook at d8 would have been from c6 or e6, and a White Knight could have been prepositioned on c6 without deterring castling. Capture would have required one move, permitting two more moves for the Knight to reach g6 or d6. But neither square could be reached with two Knight moves, so Rook capture at d6 is ruled out.

"Rook capture at e8 *appears* possible, but deceptively so. The capture would be from d6 or f6, which a White Knight could not occupy before Black castled. A first move to one of those squares would permit capture on e8 as a second move. And, following the Rook capture, the Knight could reach d6 in one move, with a Knight move to d6 one we have already counted. But *that* move from e8 to d6 could not have been the *last* move, which is the one we have counted; the Black King had to be on e6 when that last move was made. Rook capture at e8 is ruled out, leaving only f8 as a possibility.

"The capture at f8 was from e6 or g6, and the White Knight could not have been on either square when Black castled. After Black castled, a first Knight move was to e6 or g6, and a second the capture at f8. The next move has to be among those previously counted, and is - a move to g6 to allow capture of the Knight there.

"The Black King's Rook could not have made exactly one move after castling, or it would have died away from f8. And it could not have moved more than twice. It moved either zero or two times, either corresponding to even parity for that Rook.

"All available White Knight moves were used in capturing the Black King's Rook. None remain for postcastling capture of the Black Queen or Queen's Rook on the 8-rank. Both, then, were captured before the castling, when the Queen was unable to move from its home square. The Black Queen, then, was captured on d8, with even move parity.

"We now know the final location of every piece except the Black Queen's Rook, which was limited to a8 and b8. Move parity can distinguish between those possible capture locations. The White Knights at d6 and g6 are on opposite square colors, and have even move parity. With seven postcastling moves counted, it was not possible for the White King, King's Rook, or Queen to make more than the one necessary move. Castling plus those three moves adds up to even parity. The White Queen's Rook shows odd parity, as does the single Pawn move, making total White move parity *even*. Black, in check, has made one move fewer than White, and has *odd* move parity.

"Black Knights, on the same square color, have odd parity. Castling plus two King moves returns Black parity to even. The single Pawn move makes parity odd, and requires even move parity for the Black Queen's Rook. That Rook, then was captured on its home square, a8."

"One aspect troubles me, Holmes," I mused. "You have counted separately the moves required by White and Black, but have not placed them in a feasible sequence. Immediately after the White Knight captured the Rook on f8, the Black King could not move from g8 toward e8. The Black Pawn on h7 could not capture to g6, since the White Knight was not yet there. Did Black, while waiting, have to make a move or moves not previously counted?"

"That is an excellent question, Watson," said Holmes. "We often assume that what *had* to happen *could* happen, without demonstrating its feasibility." He moved some pieces to yield the position in the adjacent illustration.

"This is one possible position immediately after Black castled," he said. "White's next move, his first after castling, is the Knight from f4 to e6. Black can then make only one of his mandatory postcastling moves, the Knight from g6 to h8. White's second move is the Knight to f8, capturing the Rook there. As you have noted, Black can not then make any of his other three mandatory moves. With his Queen's Rook previously captured, Black can move only his second Knight, perhaps to c6.

"White's third move is the Knight from f8 to g6. Black may capture the Knight at g6. Now, White must wait for the Black King to reach e8 before he can make the final checking move of Knight to d6. He temporizes by moving his King to h1. Black makes his fourth postcastling move (King to f8), and White moves his Rook to g1. Black makes his fifth move, King to e8, completing his required moves, and White moves his Queen to f1. Black must make a sixth (a second previously uncounted) move, Knight to d8. Then White makes his seventh and final move, the Knight to d6 for the check.

"All seven of White's moves are moves previously identified as necessary. Only four of Black's six moves had been counted as necessary, and it is only the sequence timing that makes some additional Black Knight moves necessary, though possibly not the specific sequence we have illustrated. The need was only to show a *feasible* sequence, not a unique sequence."

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51S. MOVE FEASIBILITY
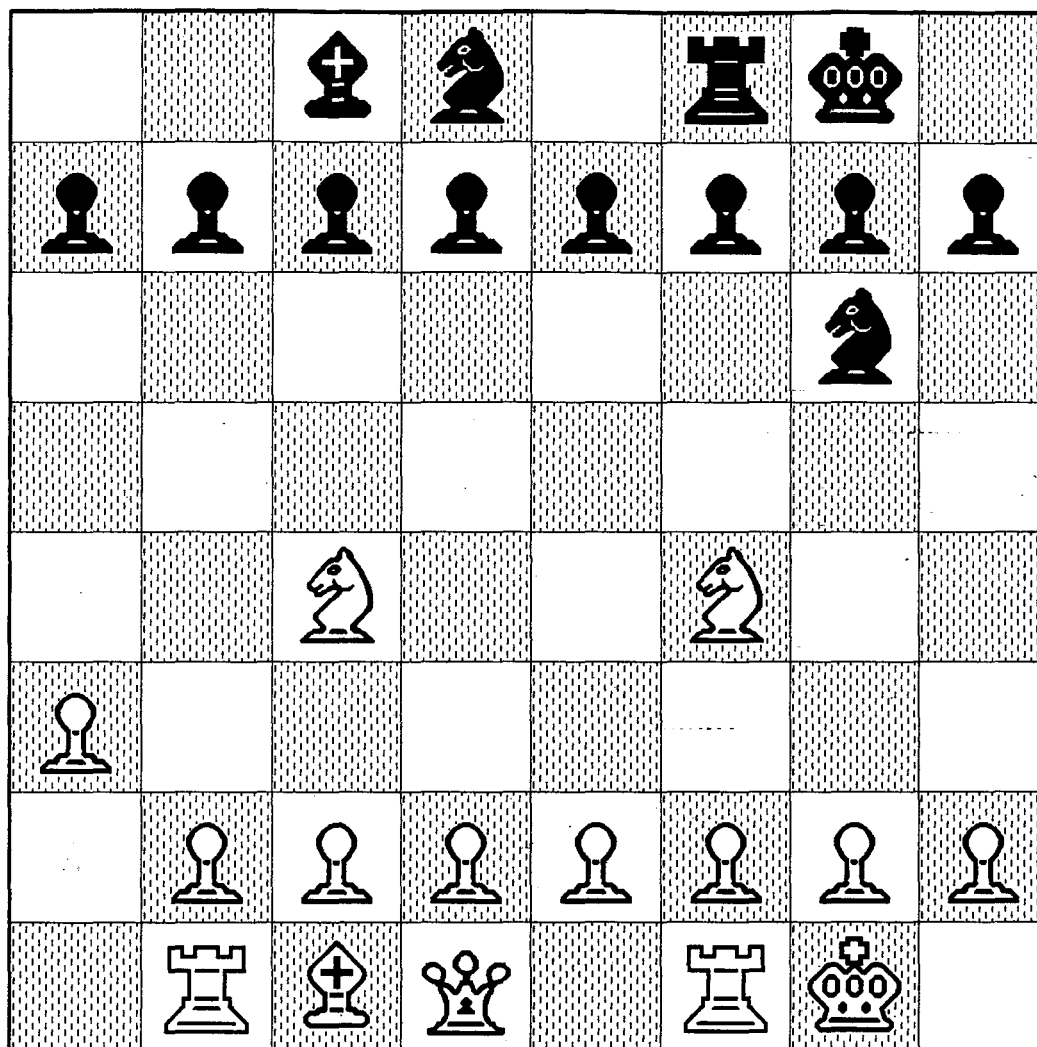
52. TWO-COLOR COORDINATES?

"Two White Bishops occupy white squares," I noted, "and two Black Bishops are on black squares. Clearly, one Bishop of each color is promoted. And it can be shown readily that no White Pawn promoted to a White Bishop.

"Such a promotion could not occur at a8 or e8, occupied continuously by the Black Rook and King. Promotion at c8 would have been through d7, giving imaginary check to the unmoving Black King. Promotion at g8 through f7 would have involved a similar imaginary check. So a White Pawn could have promoted to White Bishop only at g8 and

only through h7. That would have required two captures from f2 to the h-file, and a third to g8. Doubled White Pawns on the e- and h-files show two captures, for a total of five, with only four missing Black pieces. For that to be possible, another White Pawn would have had to promote to a Black piece without a capture. Only the White Pawn from a2 could have promoted without capturing; but the promotion would have been at a8, which has been continuously occupied by the Black Rook. The second White Bishop, then, is a promoted Black Pawn.''

''Could a White Pawn have promoted to Black Bishop to even up the score?'' Holmes inquired.

''That certainly appears possible,'' I replied. A White Pawn from d2 or f2 could have reached the e-file with one capture, and promoted on d8 or f8 with a second promotion. Together with the two captures by doubled White Pawns, the total of White captures is four. And four captures were available if each side made a reverse promotion. Visible Black Pawns have required six captures, with eight available. The remaining two captures would have permitted a Black Pawn from f7 or h7 to capture once to the g-file, and a second time to f1 to promote to White Bishop on the proper square color.''

''But there is a fly in the ointment,'' Holmes pointed out. ''The White Pawn from a2 could not have been captured by a Black Pawn as a Pawn on its home file. It would have had to promote to a White piece at b8, with an unavailable seventh capture, to make eight captures available to Black Pawns.''

''But Black Pawns did have to make at least eight captures for the reverse promotion at f1,'' I protested. ''And, if White made no reverse promotion, Black required a *ninth* capture to promote to a Black Bishop on g1, from f7 or h7.''

''True,'' Holmes smiled. ''But the reverse promotion of a Black Pawn created a seventeenth White piece, making nine captures available. The White Pawn from a2 still had to promote to a White piece at b8. But that made only three White captures in total; one Black Pawn has changed color, and 12 Black pieces remain, so the numbers do balance.''

''Then,'' I mused, ''promotion of a Black Pawn to a White Bishop occurred at f1, and a second Black Pawn promoted to a Black Bishop at g1. The promoting Pawns came from f7 and h7, and it is impossible to tell which of those made the reverse promotion.''

''And just as well,'' laughed Wendell. ''The fact is that neither of *us* can remember which Pawn promoted where.''

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53. PROMOTION ORDER

''Since the Black King has not moved,'' I noted, ''no White Pawn passed through f7 to promote at f8 or g8. Both promoting Pawns passed through b7. Promotion to the White Bishop was at c8. It is necessary to examine available captures to determine whether the White Rook promotion was at b8 or could have been at c8. Five captures were available

to White Pawns, one of those made by the Pawn originating on f2 and now on g6. The Pawn from c2 required one capture to reach b7; the Pawn from d2 required two captures to reach b7. The one capture remaining available was from b7 to c8, for the promotion to White Bishop on a white square. Promotion to the White Rook could occur only at b8, with no capture from b7.

"The Black Pawn now at b6 moved there from b7 rather than capturing from a7, since the Black Bishop had to reach a7, after promoting, *before* the Pawn on c6 blocked access. The two White Pawns promoted through b7, thus *after* the Black Pawn move from b7. So the promotion to Black Bishop was the first of the three promotions.

"Black had four captures available. The Pawn from a7 could promote on c1, to Bishop on black, with two captures. Alternatively, the Pawn from f7 could promote at c1 or g1, with three captures. Either promotion was possible, but not both, which would have required five captures against four available. There are two possible promotion locations, and would be even if we could rule out promotion of the Pawn from a7."

"Watson," Holmes asked, "what happened to the missing Black Pawn that did *not* promote to Bishop on black? White Pawns have captured all missing Black pieces except the Bishop on f8. But no capture by a White Pawn occurred on either the a- or the f-file."

I floundered for a moment. "If the Black Pawn from f7 is the one that promoted, with three captures, the Pawn from a7 could have made one more capture to the b-file."

"That won't do, Watson. The only White Pawn capture on the b-file was at b7, not reachable by the Pawn from a7."

"Then," I said, "it was the Black Pawn from a7 that promoted to Bishop, at c1 with two captures. The Pawn from f7 captured once to the g-file, an allowable third capture, and was captured by the White Pawn now at g6."

"You have identified the three promotion locations," said Holmes, "and determined that the Black promotion occurred first. You have not yet addressed the order of the two White promotions. Assume that the promotion to White Bishop preceded the promotion to White Rook. What were the two moves, by each side, following the Rook promotion? Bear in mind that the promoting White Pawns captured four of the six missing Black pieces."

"That seems a rather odd question, Holmes, and an odd reminder, but I will address it. When the promotion to White Rook occurred, at b8, some piece had to occupy c8 or d8; otherwise, the Rook would have given check, requiring its capture or move of the Black King, neither of which happened. The four captures by promoting White Pawns had been completed, and no Black piece could have occupied either c8 or d8. The White Bishop, promoted earlier, thus had to be on c8.

"The Black Bishop had been limited to shuttling between a7 and b8 after the Black Pawn move from b7 to b6. The Bishop was on a7 when the promoting White Pawn moved fom b7 to b8, and could not make the next Black move. There was no Black King or Rook move, which would have denied subsequent King-side castling. No Black Pawn now

visible made the next move. Clearly, the Black Pawn from f7 captured to g6.

"The White Rook then moved from b8 to b7, and the Black Bishop from a7 to b8. The White Rook moved to a7, its only route to the open board. The Black Bishop then could not move from b7, and no other Black move was possible! The Black Pawn on g6 remained there until the final move of the game, since White's last move was a capture, with no other piece available for capture.

"The assumption that the White Rook was the last promotion has impossible consequences."

"Yes," agreed Holmes, "so the final promotion was that to White Bishop. When the White Pawn advanced from b7 to b8 to promote to Rook, the Black Bishop was at a7, and there were still two Black pieces that could block the Rook check of the Black King -- the two pieces subsequently captured at b7 and c8 by the Pawn promoting to White Bishop. One of those pieces made the next Black move. Once the White Rook moved to b7 en route to the open board, the Black Bishop could move to b8, and remain there while the Rook moved to a7 and down the a-file.

"When the second promoting White Pawn captured at b7 and c8 to promote to Bishop, the Black Bishop had resumed its shuttling between a7 and b8; no other Black timing moves were necessary.

"The White Bishop is not now on c8, so the capture there was not the last White move. Neither was the capture of the Bishop on f8, that square no longer occupied. The only square now occupied by a White piece that could have made the final, capturing move is g6. Thus, White's last (Pawn) move captured the Black Pawn originally on f7, the capture occurring at g6."

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54. SMATHERS RESIGNS

I cogitated over the possible last moves that could produce the Rook check. "The Rook could have moved as a Rook only from b8, an imaginary check position. A White Pawn capturing at c8 and promoting to Rook could have moved only from b7, and would have given imaginary check there. The only other possibility is that the Rook check was *discovered*, the White Pawn now on c7 having just captured from b8.

"White cannot be moving down, or the capturing Pawn would have been on White's first rank before the capture. White has to be moving to the right. The left-moving Black Pawn on d7 can capture the Rook on c8. If White does not then move his Knight, three subsequent moves by the Black Queen are necessary for it to reach h1 or h2 for a checkmate. A White Knight move to e4 allows the Knight to be interposed on any rank the Black Queen selects for moving to the h-file, requiring a fourth Queen move and fifth Black move to checkmate.

"Smathers was playing White rather than the apparent Black. But, although the checkmate takes several moves, it does not appear especially tricky, Holmes, or

endangered. Why did you stress that Ayres would have to be extremely cautious?"

"Suppose, Watson," he replied, "White leaves his Knight on c5 and moves his Pawn from c7 to d7. A Black Queen capture of the Knight will then leave White with only one possible move, the Pawn to e7, threatening stalemate, whether or not the Black Queen or Bishop then captures the Pawn. After the White Pawn reaches e7, Black may have to offer up a Rook for capture by that Pawn to prevent immediate stalemate, or to remove the Rook pin of the White Pawn on g2. With foresight, Black can move his Bishop from d7 to f6, before the White Pawn reaches e7, rather than capture the Knight. Capture of the Bishop by the right-moving White Pawn prevents stalemate at little cost, as would the simple refusal to capture the Knight."

"But," I said from my freshly informed perspective, "it would be easy for Ayres to blunder by accepting the Knight capture, and delay if not jeopardize his victory. He does, indeed, have to be very cautious."

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55. CORNERED PAWN

"One promotion was obvious to me," I explained. "With Bishops of both colors on the same rank, one had to be promoted. I soon realized that the Black Pawn on f2 ruled out any promotion on the f-file. The Black Bishop had to be original, with Black moving down, and the White Bishop promoted. I needed slightly longer to realize that, with White moving up, the ambiguously placed White Pawn could not be above the Black Pawn on the f-file. The White Pawn can only be on e5 or e6. Pawns are on all files except the b- and c-files, so at most two promotions were possible. I still have to prove the necessity for a second promotion.

"Either placement of the White Pawn will block one of two apparent checks, by the White Bishop on c8 and the Black Queen on square g7. The check not thus blocked is real.

"What last move could have produced the Black Queen check? That Queen had no possible heterochromatic last move, so a Queen check would have to have been discovered. The only possible discovering move would have been a move of the Rook now on c7, from c3, and that move is not heterochromatic. There can be no check by the Black Queen; to prevent Queen check, the White Pawn has to be on e5.

"That leaves Black in check by the White Bishop on c8. Even though the Bishop is a promoted Pawn, that promotion did not occur on the last move; a Black Rook occupies the only square from which a promoting Pawn could advance. The Bishop check is discovered. The White Pawn on e5 is moving up, thus could not have discovered the Bishop check. The White Rook on f7 could have discovered the check by moving from d7 or f5, but neither move would be heterochromatic. The check was discovered by the White Knight now on e3, moving from f5 or g4.

"To prevent imaginary check by the White Queen, e3 had to be occupied before the last Knight move, by a Black piece captured on the Knight move. With the White Pawn

on e5 moving up, a Black Pawn could not be below it on e3; it was not a Pawn that the White Knight captured.

"Only a Black Queen, Rook, or Knight could have been captured at e3. But that would have been a second Queen, or third Rook or Knight, proving a second promotion, on the b-file. The promoting Pawn was Black, since the board is conventional, and the promotion was at b1."

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56. ON WHICH SIDE?

Holmes prompted me, "Assume that castling will be Queen-side and explore the implications."

"Then," I said, "one ambiguous Black Pawn must be on d7, to prevent check of c8 by the White Queen. With that Pawn placement, the Black Bishop from c8 never moved before being captured, and the Bishop on b1 is promoted. The Pawn from a7 needed one capture to promote at b1. One Black Pawn on the h-file needed two captures from f7.

"The White Bishop on a4 also is promoted, the Bishop from f1 being captured without moving. Then, four captures were available to Black Pawns. The second ambiguous Black Pawn could be on either c6, without a capture, or on d6 with a capture from c7. Assuming Queen-side castling does not fully resolve the ambiguity."

"Where did the White Pawn promote?" asked Holmes.

I considered, "The White Pawn from f2 could have promoted only at g8, a white square. It did not reach g8 through f7, where it would have given check and forced the Black King to move. It reached g8 through h7, requiring three captures. And four captures were available to White Pawns."

"What is the history of the White Pawn on h6?" Holmes asked.

"Coming from h2," I replied, "it would have needed two captures to bypass a Black Pawn on the h-file, with only one capture available to it. So it was the Black Pawn from h7 that bypassed, requiring two captures. That makes five captures by Black Pawns, even with the second ambiguous Pawn on c6 rather than d6; and only four captures were available."

"So," said Holmes, "assuming Queen-side castling leads to a contradiction, with too few Black Pawn captures available. With King-side castling, a Pawn need not be on d7; the ambiguous Pawns can be on c7 and c6 (the latter necessary to block check by the White Bishop). The Black Bishop, then, can be original rather than promoted. Black Pawns required only three captures, one from d7 to c6, and two from f7 to the h-file.

"Five captures were available to White Pawns, with three for the promoting Pawn (from f2 or h2), and two for the other Pawn now on h6."

"That leaves only the ambiguous King to be placed," I observed. "It has to be on f4, where it blocks the Rook check of f8 that would prevent King-side castling."

"Right," Holmes agreed. "Queen-side castling requires more Pawn captures than are available. Only King-side castling is feasible, and it fully resolves the three location ambiguities."

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57. FORCED FINALE

As usual with a difficult problem, I thought aloud. I knew subconsciously that Holmes would interject a comment or question if I went astray. "If it were White's move now, he could win a checkmate with a Rook move from g1 to g8. But it is Black's move, and the situation is a stalemate. Black is not in check, but cannot move except into check.

"What had to be different before White's last move that prevented the checkmate? There could have been a Black piece checking White and preventing a checkmating move, with White's last move the capture of that piece (at f1, f6, or g1). But capture has been ruled out. And, in any event, White would have had the alternative of moving his King, to leave a second Black piece capable of moving, thus denying a Black stalemate.

"The only alternative is that the Rook on g1 could not move to g8, either because the Rook did not exist as a Rook, or because the g-file was blocked by another White piece, which then made the last White move. If the Rook did not exist, it was created by promotion of a White Pawn, moving from g2 to g1 (Holmes has ruled out promotion of a Black Pawn at g1). But that move certainly was not forced. An alternative would have been to move the Knight from e8, denying stalemate by allowing a Black King move to g7."

"Good, Watson," said Holmes. "You *are* using the need for forcing as a logical element."

"Thus," I continued, "the Rook was at g1, and another White piece blocked the g-file. Three of the possible blocks were on squares below g7: King on g2, Knight or Bishop on g4. (The Pawn on h7, if White is moving up, may have captured from g6, but we are assured that any such capture was at least three moves ago and not relevant to the block one move ago.) Instead of moving the blocking piece, White could have moved his Knight from e8, thus allowing a Black King move to g7. If the block had been at g7, by the Knight now at e8, White could have moved his King, Bishop, or Knight to block the g-file below g7, allowing the Black King to capture at g7.

"The final alternative is a block at g8, by the Knight now on f6. The Knight on e8 and Rook on g1 would have prevented the Black King from moving to g7, and no single White move could have removed both of those obstacles. The Rook on h6 and Queen on h1 would both have to be removed to allow the Black King to capture the Pawn on h7. If White is moving up, both the Pawn on h7 and Rook on g1 prevent a Black King move to capture at g8. If White were moving down, only the Rook would prevent the move to g8, and White could have blocked off that Rook with an alternative last move.

"The fact that White could not prevent the stalemate, then, proves that White is moving up the board, and that White's last move was a Knight from g8 to f6."

"Well reasoned, Watson," said Holmes. "That leaves only the question of Black's last move, preceding the White Knight move."

"That move was not by the Black King," I explained. "Squares g8 and h7 were occupied by White pieces, and an impossible double check would have been involved for the King on g7. Black's last move had to be by a piece no longer on the board. With no capture on White's final move, the last moving Black piece could disappear only if it was a Pawn that made a reverse promotion. Holmes has shown that the promoting Pawn had to move from h2; that makes the White Queen the result of a reverse promotion on the only logically possible last Black move."

Holmes nodded enthusiastically. "And do you realize why Black's objectives would not have been served by promotion to a White Bishop or Knight, or to any Black piece?"

The last pieces of the puzzle fell into place for me. "Promotion to any Black piece would have left White with an overwhelming material advantage, Black with an expected loss, and no possibility of forcing a stalemate for the draw that was Black's best hope.

"With the Knight on g8, it and square g7 were protected by the Rook on g1. The only other Black King move would be capture of the White Pawn on g7. The Rook on g6 prevents that; if the Rook had made the last White move, to g6, the Queen or a Rook on h1 would still have prevented the Pawn capture. With a White Bishop or Knight on h1, White could have avoided stalemate by a Rook move from h6 to g6. The Black King could then have captured the Pawn on h7."

"And that is the beautiful aspect of the specific reverse promotion," Holmes pointed out. "To force stalemate, the promotion had to be a *reverse* promotion, and had to be to Queen or Rook.

"We have seen many reverse promotions to Queen that could be described charitably as whimsical. It is unusual, indeed, to see a reverse promotion to Queen that is rationally chosen to give the promoting player the most favorable possible outcome."

Feverel was still shaking his head disconsolately, "Meredith simply wrote me off."

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58. A CASTLING MATRIX

Holmes said, "Let me set the stage by pointing out a few obvious but highly pertinent facts. First, White Pawns clearly have captured at a3 and c3. The White Pawn on b6 came from f2, capturing at e3, d4, c5, and b6, for a total of six captures by visible White Pawns.

"Seven Black pieces are missing, including two Pawns, from g7 and h7. All visible White Pawn captures are on black squares, thus do not include the Black Bishop from c8. Visible Black Pawns have required two captures, with a maximum of three available (the Bishop on f1 died there). Neither missing Black Pawn could have captured to the e-file, thus could not be captured as a Pawn by a visible White Pawn. Thus, the White Pawns have either captured at least one promoted Black Pawn, or they have captured all six

original Black pieces that could move on black, including both original Rooks. If we could prove that White Pawns did not capture a promoted Black Pawn, Black castling would be ruled out.

"The analysis approach is to determine first the minimum number of moves in which White Pawns could capture the six original pieces. Then we examine the increase in number of moves required if a promoted Pawn is substituted for one of the Black Rooks."

"I see," I said slowly. "And the number of moves required is determined by adding the minimum number of moves for individual Black pieces to reach capture locations. For example, the Black Rook from a8 could have reached b6 for capture in two moves, through a6."

"Yes," agreed Holmes. "But we must add one move for the Pawn from a7 to a5, which was necessary before the Rook could move."

"That same Rook," I noted, "would have required four moves, after the Pawn move to a6, to be captured at c5 or d4. Rook movement to e3 would require one additional move."

"Careful," Holmes cautioned. "The Rook could have moved to a6, c6, c3, before the Black Pawns now on c6 and c5 moved there from c7 and d7, for a total of four moves including the Pawn to a5."

"But a minimum of three moves occurred before the Rook from a8 could reach *any* capture location," I said.

"Yes," said Holmes. "And now you may want to examine the table I have drawn up, verifying some of the entries to satisfy yourself of their accuracy."

Minimum Number of Moves to a Capture Square

To	From					
	R(a8)	N(b8)	Q(d8)	B(f8)	N(g8)	R(h8)
a3	5(1)	4	2(5)	1(5)*	5	4(2)
b6	3(1)*	3(3)	2(4)	2(5)	3	4(2)
c3	4(1)	4	2(5)	2(5)	3	4(2)*
c5	4(1)	2*	2(5)	1(5)	3	4(2)
d4	4(1)	2	2(5)*	2(5)	3	4(2)
e3	5(1)	4	2(5)	2(5)	3*	4(2)

NOTES TO TABLE

(1) Includes P(a7) - a5

(2) Includes P(g7) x P(h6)

(3) Includes P(d7) - d5

(4) Includes P(c7) - c6

(5) Plus P(e7) - e6 to unblock Queen and Bishop

Holmes pointed out, "It is necessary to select one combination of piece and capture square from each row and each column, so that some one piece is captured on each square, and each piece is captured somewhere, but only once. The selection marked by an asterisk in each column minimizes moves required for capture of the piece. For each piece except the Rook from a8, at least two different capture locations require equal minimum numbers of moves, so many combinations can be chosen with the same total number of moves required.

"The sum of the values marked by asterisks is 15 moves. We must add one move for the Pawn from e7 to e6. Thus, after 16 moves, all original Black pieces can be in position to be captured, or have been captured. White's final Pawn move, the last capture, could occur as early as the 17th move.

"Many move sequences are feasible. One is listed below:"

	WHITE	BLACK
1	P(h2) – h4	P(e7) – e6
2	P(h4) – h5	B(f8) – a3
3	P(b2) x B(a3)	N(g8) – f6
4	P(h5) – h6	N(f6) – g4
5	Waste	N(g4) – e3
6	P(f2) x N(e3)	Q(d8) – f6
7	Waste	G(f6) – d4
8	P(e3) x Q(d4)	N(b8) – a6
9	Waste	N(a6) – c5
10	P(d4) x N(c5)	P(a7) – a5
11	Waste	R(a8) – a6
12	Waste	R(a6) – b6
13	P(c5) x R(b6)	P(g7) x P(h6)
14	Waste	R(h8) – g8
15	Waste	R(g8) – g3
16	Waste	R(g3) – c3
17	P(d2) x R(c3)	

Only White's Pawn moves are specifically shown above," said Holmes. "Moves marked *Waste* are made by pieces other than Pawns. The only restriction is that they not interfere with Black's positioning moves, which allows enormous latitude.

"Note that Black Pawns now on c6, c5, and d5 remained on c7, d7, and e6, respectively, during the above sequence of moves. The subsequent captures to the c- and d-files were of the missing White Knight and Queen. The Black Bishop from c8 was available for waste moves, and Black Pawns were on h6 and h7. Both Pawns could promote at h1 without capturing, replacing the two original Rooks.

"This sequence, with both original Rooks captured, makes Black castling impossible. For castling to be possible, one Pawn had to promote and be captured as a substitute for one of the original Rooks. The Pawn that had captured to h6 could promote in five moves, and reach c3 in another two moves as a Rook or Queen. That total of seven moves is at least three more than would be required for either original Rook, and increases to 19 the number of Black moves necessary before White's final Pawn capture. That last White Pawn move, then, could not occur before the 20th move, as we are assured it did.

"Thus, both original Rooks were captured by White Pawns and subsequently replaced with promoted Black Pawns. With no original Rook, Black cannot castle."

"The mathematics of counting has appeared in other solutions and has to be considered within the purview of logic," I told Holmes. "But no prior use of mathematics in retrograde chess analysis has ever been so complex. I believe you have broken truly new ground."

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59. SHADES OF GREY

"With the chameleon board," I realized, "we must consider the possibility of reverse promotion. Referring to the preceding solution, we see that the White Pawn from h2 could be on h6 after four moves. With two more moves replacing waste moves (White's fifth and seventh moves), that Pawn could capture the unmoved Black Pawn on g7, and promote to Black Rook on g8, previously vacated by the Black Knight. Then the Black Rook on h8 need never move. Black moves 13-15 could be replaced by a single move, R(g8)-g3. White could then make his final Pawn capture on his 15th move. With one Rook unmoved, there is no evidence that Black cannot castle.

"Indeed, Black *could* have the option of castling on either side. The Black Pawn from h7 can promote to Black Queen or Rook at h1, then reach b6 through b1, with a total of seven moves. That is four moves more than required for the original Rook from a8 to reach the same capture square. Adding those four moves to the fifteen minimum with reverse promotion still allows White to make his final Pawn capture on his 19th move, coming in just under the wire, with *neither* original Black Rook having moved."

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60. INDETERMINATE QUEEN

"The knowledge that play was heterochromatic is vital," I remarked. "With 16 Pawns on the board, no promoted piece was available for capture. The missing original pieces are one Queen and the four Bishops. Bishops cannot move heterochromatically, and all were captured on their home squares. Except for f8, those squares are all vacant now, making a last-move capture there impossible. Even if we ignored the problem of imaginary check, the Bishop on f8 was Black, and could not have been captured on the last move by the Black King. The last move had to be capture of the missing Queen.

"Heterochromatically, the indeterminate Queen on c5 could have moved only from c4. It has to be White, since a Black Queen on c4 would have given imaginary check. That White Queen move captured the Black Queen on c5, and it is now Black's move."

Wright could hardly restrain himself, presumably impressed by my easy solution of the problem. He started to speak, but Holmes waved him off and said, "It might be advisable, Watson, to analyze the color parity to confirm that it *is* Black's move."

Obliging, I counted, "Sixteen pieces are visible on white squares, and two Bishops were captured on white squares, for an even total. That can't be right; that would make it *White's* move. I'll try again, counting pieces on black squares this time. Eleven are visible, plus two Bishops, plus the other Queen captured on c5, for an even total of fourteen. I have made some mistake; the color parity *can't* be even when it is Black's move."

"No, it can't," said Holmes. "And that means you have not correctly identified the last move. Consider possible discovered checks, Watson."

"If the ambiguous Queen is White," I essayed, "the only possible discovered check is

a White Knight move from e7 to d5. If the ambiguous Queen is Black, the discovered check resulted from a move of a Black Knight, from c3 to b5. In either case, the second Queen was captured on a white square, the present location of one of those two Knights.

"That makes the piece count on white squares nineteen, the odd total indicating it is Black's move. Thus, the ambiguous Queen, giving check, is White. The last move was the White Knight from e7 to d5, capturing the Black Queen."

Langley said, "You have homed in on it, Watson." But I realized that Wright was speaking at the same time, saying, "If at first you don't succeed..."

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61. THREE CHECKING?

"With two fallen pieces," I started, "it is possible that two of the three apparent checks are blocked. The alternative is that one is blocked, and a true double check remains.

"One fallen piece is the White King. It cannot block check by either Queen. At b5 it would be checked by the Black Knight on a3; at c6 or d7, it would be checked by a Pawn or Bishop. At f7 it would be adjacent to the Black King, and at g6 would be checked by the Rook on g1.

"The White King also cannot block check by the Rook on e5. At e6 it would be checked by the Bishop on c8, and at e7 it would be adjacent to the Black King. Since the White King is not blocking *any* check, the situation is a true double check, with the other fallen piece blocking the third check.

"The only possible double check is by the Rook and the Queen on a4, with the last move by the Rook, from b5 to e5, which *is* a heterochromatic move. A blocking piece must be on f7 or g6.

"The Pawn formation verifies that movement is up/down, with White moving up. The two White Queens reveal one promotion. With the Bishop on c8 not moving heterochromatically, the Black Rook from a8 could not reach the board, and one of the visible Black Rooks is promoted. Pawns on six files show that only those two promotions were possible. The piece on f7 or g6 is an original piece.

"No Pawn of either color can be on f7 or g6. Two Pawns (all that are permissible heterochromatically) are visible on the f-file. Of the two originally on the g-file, one promoted and the other was captured to allow the promoting Pawn to pass. With all promotions accounted for, there cannot be another White Rook or Knight occupying either blocking square. The blocking piece has to be Black and original, with only the Black Queen qualifying.

"But I see no way to determine whether the Black Queen is on f7 or g6. And the White King is not blocking a check; it could be on any one of more than a dozen squares where the King would not be in check."

"Watson," said Holmes, "before the last Rook move, some piece had to occupy e4 or

e5, to prevent imaginary check by the Rook on e3. No Black piece was available to be captured on e5; the Black Pawn on that file either promoted or was captured to allow promotion of a White Pawn; and all other Black pieces have been accounted for. So e4 had to be occupied, with only the White King a candidate. The last move was a noncapturing move by the Rook from b5 to e5.

"With the fallen White King on e4, the fallen Black Queen cannot be on g6, where it would give imaginary check. The Queen is on f7."

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62. CANCELED CHECKS

"In addition to the Knight check," I began, "Black appears to be in check from the Bishop on a4, the Rook on e5, and the Queen on h5. The two missing pieces can block two of those apparent checks, leaving a true double check. The Knight move had to discover the second check, which has to be a Bishop or Rook check. The Knight would have moved from e6 to discover the Rook check, and a Knight on e6 would have prevented Black from castling one move ago. So the Knight moved from b5 and discovered the Bishop check.

"One missing piece has to be on e6 or e7 to block the apparent Rook check, and the other on f7 or g6 to block Queen check.

"There are 13 visible White pieces, and Black Pawns have required only one capture, so the two missing pieces could be White. There are 10 visible Black pieces, and White Pawns have required five captures. Even so, the two missing pieces could be Black, since the missing White Pawn could have promoted to a seventeenth Black piece. And *any* Black piece could be on any of the four blocking squares, leaving the solution ambiguous."

"Where would the missing White Pawn have promoted?" Holmes asked.

"To limit White Pawn captures to five," I responded, "The promoting Pawn came from e2 and promoted at e8. But," I realized, "That would have required the Black King to move from e8, and would have barred later Black castling."

"So," said Holmes, "there was no reverse promotion of a White Pawn, and the two missing pieces cannot be Black. Burroughs was playing White."

"The White Pawn from e2 could have made one capture, but not more than one," I noted. "So a White Pawn cannot be on g6. A White Bishop or Queen there would be giving check. A White Rook on g6 would check g8, and a White Knight on g6 would check f8, either preventing Black castling a move ago. So a White piece has to be on f7. And only a Knight there would have permitted Black to castle a move ago. The White Knight on f7 is a third White Knight, and proves a promotion.

"Any White piece on e7 would have ruled out Black castling, as would a White Queen, Rook, or Knight on e6. The second missing White piece has to be a Pawn or Bishop on e6, the Bishop possible because the Knight on f7 blocks the Bishop check of g8. There *still* is

not an unambiguous solution."

"But, Watson," Holmes pointed out, "a White Bishop on e6 would be a second White Bishop on white squares, showing a second promotion, one necessarily the reverse promotion of a Black Pawn. That reverse promotion would have left only 15 Black pieces; 10 are visible, and five have been captured by visible White Pawns. No capture remained to permit promotion of the White Pawn from e2 without a move by the Black King. It is the unpromoted White Pawn that occupies e6.

"One White Knight is a promoted Black Pawn; with the reverse promotion creating a seventeenth White piece, one capture would have been available for the reverse promotion. The Black Pawn could have promoted from c7 without capturing, or from e7 or g7 with the one available capture.

"In any event, the two fallen pieces are a White Pawn, from e6, and a White Knight, from f7."

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63. BOTH LAST MOVES

I thought aloud, "Heterochromatic castling is possible only on the Queen side, with the White King moving to its left, or the Black King to its right in castling. It is obvious, then, that White is moving up the board, and that the Kings and their Queen's Rooks are still where they were one move ago. Each last move was a Knight move, those Knights now preventing castling by either side."

"From what squares did the Knights move?" asked Holmes.

"The White Knight on b8 necessarily moved from a6," I replied. "The Pawn on b3 rules out any promotion on the b-file, so the Knight *did* move as a Knight. If it had been on c6 one move ago, it would have prevented Black from castling, by checking square d8 through which the king would move in castling.

"The Knight on c1 was created by a promoting Pawn move from c2. The only possible move as a Knight would have been from d3, where it would have given imaginary check and prevented castling by White."

Miss Johnson was staring at me in wide-eyed wonder, clearly impressed by my thorough analysis. Masters had an expression of restrained amusement; I assumed he was sharing my satisfaction at having disproved Holmes's reservations.

A quiet question from Holmes disturbed my self-satisfaction: "Could you clear up one minor point, Watson? Where did the promotion to Rook take place?"

I considered the Pawn positions, "Except for the c-file, where the Knight promotion occurred, only files d, e, and f were possible promotion files, with no Pawns present. Promotion on the e-file would have required one of the Kings to move from its home square. But," I realized with consternation, "so would promotion on the d- or f-file, to avoid a promoting move from a position of imaginary check. There is *no* file on which the promotion to White Rook could possibly have occurred! The position is impossible!"

"It is if you insist that the Kings have always been on the e-file," Holmes agreed. "The only alternative is that the Kings were on the d-file before the last moves. For castling to have been possible one move ago, the Queens' Rooks are then on the h-file rather than the a-file. The King on d1 and castling to its right had to be Black, moving up the board. The White King, on d8, would have moved to its left in Queen-side castling."

Holmes wrapped up the analysis, "With both Kings on the d-file before the last move by each side, no promotion could occur on the c- ,d- , or e-file. The promotion to White Rook could, and had to, occur on the f-file. The Knights on the first and eighth ranks moved there some time before the last moves by the two Kings.

"Ambiguous piece color White-Down represents White, moving down. White-Up is Black, moving up. Each side, by its last move, of the King from the d- to the e-file on its home rank, voluntarily sacrificed its ability to castle."

"A remarkable analysis, Holmes," said Masters. "I would have sworn it was impossible to deduce the move direction. Can you tell whether the promotion to White Rook was a reverse promotion?"

Holmes abruptly changed the subject.

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64. DISPLACED TRIO

Accounting for the checkmating move appeared to be the first order of business. I cogitated, "A Rook move to h1 could only have been from h2, ruled out by the imaginary check. The Black King could not have discovered the check, since it could not have occupied h2, adjacent to the White King. No other Black piece could have moved to discover the check. The only possibility is that a Black Pawn moved to h1 on the last move, promoting to the Rook. Black is moving down.

"The Pawn now on b6 could have reached there from a7, b7, or c7. A Pawn originating on a7 or b7 could not have reached any of the other squares now occupied by Black Pawns, regardless of the resolutions of the ambiguities; sidewise displacement of a Pawn cannot exceed its filewise displacement. For that same reason, the Pawn from a7 could not have reached h1. It can only be on b6.

"The promoting Pawn had to originate on b7, and to capture on every move. Thus, the promoting Pawn captured all of the six missing White pieces that could occupy white squares. The Pawn on b6 came from a7, capturing the White Bishop on black.

"Those seven captures by Black Pawns account for all missing White pieces. There could have been no further capture to create a doubled Black Pawn on any file. To avoid a doubled Pawn, each of the ambiguously located Black Pawns belongs in the leftmost of the two possible files. That is, we have Black Pawns on c7, d6, and e5, in addition to the unambiguously placed Pawns on b6, f6, g7, and h7. Only the a-file contains no Black Pawn.

"The White Pawn formations require at least seven Pawn captures, with only seven

missing Black pieces. And that minimum assumes that the missing White Pawn is from c2. With a Black Pawn on c7 throughout the game, the missing Pawn could not promote without capturing, and had no capture available.

"The missing White Pawn, then, was captured as a Pawn, and by the Black Pawn from b7 that made all possible captures on white squares. That capture was possible only on c6, as the first of the six captures by the Black Pawn en route to promotion."

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65. PLAY BEFORE MATING

"Holmes," I said, "before you introduced a prior Bishop check, I could see two possible last White moves. The White Bishop did not move as a Bishop; it would have given imaginary check on any square from which it could have moved. So the check was discovered. With White moving up or left, no White piece on the board could have moved from a square on the diagonal between the Black King and the White Bishop.

"With White moving to the right, the Pawn now on f6 could have moved from e6 to discover the Bishop check. And, with White moving down, that same Pawn could have moved from f7 to discover the check. I can find no rationale for eliminating either possibility. And I see no third White move that could discover the check."

"Neither could I, Watson," Holmes replied, *"unless* the preceding White move also was a Bishop check. The solution would have been indeterminate, with the two possibilities you have described. But, knowing now that Black's last move was in response to a Bishop check, you should be able to identify that Black move."

"It is difficult to visualize two consecutive checks by the Bishop on g8," I said. "We can assume that there was a second White Bishop giving the earlier check. It would have shared the square color of the Black King and of the White Bishop on g8, requiring a Pawn promotion; with eight White Pawns on the board, it would have been a reverse promotion of a Black Pawn. Since a second White Bishop is not on the board now, it was captured on Black's last move. If the last Black move was by the King, the earlier checking Bishop was on b3 and was captured by the King move to b3. To be in check from a Bishop on b3, the King had to be on a2, c2, a4, or c4. In any of those locations, the King would have been in check by at least one other piece in addition to the Bishop on b3, with no possible prior Black move creating the double or triple check.

"So the Black King did not make the last Black move. The second White Bishop had to be checking b3, and had to be captured by some Black piece other than the King. But no Black piece is currently on a square from which a Bishop could check b3. And the Bishop on g8 is the only White piece on such a square; since it did not make the last White move, White did not capture a Black piece that had captured a second White Bishop. There *was* no second Bishop. Both White Bishop checks have been by the Bishop on g8."

"Good, so far, Watson," said Holmes. "Now, what was Black's move following the first of the two Bishop checks?"

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"We have seen that the Black King could not have made the last Black move," I said. "Obviously, the check was not removed by capture of the Bishop, which is still on g8. The only alternative is that Black interposed a piece on the diagonal between his King and the checking Bishop."

"What Black piece was interposed, Watson? And what happened to it?"

"No Black piece is on the pertinent diagonal now," I noted. "And there is no White piece on the diagonal that could have captured the interposed piece. It had to be captured by a White piece that does not occupy the square that was occupied by the captured piece. And *that* uniquely defines the piece that was interposed. A Black *Pawn* was interposed to remove the first Bishop check; and that Pawn has been captured *en passant!*"

"Precisely so!" agreed Holmes. "And you will find only one White Pawn that could have made the en passant capture of a Black Pawn that was between the Black King and White Bishop."

"The Black Pawn involved had necessarily just made a two-square advance from its second rank, with the White Pawn capturing en passant on Black's third rank," I pointed out. "With Black moving to the right, a Black Pawn from b5 could have advanced to d5 to block the check; but the absence of a White Pawn on c5 rules that out. With Black moving left, the White Pawn on f6 could have captured a Black Pawn that had moved from g6 to e6; but the Knight on g6 rules that out. With Black moving down, the Black Pawn from d7 would have advanced to d5 to block the check; the Bishop on d6 rules that out.

"That leaves the White Pawn on c3 as the only possibility. Black was moving *up*, and the Pawn from c2 blocked by advancing to c4. A White Pawn on b4 captured en passant at c3 to discover the second Bishop check."

"Yes," agreed Holmes. "And you can see now that there was only one possible move to discover the first Bishop check. With White moving down, the White Pawn now on f6 moved there from f7 to discover that check."

"Holmes," I admitted, "I have never before seen a situation in which an en passant capture was so essential, with that fact so subtly concealed. But what led you to suspect two consecutive Bishop checks?"

He laughed, "It was not a suspicion, but a fervent hope once I saw the possibility of the en passant capture. Like a bridge player who assumes the only possible opposing distribution that makes a successful finesse possible, I sought the only circumstances that could make the en passant capture necessary."

I am not certain whether it was Chuzzlewit or Micawber who exclaimed, "The dickens, you say!"

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66. TRUE COLOR

I mused, "The White-Down Queen on f4 checks the White-Up King on c1. Thus, White-Down made the last move. We must determine whether the speaker told the truth or a lie, in order to know whether he played White-Up or White-Down. And we must determine which ambiguous color pattern represents White.

"Move parity can tell us whether White or Black moved last. I count 16 pieces on white squares, including the Bishops necessarily captured on c8 and f1. The even color parity, matching the initial color parity, shows that total move parity, by White and Black combined, also is even, so Black moved last. The White-Down pieces represent Black.

"Basset has made three statements. Either all three are true or all are false. Only one of those statements appears to be directly analyzable, the statement concerning relative numbers of Queen and Rook moves. I feel that evaluation of move parity for the Queens and Rooks separately should be relevant. But I can't carry out that evaluation, being unable to decide which of the doubly-ambiguous Rook patterns is White and which is Black."

"Watson," Holmes observed, "you have successfully applied the only principle necessary in determining that Black moved last. You determined total move parity without having to resolve the colors of those two Rooks. You need only to recognize what you have done, while describing it more broadly.

"You have considered a specific aggregation of pieces which individually make heterochromatic moves, the aggregation of all 32 chess pieces. When any piece within that aggregation moves, the color parity of the aggregation changes. That same principle applies to *any* distinguishable aggregation, including the one composed of two Queens and four Rooks."

A light turned on for me. "The game began with one of the two Queens on white, together with one Rook of each color. For the aggregation of Queens and Rooks, *initial* color parity was odd. The present color parity also is odd, with three Rooks and no Queen on white squares. When the present color parity equals the initial color parity, pieces making up the aggregation have made an even number of moves.

"With the total move parity of the aggregation even, the included White-piece and included Black-piece aggregates have the same move parity, both odd or both even. Any difference between the actual move counts can only be an even number, *not* three. So Basset lied. Then he did not make the last move, a Black move, and had to be playing White."

"Bravo," Holmes applauded. "That also means the last move was not a Queen move. What was it?"

"The Queen check was discovered," I replied. "And the only possible discovering move was by the Black Knight now on e4, moving from d2."

"Right," Holmes complimented me. "The information available to us does not permit

us to resolve the colors of the two Rooks of doubly-ambiguous patterns. But we can and have established move parity, both for the total of 32 pieces and for a set of 6 pieces that one normally would not think of as a meaningful aggregation."

"Holmes," I said, "before Basset spoke, you seemed to have solved a problem to your satisfaction. Was that the problem of who moved last?"

"It was, indeed, Watson. The position alone contained all the information necessary to determine possession of the move, as you did so ably. Basset's statements merely broadened the problem to include his veracity and the precise last move."

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67. MORIARTY'S QUESTION

"Let us work through the feasible combinations of Pawn color and location, and see what each implies in terms of numbers of promotions," said Holmes. "But first, Watson, would you help set the stage by analyzing the two obvious promotions and the capture implications?"

"The Black Bishop on c8 and the White Bishop on f1 have been hemmed in by Pawns, never moved, and were captured at home, a first capture by each side," I said. "The Black Pawn from a7 could have promoted to Bishop at b1, or the Pawn from either c7 or e7 could have promoted at d1, in each case requiring one capture. The White Pawn from either d2 or f2 could have promoted to Bishop at e8, or the Pawn from h2 at g8, in each case requiring one capture. Promotions at other locations would have prevented a Bishop from moving after promotion. Thus, the promotions to Bishop account for a second capture by each side.

"Doubled White Pawns on the b-file and doubled Black Pawns on the f-file prove a third capture by each side. The side to which the ambiguous Pawn belongs could have made a fourth capture; the opposing side has exhausted its maximum of three captures. It goes almost without saying that White is moving up the board, too few captures available to support Pawn bypasses."

"Fine!" Holmes approved. "Now, could the ambiguous Pawn be on e5 or d4?"

"If it were on e5," I observed, "both Kings would be in check simultaneously, from the White bishop on f3 and the Black Rook on d6. If the Pawn were on d4, Black would be in check from the White Bishop on f3, with no possible last White move. The Bishop could have moved only from d5 or e4, both imaginary check positions, and no White piece could have discovered the Bishop check by moving from d5 or e4. We are left with two possible squares for the ambiguous Pawn, d5 and e4."

"We can quickly rule out a White Pawn on d5," said Holmes. "It would be giving check, thus would have made the last move. That move had to be a capture from e4, or the White Bishop would have given imaginary check."

"That would have been a fourth capture by White," I observed. "But, with the ambiguous Pawn White, a fourth capture was allowable."

"But not a *fifth*," replied Holmes. "With a Pawn on e2, a second Pawn on the e-file had to capture from the d- or f-file."

"So we are down to three possible combinations," I noted, "a Black Pawn on d5, or a Pawn of either color on e4. Can any of *those* be eliminated?"

"Not before the final step of the analysis," Holmes replied. Let us consider the Black Pawn on d5. It originated on c7 and made a fourth Black capture to reach the d-file (the Pawn from e7 is one of the doubled Pawns on the f-file). The Pawn from a7 promoted to Bishop at b1. What happened to the Pawn from h7? It had to be captured, but not on the h-file and not as a Pawn. It promoted at h1. If not subsequently captured, it replaced a Black Queen, Rook, or Knight that was captured by a White Pawn.

"With no further captures available to White, no second White promotion was possible; the Pawn from f2 was captured by the Black Pawn from e7. Thus, if the ambiguous Pawn is on d5, the Pawn is Black and three Pawns have promoted - one White and two Black."

"Let me try the next combination," I pleaded, "a Black Pawn on e4. With at most one Black capture not yet counted, the Pawn had to come from e7 without a capture. In that case, the Pawn on f5 came from h7 rather than e7, increasing the capture count by one, to the maximum allowable four. The capture on g6, a white square, was not of a Pawn as such, so White required a second promotion. That promotion was possible for the Pawn from h2, promoting at h8 without a capture; the promotion to Bishop then was at e8, by the Pawn from d2. Neither of the two White Pawn captures was of a Black Pawn; one was the missing Bishop on black, while the second proves a second Black promotion (the Pawn from a7 promoting at a1 without capturing). Thus, a Black Pawn on e4 proves four promotions, two by each side.

"With the Black Pawn on e4, White is in check, from the Rook on d6, which could not have moved last. It looks as if the Black Bishop could have discovered check, by moving from d3 to b5. But the Bishop would have to capture at b5, or the White Queen would have given imaginary check. That capture would have been an impossible fifth Black capture. The only possible last move was by the Black King from d5 to e6, discovering the Rook check."

"Well done, Watson," said Holmes. "Finally, a White Pawn can be on e4, after a fourth White capture from the d-file. Again, White is checked by the Rook on d6, the check necessarily discovered, again by the Black King move from d5 to c6. Before the King moved, it was in check by the Pawn on e4, which therefore made the preceding move, That Pawn move was a capture from d3 to e4, to prevent imaginary check by the White Bishop. What did White capture at e4? Not the Pawn from e7; with only three captures available to Black, the Pawn from e7 had to capture to the f-file, to account for the doubled Pawns there. White had to capture, on e4, a promoted Black piece or a piece replaced by a Black promotion, proving a second Black promotion.

"The White Pawn capture on the b-file was not of a Black Pawn; the Pawn originally on b7 is still there, and no further Black capture was available to place a Pawn from a7 or c7 on the b-file. Also, the White Pawn from h2 captured at g7 or g8 in promoting to

Bishop, and could not have captured a Pawn in either location. One of those two White Pawn captures was of the Black Bishop on black squares; the other capture proves a *third* Black promotion. Pawns from a7 and h7 promoted without capturing, leaving the Pawn from c7 to make the one capture we have allocated to the Pawn promoting to Bishop.

"In this case, all White Pawns are accounted for. Of the two missing Pawns, that from f2 was captured at f5, while the Pawn from h2 promoted to Bishop at g8 (capturing at g7 before the Black Pawn from h7 made *its* first move toward promotion.

"A White Pawn on e4, then, requires one White and three Black promotions - unequal numbers. The Black Pawn on d5 requires two Black and one White promotion, again unequal numbers. We have seen that the Black Pawn on e4 requires two promotions by each side. Two of the three possibilities involve unequal numbers of promotions; one involves equal numbers."

"But," Professor Adler objected, "you don't know whether the numbers actually were equal or unequal."

"Actually, I do," said Holmes. "If Moriarty had been told that the numbers of promotions were *unequal*, two possibilities would have remained, and he would have been unable to prove a unique solution. He would have known *neither* color nor location. Only one of the two possible answers permitted a unique solution. The fact that Moriarty had enough information for a unique, complete solution proves he was told that Black and White had had *equal* numbers of promotions. The ambiguous Pawn is Black, and belongs on e4.

"I *do* know what answer Moriarty received, but had to solve the problem in order to know that answer. If that is a cart before a horse, so be it. Let the horse push."

"Then," Adler asked, "weren't you being unduly modest last night, in saying you *might* be able to solve the problem?"

"Not really," Holmes replied. "For the problem to be soluble with the information available, one possible answer to Moriarty's question had to leave the solution ambiguous. You will note that, of the eight combinations of Pawn color and location, we have ruled out five before considering the question of equality of promotions. I had not completed that paring down when I spoke last night. There was a possibility that *six* of the eight could be thus ruled out. In that case, each possible answer to Moriarty's question would have corresponded to a different unique solution, and a single solution could be reached only by knowing the answer to that question."

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68. THE ROOK ON H7

"The third Black Rook proves a promotion," I started. "Since all eight Black Pawns are still on the board, it was a White Pawn that made a reverse promotion, reducing the number of White pieces to 15. Black Pawns have made four captures, ten White pieces are visible, and an eleventh is checking the Black King. All White pieces are thus accounted for, and two of the fallen pieces have to be Black."

"Where did the reverse promotion occur?" Holmes asked.

"We must consider the captures available to White Pawns," I said. "Including the two fallen pieces, sixteen Black pieces are on the board. The reverse promotion created a seventeenth Black piece, allowing one capture by a White Pawn. One capture would not permit the White Pawn from the b-file to promote. It is the Pawn from g2 that promoted, with one capture, at f8 or h8. Promotion at f8, through f7, would have required the Black King to move from e8, denying castling. So the reverse promotion was at h8. The original Black Rook vacated h8 to permit the promotion, and Black cannot castle King-side. The castling anticipated has to be on the Queen's side."

"And," added Holmes, "since no other promotion was possible, all of the fallen pieces are original. The White piece is not a Pawn on the b-file, which could not give check, but is a Queen, Rook, or Knight. The two fallen Black pieces are drawn from the second Knight, the Bishop on white squares, and the Queen."

"Tracking down that one Rook told you a great deal," said Crenshaw, clearly impressed. "Does the potential for castling lead to any further conclusions?"

"For Black to retain a castling option," I said, "he must be able to capture the checking White piece, or to interpose, rather than move his King. Also, the apparent check of c8 by the White Bishop on g4 must be negated to permit castling. Moreover, the White Pawn on a6 threatens to capture on b7. While on b7 it checks c8 and denies castling; and it threatens capture of the Rook on a8. Either that capture or a Rook move to avoid capture would permanently deny Black castling. Black must accomplish much on his next two moves, to permit castling on the third."

Holmes resumed questioning, "What can you say about the ambiguous piece on d6, Watson? As a start, can it be White?"

"If the unknown piece on d6 is White," I replied, "it must be a checking Knight, which must be captured to eliminate the check. The missing Black Bishop moves on white squares, and cannot occupy d6. The capture must be by the Pawn now on c7 or by a Black Knight or Queen. Capture by the Black Knight has an obvious advantage; it checks the White King, forcing its move to d3, and delaying a White Pawn capture on b7.

"Black's second move also must be a forcing check, or the Pawn capture from a6 to b7 will prevent third-move castling. The Black Queen or second Bishop could move to check the White King's d3 location; but that move must also prevent the White Bishop on g4 from checking c8. Checking and blocking simultaneously requires the Black Queen or Bishop to move to f5; but the Bishop on g4 can capture the checking Black piece,

reestablishing its check of c8 and preventing third-move castling. Thus, capture of a White piece on d6 by a Black Knight does not accomplish the objective.

"If Black captures the White Knight with his Queen, White is not in check, and can make the Pawn capture from a6 to b7. The second Black move must capture the Pawn on b7; either a Bishop or Knight could make that capture. But then there is no piece of either color between the White Bishop on g4 and square c8, and Black still cannot castle on his third move.

"Thus, there is no solution with a White piece on d6. The piece there can only be the Black Queen (a Black Knight on d6 would place both sides in check simultaneously, and the missing Black Bishop cannot occupy the black square). The White piece giving check is a Rook or Queen, since all other squares from which a Knight could check the Black King are occupied."

"Well thought-out, Watson," Holmes commended me. "What are the implications of possible White Rook checks?"

"The White piece giving check did not capture on its last move, all captured Black pieces (one) already accounted for. A Rook would have to be on d8, e5, e6, e7, or f8 to give check. A White Queen could also be on c8 or g8 to give a Rook-type check. But only the positions on the e-file allow for simultaneously nullifying the check and checking White, with capture by the Black Queen from d6 in each case. The White King could move to d3 (if to f5, with the Black Queen on e7, Black could castle on his *second* move). Again, no second Black move is available, whether the second Black piece is a Bishop on white or a Knight, that both checks to force White play, and blocks the White Bishop check of c8, except a Black Queen or Bishop move to f5. The White Bishop on g4 could capture the checking piece on f5, reestablishing its check of c8, and preventing third-move castling. And, of course, a noncaptureing second Black move would permit the Pawn capture at b7 to prevent third-move castling.

"With a White Rook or Queen on e5, a Black piece could be interposed on e6, temporarily negating the check and also blocking the Bishop check of c8. But the Bishop on g4 could capture that blocking piece (and a potential second blocking piece with a second White move). If Black's second move captures the White Bishop, White's second-move Pawn capture at b7 denies third-move castling."

"The only alternative," Holmes pointed out, "is that the checking White piece is a Queen giving a Bishop-type check. You should now be able to determine precisely where the checking White Queen is located, and the identity and whereabouts of the second fallen Black piece."

I nodded and continued, "The Queen, if on c6, d7, f7, or h5, could be captured, but the capturing move would not be forcing, and White's next move could be a Pawn capture at b7. Black capture of that Pawn would leave the Bishop on g4 free to capture any remaining Black piece that blocks its check of c8. The White Queen at g6, giving a Bishop-type check, is a different story. Negating the White check and simultaneously checking the White King is possible by capture of the Queen, from h5, by the Black Bishop on white squares. (Not from f7, where the Bishop would have blocked the Queen

check, and not from f5, where the Bishop would have given imaginary check.)

"No move is available to the White King, but either of two pieces can be interposed on f5. If the Rook from f4 is interposed there, the Bishop check of c8 is blocked, and Black can castle on his *second* move. If the Bishop from g4 is interposed at f5, it is captured by the Black Bishop, again checking. The Rook from f4 must then capture the Black Bishop, and Black can castle on his third move.

"Thus, the White Queen is on g6, giving check, the second Black Bishop is on h5, and the ambiguous piece on d6 is the Black Queen. Black can castle no later than his third move, against any White play, but can be prevented from castling earlier."

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69. TERMINAL MOVE

"Black is in check by the Rook on h3," I observed. "If White were moving down or right, the Black King would also be checked by the Pawn on d4, and that Pawn could not have made a last move that discovered the Rook check. If White were moving up, Black would be in check by a White Pawn on f2 that had *never* moved. White can only be moving left. The White Pawn on f2 checks the Black King, and made the last move, a capture from g3 that discovered the Rook check. That's the entire solution, unless you insist that I identify the Black piece captured by that last move."

"Oh, I *do!*" Holmes laughed. "That is the whole point of the problem. Examine the promotions and count captures, to determine what *could* have been captured at f2."

"With White moving left," I said, "there are two Black Bishops on white squares, one of which has to be promoted. Both White Bishops on white squares are promoted, since the original Bishop on h6 was hemmed in by Pawns and captured at home, not by a Pawn.

"White had five Pawn captures available. The Pawn on f2 has captured once. Since that Pawn originated on g3, one of the doubled White Pawns on Row 4 came from Row 2 or Row 6, with two captures. The other of those two Pawns could have promoted on a white square with one capture, at a3 from Row 2, a5 or a7 from Row 6. The White Pawn from g8 could have promoted to Bishop at a7, using the last available Pawn capture. Further, the White Pawn from g1 could have promoted at a1, without a capture, but a Bishop created there could not have left a1. Thus, White Pawn promotions could account for two of the three promoted Bishops, but not for the third.

"Black Pawns on Rows 2 and 7 show two captures, with up to three captures available to Black Pawns. A Black Pawn from Row 1 could have promoted at h2, or one from Row 5 at h4, or that from Row 3 at h2 or h4, with one capture. With no other capture available, a second missing Black Pawn, if from Row 1 only, could promote on black square h1, thus not to one of the three obviously promoted Bishops. Only one of the three promoted Bishops can be a promoted Black Pawn."

Holmes interjected, "Note that, before White's last move, Pawns on g3 and g5

hemmed in the White Bishop on h4. Therefore, that Bishop was created by the reverse promotion of a Black Pawn, from Row 3 or Row 5. The other of those two Black Pawns, with no capture available, could not promote, and was captured as a Pawn. All available captures of Black pieces have been by White Pawns. We have seen that a White Pawn captured on one of those two rows, en route to Row 4. The other Black Pawn from Row 3 or 5 promoted at h4. The Black Pawn from Row 1, then, has captured to Row 2 rather than promoting.''

"In that case,'' I noted, "one of the White Pawn promotions to Bishop also was a reverse promotion, and the availability of captures remains unchanged. The Black piece captured at f2 could not have been another Bishop on white. And no Black Pawn could have reached f2. So the piece captured on f2 was a Black Queen or Knight. A Rook on f2 would have been a third Black Rook, created by reverse promotion at a1. And that second White reverse promotion, without a second Black reverse promotion, would have left too few captures available to Black.''

"For the moment,'' said Holmes, "let us assume that the piece captured on f2 was a Black Queen. The Queen on f2 checked White before the Pawn capture. How did that check develop?''

"The check was not discovered,'' I replied. "No Black piece except the King could have moved from f3 or f4. On f4 the King would have been doubly checked by unmoved Pawns on g3 and g5. On f3 it would have been checked by the Knight on g1; and, with the King on g3, there was no square from which that Knight could have made the preceding White move. So the Black Queen on f2 had just moved from g2. Before that move, f2 had to be occupied by a White piece, or Black's move would have been from imaginary check by the Black Rook on f1. The Black Queen, then, would have captured on f2 upon moving from g2.''

"Captured *what*?'' asked Holmes. "We have already accounted for the three captures available to Black. No White piece would have remained to be captured at f2 by a Queen.''

"Then,'' I said, "the piece captured at f2 could only have been a Black Knight and necessarily an *original* Black Knight.''

"Yes, it was,'' Holmes agreed. "To complete the promotion picture, the White Pawn from g1 necessarily promoted at a1 without capturing; it had to be captured by a Black Pawn, and could not be captured on Row 1.''

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70. BLACK'S GAFFE

Holmes asked, "What was Black's last move, Watson?''

"The checking Bishop could not have made the last move,'' I realized. "It could have moved only from a position of imaginary check at f7 or g6. The check, then, was discovered. The only possible discovering move was by the Knight, from f7 to g5.''

"Right. And what more lucrative Knight move was available to Black?"

I examined other possible Knight moves from f7. "Moves to h8, d8, or e5 held little promise. But a Knight move from f7 to d6 would have given double check, by the Knight and by the Bishop on h5. The double check would have prevented capture of the Black Knight. And a White King move would have allowed the Knight to capture the White Queen.

"Black *does* seem to have missed an opportunity for a better Knight move. I suppose he *should* have moved to d6. But we have certainly seen many players, even very good players, make moves more disastrous. Why is this slight misplay so disturbing?"

Holmes replied, the tongue in his cheek on the side away from the ladies, "Don't you realize what Black did? Seated between Mrs. Post and Mrs. Vanderbilt, *he used the wrong fork!*"

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APPENDIX

● PURPOSE AND SCOPE

Many years have passed since the 1981 publication of the last widely distributed book of retrograde chess analysis problems, Raymond Smullyan's *The Chess Mysteries of the Arabian Knights*. Even many devotees might benefit from a brief refresher on fundamental disciplines applicable to retrograde analysis. Readers to whom the field is completely new could obtain the best grounding from Smullyan's 1979 *The Chess Mysteries of Sherlock Holmes*, if they could find it. Though no more than the second-best introduction, this Appendix may be the only preparation available to some before they dive into the present book. The body of the book includes simple problems as review of move parity before extending its application. but assumes some background in counting Pawn captures, evaluating promotion possibilities, addressing castling issues, and applying the constraints of the homochromatic (or monochromatic) game form.

Even for review purposes, these principles are most readily communicated in the context of specific position examples. A similar example serves as an introduction to or refresher on resolving ambiguities of board color, piece color, piece location, and piece identity.

● COUNTING PAWN CAPTURES

Figure A1 illustrates the counting of Pawn captures, and its application to questions of promotion and castling. This discussion assumes we observed the last move leading to the position, such as a White Pawn move from h4 to h5, thus know that White is moving up the board.

Counting the pieces remaining on the board, we find 11 Black and 10 White pieces. Then, White Pawns can have captured at most five Black pieces, and Black Pawns made a maximum of six captures. Present Pawn positions, together with known promotions, can establish the minimum numbers of captures required.

The h-file at the far right contains two White Pawns ("doubled Pawns"). One had to capture from farther to the left. Backing one Pawn up to the g-file results in doubled Pawns there, with one coming from an earlier file. We have to back up to the d-file before we run out of doubled White Pawns. Four captures were required for the Pawn to move from its origin on d2 to its present position on h6,

We can be certain the Pawn from d2 did not wind up on any square occupied by a White Pawn on the e-, f-, or g-file. A Pawn captures along a diagonal or above it (the latter when it makes a noncapturing advance between captures). It can never wind up displaced to the right or left, but below the 45° diagonal. We are equally certain the Pawn on h6 did not originate to the left of d2 (say, at c2), since the 45° diagonal from c3 would pass above h6.

Thus, the Pawn now on h6 has made precisely four captures, each capture on a black

square. Pieces captured consist of the missing Black Knight and three of the four missing Black Pawns, either as Pawns or as promoted pieces. The fourth missing Black Pawn promoted to the second Black Queen now on the board. The missing Black Bishop moved on white squares and could not be captured by the Pawn en route from d2 to h6.

The White Pawn from c2 is missing, and a second White Bishop on black proves it promoted, on a black square. The Pawn from c2 needed one capture -- the last available to White -- to reach the d-file and promote at d8. Two White Pawns together made five captures, the maximum available to White. No other White Pawn or piece captured anything.

A1. PAWN CAPTURES
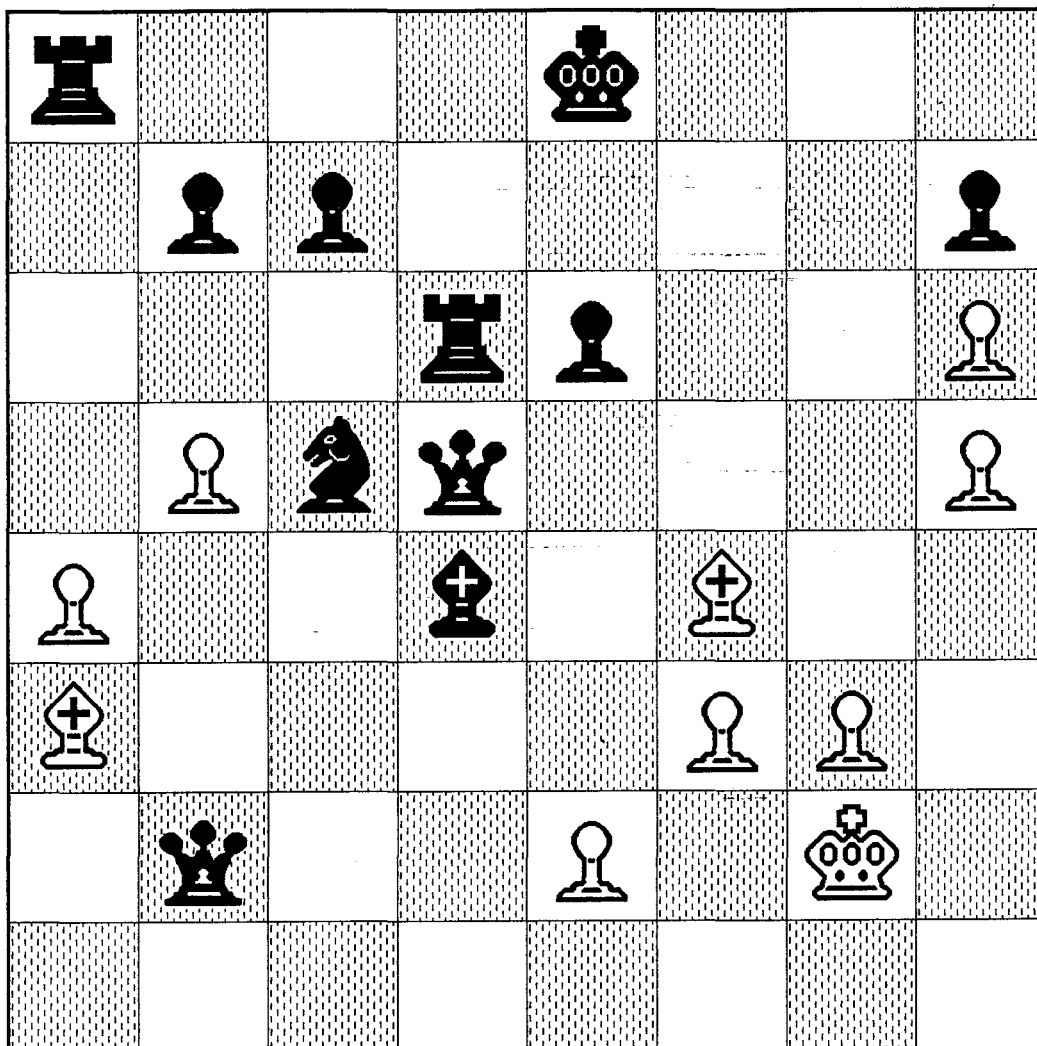
After capturing on the d-file, the promoting Pawn moved straight up that file, through d7 to d8. At d7, it would have checked the Black King on e5 unless the King had previously moved from that square. Since the checking Pawn was not captured, the check could only be negated by a King move (or avoided by an earlier King move). The Black King *has* moved, and Black cannot castle.

All four missing Black Pawns *could* have promoted, needing only four captures: one by the Pawn from a7, promoting at b1; none for the Pawn from d7, promoting at d1; two for the Pawn from f7, promoting at d1, f1, or h1; and one for the Pawn from g7, promoting at h1. With another capture, other promotion sites were possible. At least one did promote, to the second Black Queen on the board, and no second Black promotion was required. Black Pawns from f7 and g7 could be captured on their own files as Pawns, thus did not need to promote; the Pawn from d7 could have captured once to the e-file, to be captured as a Pawn on e3.

The "count" of captures by Black Pawns was 1 to 6, carrying no punch. The count for White Pawns was precisely 5; that told us where the only White promotion occurred, and allowed proof that Black could not castle. Pawn counting is extremely fruitful only when the minimum number of Pawn captures required is at least very close to the maximum number of captures available.

Can that coincidence or near-coincidence of minimum and maximum occur often? Probably not in the real world of chess, but look for it extremely often in the artificial world of composed problems. The problem formulator has to make his problems soluble.

● PASSAGEWAYS

Figure A2 has two prominent features. First, it appears possible that Black can castle on his Queen's side. Secondly, two Black Bishops on black squares prove promotion of at least one Black Pawn. The players inform us that the promoted Bishop has moved an odd number of times as Bishop. Let us explore the castling potential, in order to illustrate constraints of a different kind.

We note that Black *could* be moving up, which would rule out castling. With that move direction, White has four captures available, and Black six captures. Four files contain White Pawns below Black Pawns. At most eight captures, two per file, would permit the bypassing of Pawns on those files; half of those bypassing captures could be made by each side. That would allow Black another two captures to bypass once more in order to promote to Bishop at b8 or d8.

If Black is moving down, as is necessary to make castling allowable, there is no evidence that White has promoted a Pawn, with moves by the promoting Pawn forcing the Black King or Rook to move from its home square. If castling is to be ruled out, it must be for an entirely different reason.

There *are* additional constraints. White Pawns on b2 and d2 have not moved, and prevented the White Bishop from leaving c1. That Bishop was captured at home, and not by a Pawn. Only five missing White pieces could have been captured by Black Pawns.

The promotion of a Black Pawn to Bishop was necessarily at g1. If promotion had occurred at a1, c1, or e1, the promoted Bishop could never have left that square because of blocking White Pawns. It appears that the Black Pawn from d7 could have promoted at g1 with 3 captures, the third capture to g2 after move of the White Pawn to g3. But then the promoted Bishop could have moved only by shuttling between g1 and h2, and would be on h2 after any odd number of moves. The Bishop now on g1 *may not be* the promoted piece! For it to leave and return to g1 with an odd move total or for the two Bishops to exchange places, there was movement through g3 after the promotion. The promotion was through h2 before the Pawn from g2 advanced to g3. The Black Pawn from d7 made five captures to reach g1 through h2, exhausting available captures by

A2. TRAFFIC BARRIERS
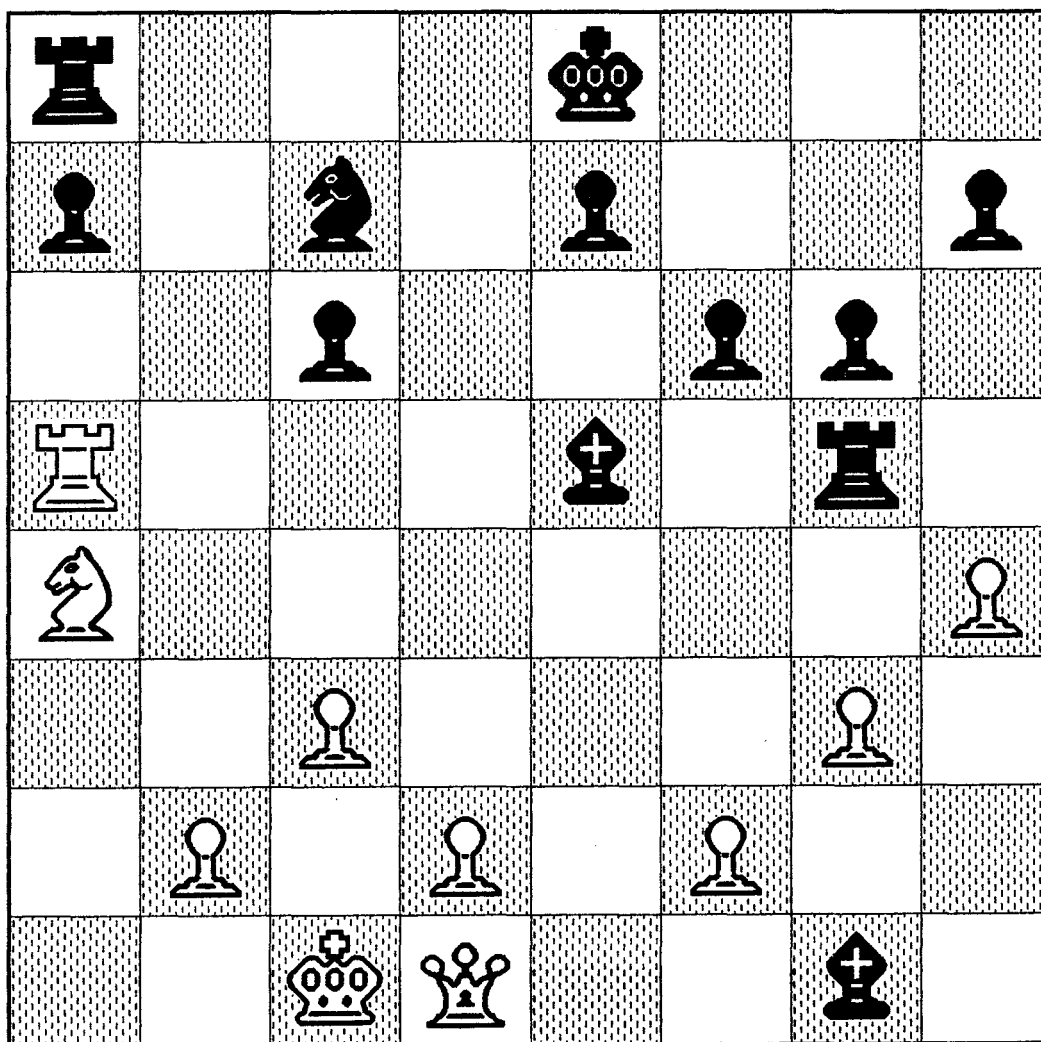
Black Pawns. The Black Pawn from b7 could not promote without capturing to bypass the unmoved White Pawn on b2, so there was no second Black promotion.

Since the Black Rook on g5 is not promoted, how did it get to that square? If the Pawn on f6 had captured from g7, and that on g6 from f7, the Rook could have moved down the f- or g-file after the first of those captures and before the second. But the promotion to Bishop exhausted the captures available to Black Pawns. Without that cross-capture, the Rook from h8 could reach g5 only by way of the b- or d-file, passing through e8 en route, with the Black King then on some other square. The Black King having moved, Black cannot castle.

Once more the counting of Pawn captures has played a key role. But so has a second element -- Pawn formations as barriers to movement, for both the promoted Black Bishop and the original Black Rook. Pawn barriers of both colors were vital to the analysis.

The importance of Pawn formations as barriers to movement is by no means limited to questions of castling. They can limit moves by other pieces, including Knights and Kings, the latter especially in homochromatic play.

● HOMOCHROMATIC (or MONOCHROMATIC) PROMOTION

We are told that Figure A3 represents a position reached in homochromatic play. What can we determine concerning promotions? Is the move direction ascertainable?

There are Black Knights on both g1 and b8. A Knight cannot move homochromatically (between two squares of the same color). Thus, any Knight present either is on its home square or is a promoted Pawn. One of the two Knights is promoted; determining which requires establishing the move direction.

For either move direction, one of the White Rooks is promoted, on a black square. Homochromatically, an original Rook can reach only squares on its first, third, fifth, or seventh rank. With White moving up, the Rook at h6 is on White's sixth rank. With White moving down, the Rook at g3 is on White's sixth rank and is a promoted Pawn.

In homochromatic play, a Pawn can make an initial two-square advance. All subsequent moves by that Pawn have to be captures. Then, a Pawn must make at least four captures in order to promote. Note that one capture can be an *en passant* capture of an opposing Pawn moving on the other color. But three captures are necessarily of opposing pieces moving on the promoting Pawn's own color. Missing Black pieces on black squares are the Queen, one Rook, and four Pawns. But one Black Knight is a promoted Black Pawn, leaving only three Black Pawns on black squares available for capture. The five available captures would be more than enough to permit promotion of one White Pawn on black.

But the White Pawn on h4, moving down, has made at least one capture on black; the Pawn on a3 has made at least two captures, neither an en passant capture, with a Black Pawn still on a2. Six Black pieces on black squares had to be captured by White Pawns, exceeding the five captures available. (If that fact did not rule out White movement down,

it would be necessary to ask whether all missing Black Pawns on black could reach positions permitting a promoting Pawn to capture them.)

White, then, is moving *up*. The Pawn on a3 captured once, from b2; the Pawn on h4 required no capture with movement up. Including the promoting White Pawn, Pawns required only four captures of Black pieces on black squares. One feasible sequence has the White Pawn from d2 promoting. It makes an initial two-square advance to d4. At e5 it captures the Black Pawn from e7, which could reach e5 without capturing. At f6 it captures, en passant, the Black Pawn from f7, which had advanced to f5 without a capture. At g7 it captures an unmoved Black Pawn, and at h8 the Black Rook originally there. The Black Queen remains available for capture on a3 by the White Pawn from b2.

A3. MOVE CONSTRAINTS
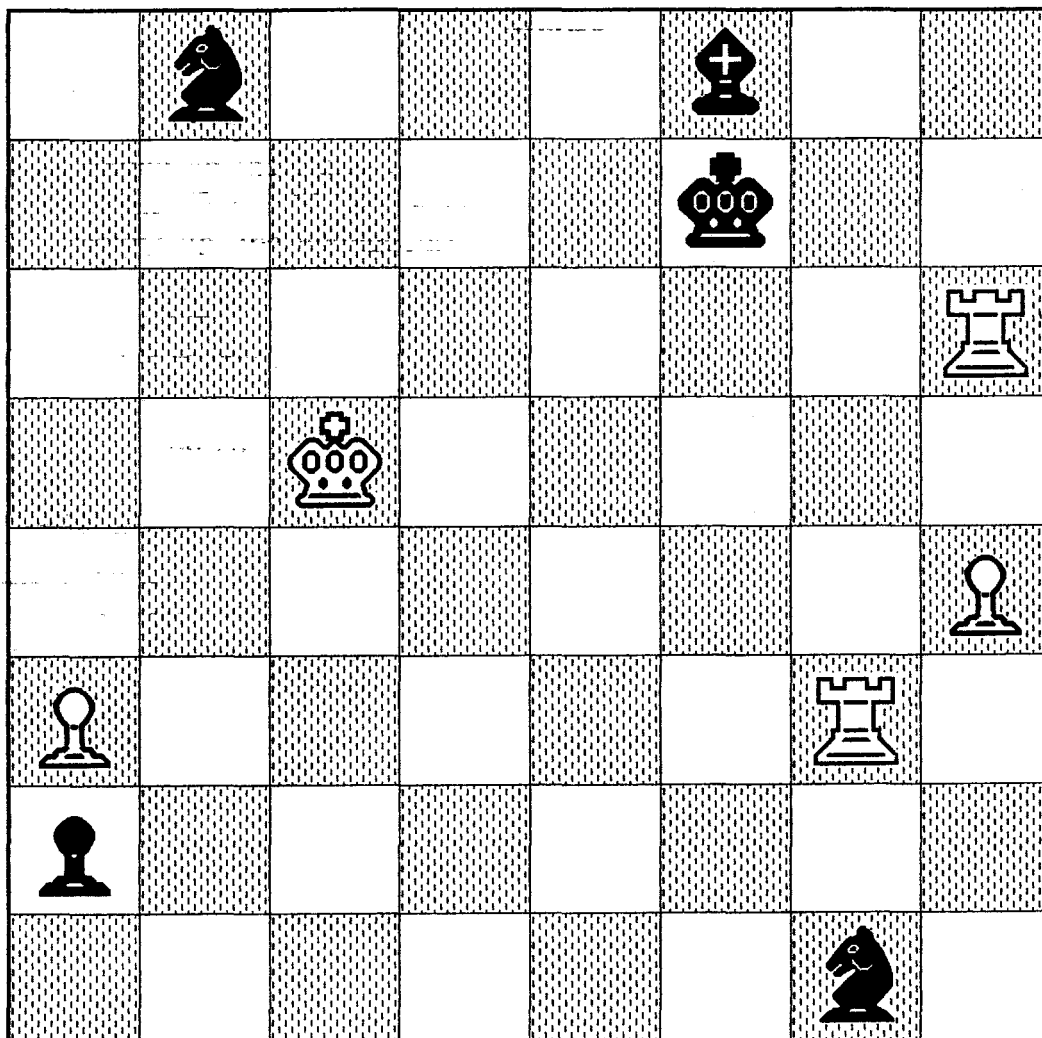
Thus, White can be and necessarily is moving up; the White Rook on h6 is a promoted White Pawn, and the Black Knight on g1 a promoted Black Pawn. (That Black Pawn, from c7, could have captured a White Bishop on d4, the White Pawn from e2, en passant at e3, the unmoved White Pawn on f2, and the unmoved White Knight on g1.)

The Black Pawn on a2, moving down, has made three captures on white squares; with a White Bishop, Rook, and Queen available on white squares, it is not even necessary to consider captures of White Pawns.

Notice that we used homochromatic constraints on Knight and Rook moves to prove two promotions (the easy part of the problem). Establishing the move direction was more complex in practice, but the only principle involved was the counting of Pawn captures against available captures.

These same homochromatic constraints, and those on King and Queen moves, are frequently applicable to the question of the last move, though they obviously are not needed here. They are particularly effective in discriminating between direct and discovered checks, or between different potential discovering moves.

● COLOR AND LOCATION AMBIGUITIES

In Figure A4, a Knight of uncertain color is also uncertainly located, belonging on either b3 or c3. A Pawn of uncertain color may belong on e4, e5, f4, or f5. The board is of ambiguous colors, the move direction not immediately obvious. To complicate the situation, a White Pawn is on h8. What is happening? What colors are the ambiguous Knight and Pawn, and where does each belong? Can the move direction be determined?

Despite the multiple ambiguities, the problem is fundamentally simple. Let's begin by examining that White Pawn on h8. How can it be there legally?

The only possible answer is that White made the last move, of a Pawn from g8 or h7 to h8, and was interrupted before he completed the move by exchanging the promoting Pawn for a piece. That conclusion vastly simplifies the rest of the problem.

White could not have moved that Pawn if either side had been in check. To prevent check of the Black King by the White Bishop, the Knight of unknown color has to be on b3 rather than c3. On b3 it has to be White, or it would check the White King.

The Pawn on a corner has to be on the e-file to block check of the Black King by the White Queen. And it must be on e5 rather than e4 to block check of the White King by the Black Bishop. Only its color remains to be determined.

The last Pawn move showed White to be moving either up or right, leaving Black moving down or left. The Pawn now localized to e5 can only be White, since a Black Pawn there would check the White King for either move direction.

Finally, no, the move direction cannot be determined uniquely. Two of the four move directions remain possible, but that uncertainty did not prevent resolving the other ambiguities.

This problem is *overdetermined*, i.e., contains redundant forcing elements. Removing the White Queen would leave the problem soluble. And a board uncluttered with extraneous pieces makes it easier to focus in on the essentials. Still, it conveys a broadly applicable lesson on resolving ambiguities.

We could have viewed the problem as one of choosing a correct solution from among 128 combinatorial candidates (the combination of four move directions, four combinations of Knight color and placement, and 8 combinations of Pawn color and placement). We could have asked, for each of the 128, "Is the position legal?" The answer would have been "Yes" for only two of the 128 combinations (one combination of piece colors and placements with two possible move directions). But our reasons for rejecting

A4. RESOLVING AMBIGUITIES
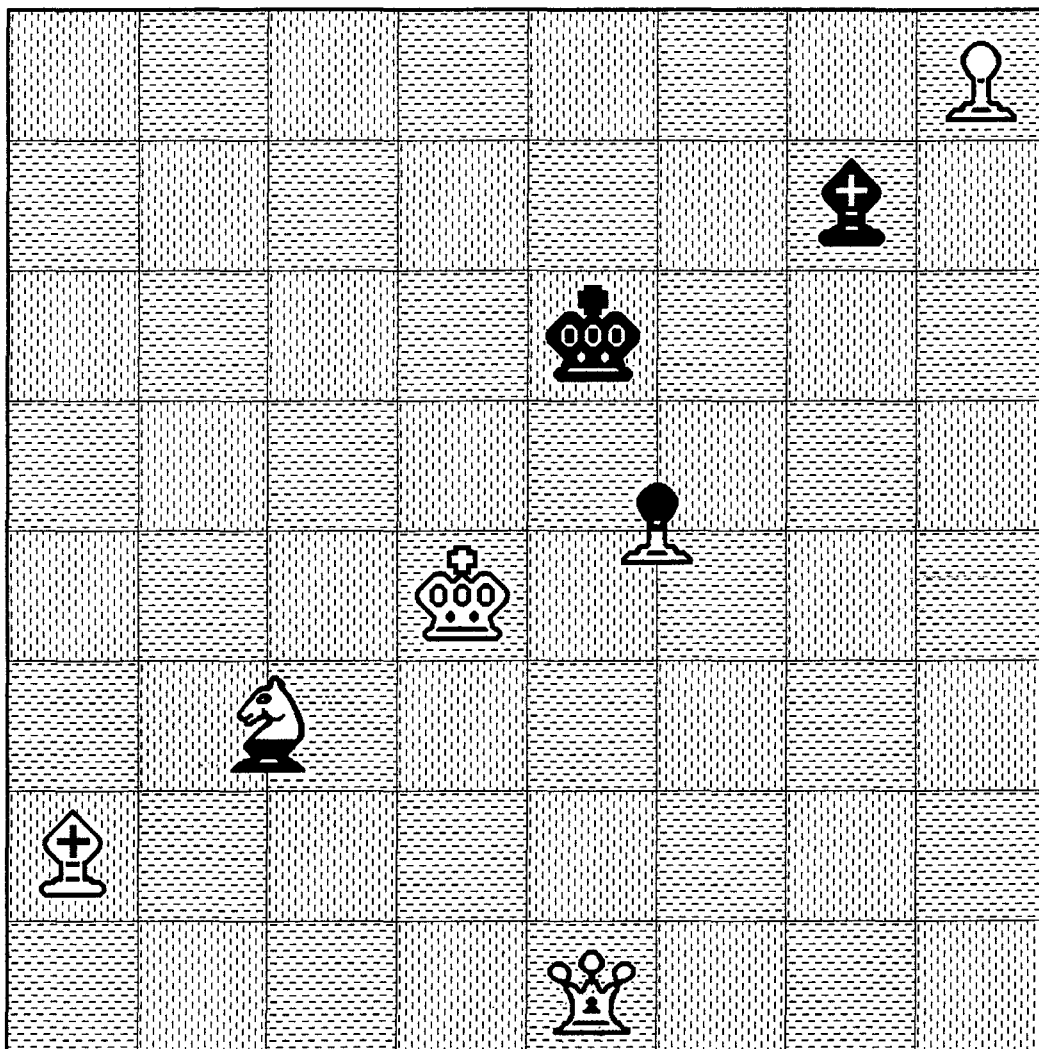
combinations would have been highly repetitive. It is more efficient to eliminate half or three-quarters of the combinations at a time, as we did by first questioning piece placement independent of color.

We can imagine replacing the ambiguous Pawn with a piece of unknown identity as well as uncertain color and location. The correct location would still be determined quickly. On e5, the piece could not be a White Queen or Rook, which would check Black, or a Black Queen or Bishop checking White. In addition to the White Pawn, the piece could be a White Knight or Bishop, or a Black Knight or Rook. We could add a second White Knight to the board, a White Bishop on vertical grey, two Black Knights and two Black Rooks. Then, a White Knight or Bishop, or Black Knight or Rook would Prove a Pawn promoted. We could construct Pawn formations that would rule out promotions other than the one in progress at h8, limiting the piece on e5 to a White Pawn.

That is how complex retrograde analysis problems are *constructed*, by adding constraints sequentially until only one solution is possible. And they are *solved* by *applying* the applicable constraints sequentially to narrow the possibilities down, ultimately to one.

● WHAT IF THE POSITION IS "IMPOSSIBLE"

Please, don't insult the problem formulator. If the position *appears* impossible, you are overlooking something. That may be:

An *en passant* capture, making the position look unfamiliar to even a highly experienced analyst.

An interrupted move, whose completion would restore sanity to the position, as in Figure A4.

A last move by a piece no longer on the board, which can only be a Pawn that promoted. That possibility is especially easy to overlook if the promotion was the less familiar reverse (color-changing) promotion.

A board from which a piece (or pieces) fell without replacement. Usually you will be informed at least that something has fallen, but not always, and not usually with abandoned games. You should search for positive proof that something has fallen. A missing King of one color is usually accepted as positive proof. In some instances, particularly in homochromatic play, it is provable that a missing piece neither moved nor could have been captured; with no other alternatives, that piece *has* fallen from the board.

But don't be too quick to judge a position impossible in the absence of a fallen piece (as Watson has been known to do), until you have ruled out the earlier-listed alternatives.

About the Author

William P. Murden matriculated at the College of William and Mary in 1941, had his education interrupted by World War II service (Central Africa and the Caribbean), and acquired two engineering degrees at Virginia Tech. He performed research in analytical mechanics in Texas, and military operations analysis in Virginia, California, Hawaii, and Washington, DC. He joined McDonnell Aircraft Company, in St. Louis, Missouri, as Manager, Operations Analysis, and built that department into what government defense analysts described as "the best in the industry." He contributed significantly to the conceptual design of many fighter aircraft, and most especially to the F-15 Eagle. He retired in 1985 as Director, Engineering Technology for McDonnell, a division of McDonnell Douglas.

Murden had been intrigued by the discipline of retrograde chess analysis since the 1979 publication of Raymond Smullyan's first book in that field. He generated many problems in retrograde analysis before retiring, but pursued an across-the-board understanding in his years of retirement. Retirement also provided time to explore computer graphics, developing programs for high-fidelity representation of the sometimes-strange boards seen in retrograde analysis problems.

Mr. Murden lives in a St. Louis suburb, with his wife of more than fifty years.

Bill Murden passed away June 8, 2008.

– Fred Stahl